bridge**course**

Physics

A Self Study Course after **Class 10** Board Exams

to Bridge the gap between Class X & XI &
to Build Foundation for Engineering/Medical Entrances

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ARIHANT PRAKASHAN (SERIES), MEERUT

PREFACE

Being involved in preparing the students for JEE for the last many years, I personally felt that when students enter in class XIth they feel a lot of difficulty in maintaining their spirit and enthusiasm of High School. The reason being that there is a vast gap between the levels of the two classes. Almost everything taught to them in 11th class is either new to them or has been taught to them in a different manner. Moreover, the vast syllabus and the big fat books frightened them, as a result of which even the very bright students of High school tend to loose their confidence in the initial stage of their intermediate. There are many good books available in the market which serves the purpose for curriculum of classes 10th and 11th but there is no book available in the market which can bridge the gap of these two classes and help the High school students to become successful student in their intermediate also.

The book in your hand is written with the motto-"Maintain the pace and spirit of our young minds for Towards JEE. The idea behind the introduction of this product is of Mr. Deepesh Jain (Director, Arihant Prakashan). With this motto in mind I completely went through the syllabus of class 8th, 9th, 10th and talked to various pioneer teachers of High School and those involving in IIT-foundation program about their curriculum, level of students and teaching methodologies. With this small research I started to develop the content of Towards JEE.

The major problem what I faced is that up to High school Algebra based physics has been taught and in 11th class calculus based, moreover the students came to know the things about calculus in the 2nd year of their intermediate, so to develop the content of this book I can't use calculus and it is also a proven fact that without calculus it is very difficult to explain certain concepts in physics, still I tried to explain those concepts in a different way by taking certain examples from daily life or I skipped those concepts. To understand the content of this book you do not need any previous concepts of physics. The book is complete in all sense. The only thing which I feel, is required, is the basic receptive mind, curiosity to learn, habit of understand the things from core and little mathematical skills.

This book can be useful for students of 8th, 9th and 10th class or for 10th to 11th moving students or by those who are having fear in physics or not having any interest in physics. This book will try to keep you in race for JEE.

I have put in my best sincere efforts to develop the content of this book and tried my level best to present the theory in a very simple and interesting way. Even though I have tried my best to provide the most accurate matter, the mistakes/errors might have gone unnoticed. If you find some, then please bring it to my notice. It would be highly appreciated.

I feel that this book will serve the motto ["In maintaining the pace and spirit of our young minds'] with which it has been written. Still to serve the student community in a better way-feedback and suggestions of students and teachers are invited.

Vikas Jain

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CONTENTS

TO THE STUDENTS

To get the maximum benefit from any book it is very important to know its organizational structure and brief overview of every chapter. So, here we are providing you the same.

This book is divided into 17 chapters and is organized in the following way

FIRST TWO CHAPTERS ARE

1. **An Introduction-Realm of Physics** In this chapter a brief introduction of physics is provided in very interesting manner. After going through this chapter you will be able to realize that how science/physics is developed and what is your task for next few years.
2. **Units and Measurement** This chapter deals with some of the basic thing about measurement, units etc. this chapter you go through once and even if you won't understand it completely don't get panic because it includes many terms which you will know after going through later chapters. But it is strongly recommended that once you go through this chapter before proceeding further.
   - **Chapters 3 to 9 Constitute the Mechanics Part of the Physics** In these chapters (exclude 10th chapter) the kinematics and dynamics of one dimensional and two dimensional motions are given and remember mechanics is the backbone of physics. This branch constitutes approximately 30% of the entire physics syllabus for JEE. In 10th chapter basic information about circular motion is given, here in this book we have not discussed about rotational motion in detail and simple harmonic motion we have not touched at all. These I am leaving for regular intermediate classes as they require some basic pre-requisite and use of calculus. It is strongly recommended that these 7 chapters you study in sequence, this will give you a greater insight into the subject.
   - **Chapter 10 is Gravitation** This chapter deals with the concept of gravitation, although in earlier chapters you will encounter gravitational force due to earth frequently but this chapter is mainly devoted to gravitational effects. Without breaking the continuity of the subject you can skip this chapter but this is better to learn this to have more insight into the gravity force due to earth and about gravitational potential energy.
   - **Chapters 11, 12 and 17** Constitute three different and somewhat independent branches of physics Waves, Thermal physics and Optics respectively. From JEE point of view these topics are important and rewarding, but these are very conceptual also. So, while going through these chapters pay more attention and try to understand very small points also.

- **Chapters 13 to 16 Constitute the Electrodynamics Part of the Physics** These chapters comprise another major portion of the physics. In these chapters you will learn about electrical and magnetic phenomena. The 17th chapter is electromagnetic induction which is not explained in detail, moreover two main electrical circuit components capacitors and inductors are not included in this book.
- **Basic Mathematics for Physics** In this chapter I have tried to brush up you're the basic concepts of maths and some new to you topic of maths like vectors and calculus. Remember without maths; wish to learn physics is like having the motorbike without fuel.

This chapter is one of the most important chapter but the fact is that you will not be able to grasp the content of this chapter in one go. So, I will suggest that once you go through this chapter shortly and after that refer this chapter as and when needed.

Most of the chapters in the book are provided with detailed theory and concept building illustrations fully solved. It is advised that after going through the theory try to solve the illustrations first by yourself and then see the solution and then give a thought process to the question and solution to understand the concept behind the question. It is always better to reason the answer after getting the answer whether it is right or wrong. In the end of the chapter **Proficiency in Concept** problems are given with full detailed solution. It is better to solve these questions first by yourself and then check your solution.

In last every chapter is having exercises containing questions for practice, exercise-1 contains the discussion questions, numerical type, fill in the blanks and true false it is strongly recommended that you follow the same sequencing. Solutions are provided for typical ones, you can refer them but only after trying your best. Exercise-2 contains different pattern questions similar to asked in JEE, like objective questions having single correct option, more than one correct option, comprehension linked passage followed by objective questions, matrix type match the following.

Acknowledgements

It is very difficult for an individual to complete a new and big work. Development of this book is also not an exception. As the content development of this book was going on, many people have put their suggestions directly or indirectly. I am thankful to all of them. Personally I thank Mr. Chandrawardhan Sir (Warangal) for his valuable suggestions on sequencing of chapters, Mr. Vijay Sir (Warangal) for helping me in making decision of level of contents, Mr. Nitin Jain (Jaipur) for contributing Mathematical tools chapter, mine students G Vinod Reddy (IIT Bombay), Krishna Sawanth (IIT Bombay), N Varun (IIT Chennai), Praveen Reddy (IIT Bombay), Faheem Siddiquee (NIT Warangal) and many more who have helped me a lot in giving the ideas to present the theory in a simple and interesting manner.

My sincere thanks to Mr. Deepesh Jain and the entire team of Arihant Prakashan for bringing out this book so beautifully. I would be failing in my duty, if I forget my guru, my parents, grandmother, sisters and brother at this moment of my life, I owe all my success to them, I express my sincere gratitude to my maths teacher Mr. Jeetendra Sharma (Jitu Sir) and all my family members who taught me the values of life and who are behind the success of mine.

In one line I can't express my thanks to my wife for giving tremendous support to me, but thanks to her for everything and being with me.

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Chapter 1

The Realm of Physics

The First Steps' Learning

- What is Science?
- What is Physics ?
- Why to Study Physics and How to Study Physics?
- Limitations & Assumptions?
- What to Study in Physics?
- Physics & Mathematics

As you begin your journey of revealing the concepts and laws of physics you should know exactly what lies ahead of you. Physics, the most fundamental of all the physical sciences, is concerned with the basic principles behind all physical phenomena in our universe. From chemists to micro-biologists, scientists of all disciplines make use of the ideas of physics. It is the foundational structure on which engineering, technology and the other sciences—astronomy, chemistry, geology, biology etc are placed. The beauty of physics lies in the simplicity of its basic concepts, fundamental theories and their vast and varied applications.

The blue sky, motion of planets around the sun, appearance of rainbow, pleasent waterfalls, disastrous earth-quakes, the beautiful night sky with bright celestial objects are few of the manifestations of the endless lists of natural phenomena. Directly or indirectly the explanation of these facts (natural happenings) belongs to the Realm of Physics. These are far reaching facts and questions, and in this Physics book we are to only make a modest beginning of answering/ understanding them. In fact, our goal is to give you a background of the fundamentals of physics, so that your study of physics later will be more effective, and you would be able to understand all these facts and laws by yourself. In this chapter, we shall delve into certain ideas about the science, the physics, and the 'realm' of physics.

What is Science ?

The word science originates from the latin word "*Scientia*" which means "*to know*". Now, the question that may arise to your mind is —"To know", but about what ? In somewhat more informative manner, science means "To know—to understand a natural phenomenon". But now the question arises, how to understand the various natural phenomena ? Are there some laws which describe these natural phenomena ? Are these laws (if any) stated somewhere ? Who derived these laws ? Answers to all these questions lies in the basis of science, which is—"*The test of all knowledge is experiment*". Experiment is the sole judge of any scientific truth. And the more interesting thing is that, an experiment itself is the source of knowledge, and from the results of an experiment, some conclusions have been drawn. These conclusion help us in producing the laws. Then again experiments are done to test the laws so stated, to have an idea about their applicability in different situations.

Broadly speaking, science is a systematic attempt to understand all natural phenomena by performing experiments, observation of results thus found and then by using the knowledge so gained, logical thinking and imagination, scientists try to explain the related natural phenomena. The conclusions made by the scientists may be in the form of statements, laws, limitation on applicability etc and we, the curious few are going to learn the works done by these scientists.

'Values' of Science

To pursue an effective study of science you need to cultivate the following sets of values, earnestly:

- Being curious about things and events around you.
- To have the courage to ask the question against established beliefs and practices.
- Always asking 'What', 'Why' and 'How', and try to find the exact answers by critical observation, experimentation, consultation, discussion and reason.
- To record honestly your observations and experimental results in your laboratory or outside it.
- To repeat experiments carefully and systematically if required but without manipulating your results under any circumstances.
- Being only guided by facts, reasons and logic. Not to be biased in one way or other.
- To aspire to make new discoveries and inventions by sustained and dedicated work.

What is Physics ?

Physics is the branch of science, which deals with the basic laws of nature related to energy and matter, and their applicability to different physical and natural phenomena. Physics is an experimental science, physicists observe the phenomena of nature and try to deduce certain theories and principles which explain these phenomena. The theory and principles developed by physicists, when well established and of universal acceptance are termed as physical laws or principles of physics. If one wants to be a physicist, he/she has to first go through the works of great physicists like Galileo, Newton, Maxwell, Faraday, Einstein as well as others in depth and then look for some happening in nature which is still unexplored or needs refinement of the existing laws and/or theories to explain it fully. But for you people—the students, the first step is to understand the existing theories first at very conceptual level and then try different varieties of questions to grasp the subject fully, and only after that you could try to discover the things.

In words of Great physicist and **Nobel laurate, Richard P. Feynman,** "the world"—is something like a great chess game, being played by the gods, and we are observers of the game. Let us discuss this analogy of the great physicists in detail.

Take a game of chess, which you don't know how to play, neither you are allowed to play, the only thing is that you are allowed to watch the game being played by other players. Just imagine this situation and think what happens. It is quiet obvious, that after watching the game for a long time, you would be able to understand a few rules of the game, however, they may be incorrect or incomplete in same or other way. For example, you are watching the movement of elephant continuously and you deduce "that it would be always moving straight and can cross any number of squares but always remain on the corner column", all of a sudden the player changes the course of motion of elephant along the row and then you have to make your rule more complete by adding "that elephant always moves along a straight line either along a row or column and can cross any number of squares". Now, let us consider that after watching the game for a long time, you are able to deduce many of the rules or almost all, *ie*, you become aware of all the rules of game. Can you play the game effectively after knowing all the rules ? The answer is no because you may know all the rules, but still you don't have that much expertise which let you understand all the moves and that's why you may not be able to make the best move in the given situation. *And, exactly in the same way the physics works!*

Now make any analogy between chess game and physics.

Analogy between the Game of Chess and the subject matter of Physics

Play Field	Chess Board	Nature/Universe
Master	The Player	The God (If you are not an atheist!)
Centre Person	You	The Physicist
Happenings	Moves	Natural phenomena
Ways	Rules	Laws, principles and concepts of physics
Further Ways	Extension or Modification of Rules	Change or modification in existing laws and theories so that a new situation is explained fully, and to get the limitations of existing theories and principles
Third Party	Neutral spectators of the game	The common man

In short we can say that physics involves keen observation of natural and physical phenomena. With the help of the knowledge, logical thinking and imagination, we know the answer of (why? and how?) related to a concerned phenomenon. Once the natural phenomenon has been understood, some fundamental theory would be developed and then forever, we make use of this theory in various applications. Experiments would be carried out to check the validity of this theory in different domains or to find some degree of incorrectness or incompleteness, in the existing theory.

Why to Study Physics and How to Study Physics ?

You might ask, why to study physics ? What is the need of physics in our daily life ? There are numerous reasons, one can count in support of studying physics. Here we will find a few.

First one, we can't imagine our life without physics. Look around you—the book in your hand, you are able to read (see) only because of physics. The notebook in which you are making notes, is useful only because of physics, if there would not be any friction between paper and pen tip, then you would not be able to write. If you are able to grasp the contents of a lecture or are discussing a question with your friend then you are able to hear your friend because of sound waves. From a few of the applications of physics in our daily life mentioned above, can you imagine life without physics!

The second one, almost all technical and engineering disciplines are based upon physics, whether it is mechanical, electrical, electronics or telecommunication.

The third one, physics is an adventure. When you study physics, you will find it challenging and interesting, sometimes frustrating and painful, but immensely rewarding and satisfying.

And for you—the students, the most important reason to study physics is—that in almost of all of the basic courses in science, physics is compulsory. Moreover, any discipline of life includes physics directly or indirectly. So, without having basic understanding of physics and its laws it is difficult to get success in any field.

Now, you can ask, if physics is so important for me, then how should I study physics ? As such we hope that you understood how the discipline of physics has developed and your aim is to understand and learn the concepts, physical laws and fundamental theory of physics developed by our eminent physicists. So, what is best way to learn physics ? Simply, first learn all the physical laws and then see how they fit (work) in different physical situations. But we cannot do it in this way for two reasons, first we do not yet know all the basic laws, the search is still going on and it will continue for forever to find the unknown ones. And second, the correct statements of the laws of physics involve some very unfamiliar ideas which require advance mathematics for their description *ie*, one should require considerable amount of fundamental knowledge even to understand what the various terms mean in physical laws. Hence, the best way to start physics is to do it bit by bit. What we mean by a bit is that first we consider a simplified model of the actual situation and when we have learnt the simplified model and other requirements to understand the exact situation then we will go for the learning of the actual and exact situation. For example, if one has to analyse the motion of a ball under gravity, then it is advantageous to first analyse the motion of the ball neglecting air friction etc because under this assumption we can treat the ball as a point mass and can neglect the rotational and spinning aspects of motion, and when we have become well acquainted with rotational motion, we can look for the exact picture. The same methodology we are going to

adopt in this book, we will start from very beginning at very basic levels and slowly will go ahead to mix the concepts to understand exact situation or the more approximate situation.

As such, we can say that each bit, or part of the whole of nature is always an approximation to the complete truth or the complete truth as we know it. In fact, every thing that we know is an approximation of some kind because we do not know all the laws yet.

Limitations and Assumptions

As we are going to explore the concepts of physics, it is very important to understand the meaning behind the word "assumption", which we use very frequently.

What to Study in Physics ?

Initially, the physics were roughly divided into mechanics, heat, gravitation, electricity, magnetism, quantum mechanics, optics etc. However, the aim of physicists is always to see the nature as a whole *ie*, to interlink various branches of physics. This is another major challenge for physicists to combine various branches, in addition to find the laws behind experiments. The process of amalgamation (interlinking) of various process is continuous one because as the time passes newer things are being found.

Broadly speaking, physics can be divided into two : *classical physics* and *relativistic physics*. The relativistic physics (Major contributor of the relativistic physics is Einstein) is the correct one or complete, in the sense what we know till now. While classical physics (Mainly given by Galileo, Newton and others) fits only in some situations.

Relativistic physics is more correct, interesting and contains unfamiliar ideas and is difficult to understand, while classical physics is approximate and doesn't include difficult ideas but still the interesting one. Now, the question arises what should we study first—Relativistic physics which is more accurate or the classical physics which is only an approximate one. The first you are going to learn the classical physics because knowledge of classical physics is the first step to learn the relativistic physics.

In this book of physics we will mainly emphasize on classical physics and won't talk about relativistic physics. We will study in the sequence as

1. Mechanics
2. Heat and Thermodynamics
3. Properties of Matter
4. Wave Motion
5. Optics
6. Electromagnetism
7. Modern Physics

Physics and Mathematics

Physics involves the laws related to nature, which has been developed after a series of experiments have been conducted. These laws can be expressed in statement (words), but physics always involves some physical quantities. So, it is always advisable and beneficial to express physical laws in a mathematical form. Can you imagine your life without words, language, expressions etc, something like same would happen in physics if mathematics won't be there. In short, we can say mathematics is the language of physics.

Chapter 2

Units and Measurements

The First Steps' Learning

- Physical Quantities
- Units
- Fundamental and Derived Quantities
- Systems of Units
- Units of Derived Quantities
- Interconversion of Units
- Dimensional Analysis
- Scalar and Vector Physical Quantities

In this chapter, we are to explore the methods to understand the concepts and laws related to physical quantities. These are units, measurements, classification etc. These methods which you would learn are fundamental to the whole of the physics. But, unfortunately you would be asked to learn these methods with respect to the conceptual terms spanning over the whole physics course at this level. Then, why this chapter at the beginning of the book ? The answer lies in the fact that this chapter deals with the methods of measurements, scaling and classification, and these methods are required for concept-formation in throughout the latter chapters. That is why, this chapter is traditionally placed at the beginning of all physics books at all levels. Be clear that here you have to master only the methods-and not the concepts which you will be doing in all the chapters that follow.

Physical Quantities

Physics, the most fundamental of the physical sciences is by its nature an experimental science. Physics as a science bases itself on experimental observations and mathematical analyses, to provide a quantitative understanding of all natural phenomena. From the experiments conducted, analyses and observations of experimental results, the physicists derive the theories and these theories are in forms of laws of nature, statements etc. While dealing with physical laws, very often we have to deal with the situations of measurement and comparison, like which is heavier—an elephant or an ant? Who is taller, you or your friend ? When can you ride your bicycle faster—in the presence of wind or in absence of wind ? How far is Hyderabad from Delhi by road, rail or air ?

These questions are very easy to understand, but what actually the key words 'heavy, tall, fast, farther, near etc" signify in above situations ? We know that an elephant is heavier than an ant but what if we ask from ourself why it is so ? Then, we can satisfy ourselves by saying that "mass" of an elephant is greater than mass of an ant and hence, the elephant is heavier than ant. Now, the word "mass" is playing the important role—To tell that which object is heavier we must know which is having more mass. In the same way to identify who is taller among two friends, we require height, thus we can have an endless list of illustrations. These terms which we use to measure and/or compare some physical properties of any object are termed as **physical quantities.** Some of the most common physical quantities of which you are aware are—mass, time, distance, speed, acceleration, velocity, force, work, energy, power etc.

Every physical law makes use of some physical quantity. For example, the law of conservation of energy—the most fundamental one, according to which "for an **isolated system** energy can neither be created nor be destroyed." Here, in this law the description about physical quantity "energy" is given under some conditions.

We must be clear that all our conversation in physics is going to take place on the definition of physical quantities expressed in forms of various laws and principles concerning different natural phenomena.

Units

We have seen in the previous section that physical quantities are needed to describe the laws of nature. These physical quantities **involve measurement**, and to measure physical quantities we need some **standards.** Let us try to understand the meaning of word standard.

If we ask you that how much heavy is your school bag, then how will you answer this ? Assume that you don't possess any weighing

instrument. Then what you can do is to find out how many copies of your physics text book are required so that when these books and your school bag are placed on the two pans of a weighing balance (beam balance), and see that balanced arms are horizontal. Then you count the number of books placed on the pan, let it be 15, then what you will say that—my school bag is 15 times heavier than my books. Someday when some classes are free your bag may be lightier and you can say that today my bag is only 5 times heavier than my books. Now you can ask why only books we have been considered ? Can't we take some other books or even some other objects say chalk piece, pencil box, writing pad, pair of dresses etc, as the standard. Surely you can take anything as a standard because we all are living in a democracy and we are having the freedom to do what we like. Imagine the situation, when we all are free to measure the mass of your bag in our own way. Being a teacher, I can take chalk piece as the standard and after comparing mass of your bag with chalk pieces, I will say that your bag is 1500 times heavier than my chalk pieces or I will say that mass of your bag is 1500 times of my standard. From your point of view the mass of your bag is 15 times of your standard. Some of your friends can take their pencil boxes as the standard and can measure the mass of your bag as 23 times of their chosen standard. In this way, different people can estimate the mass of your bag in terms of different standards. So, what is the mass of your school bag ? You will reply 15, I replies 1500 and your friend replies 23. Although we all are correct from our own point of view but for others, the situation becomes confusing and leads to contradiction.

The only way to avoid this contradiction is that we all should agree to use one standard. Now the question arises, whose standard would be accepted and why ? The first thing is that it should have universal acceptance, everybody should agree to use it as a standard. One of the three intergovernmental treaty organizations is **Conference General Despoindset Measures** General conference on weights and measures, **(CGPM)** which works under the terms of metre conversion, 1875. It is entrusted with the work of maintaining the SI system of units. The CGPM currently meets at Sevres near Paris in France every four years.

The standard of the physical quantities decided by CGPM is termed as unit of the concerned physical quantity. For example, the unit of mass is kg (kilogram). Kilogram is the standard of mass and 1 kg is the mass of a cylinder made of platinum-iradium alloy kept at International Bureau of Weights and Measures. Now, if you want to find out the mass of your school bag then you will try to figure out how many kilograms your bag is, let us say it is 8, then the mass of your bag is 8 kg.

While choosing any standard unit for any physical quantity, following points should be kept in mind :

(a) The standard unit should be invariable *ie,* shape, size and other properties of the standard unit must not change with time, place and other surrounding conditions.
(b) The standard unit should be easily available.
(c) The standard unit should be easily reproducible, at any other place in the world.

Fundamental and Derived Quantities

As we proceed to study physics, we will encounter various physical quantities, and all physical quantities need to be measured, and for the measurement of a physical quantity we need a standard unit. We would have thousands of physical quantities, then we require thousands of measuring

standard units which is somewhat difficult to handle. If we would be able to divide all the physical quantities into two groups in such a way that in one group those physical quantities are there which are independent of each other and the quantities of the second group can be expressed in form of one or more quantities of the first group, then we need the standard units only for physical quantities of the first group. For example, if we have the unit of length, then we can find the unit of area and volume in terms of unit of length.

All the physical quantities can be classified into two categories—

(a) **Fundamental Quantities** Those quantities which are independent of each other are termed as fundamental quantities. The units defined for fundamental quantities are termed as fundamental units. Fundamental quantities are also called the base quantities.

(b) **Derived Quantities** All other quantities which can be expressed in the form of fundamental quantities are termed as derived quantities. The units defined for derived quantities are termed as derived units.

It has been found that the number of fundamental quantities is only seven, and all other quantities may be derived from these quantities by multiplication and division.

The seven fundamental quantities are :

1. Length
2. Mass
3. Time
4. Electric Current
5. Thermodynamic Temperature
6. The Amount of Substance
7. Luminous Intensity

C-BIs

Concept Building Illustrations

Illustration | 1 *Derive the relation for velocity in terms of fundamental quantities length and time ?*

Solution We know velocity $= \dfrac{\text{displacement}}{\text{time}}$

Displacement is measurement of length, so

$$\text{velocity} = \frac{\text{length}}{\text{time}}$$

Systems of Units

Several systems of units are in use over the world and in each system the units of fundamental quantities are defined in a different way. The four commonly used systems of units are mentioned below along with the units used for fundamental quantities.

(a) **MKS System** In this system of units, for length the unit is metre (m), for mass the unit is kilogram (kg) for time the unit is second (s).

By taking the first alphabets of the units of length, mass and time, this system of units is termed as MKS.

The unit of velocity in this system of units is ms^{-1}.

(b) **CGS system** In this system of units, for length the unit is centimetre (cm), for mass the unit is gram (g), for time the unit is second (s).

The unit of velocity in this system of units is cms^{-1}.

(c) **FPS system** In this system of units, for length the unit is foot (ft), for mass the unit is pound (P)*, for time the unit is second (s).

The unit of velocity in this system of units is fts^{-1}.

(d) **System international units (International system of units, SI system)** This is the most common and most widely used system of units. In this system of units, for length the unit is metre (m), for mass the unit is kilogram (kg), for time the unit is second (s). *ie,* same as that in MKS system of units. In this system of units the unit of velocity is ms^{-1}.

The basic difference between MKS system of units and SI system is that in SI system of units, there is a provision to assign special names to units of derived quantities while it is not so in MKS system of units.

For example, in SI system, the unit of force is newton (N) in honour of the name of Sir Issac Newton, while in MKS system the unit of force is $kg\text{-}ms^{-2}$.

In this book, mostly we shall use SI system of units.

In SI system, not only for length, mass and time but also for other fundamental quantities unit has been defined which we mention below :

S. No.	Quantity	Name of the unit along with the symbol
1.	Length	Metre (m)
2.	Mass	Kilogram (kg)
3.	Time	Second (s)
4.	Electric Current	Ampere (A)
5.	Thermodynamic Temperature	Kelvin (K)
6.	Amount of Substance	Mole (mol)
7.	Luminous Intensity	Candela (cd)

Units of Derived Quantities

We know that, derived quantities can be expressed in multiplication and/or division form of base quantities. Once, we know the relation of derived quantities, we can write down its unit.

For example, velocity $= \dfrac{\text{displacement}}{\text{time}}$

$= \dfrac{\text{length}}{\text{time}}$

As unit of length is metre and that of time is second, and unit of velocity is unit of length divided by unit of time, so unit of velocity is $\dfrac{\text{metre}}{\text{second}}$ *ie,* m/s or ms^{-1}, read as metre per second.

C-BIs

Concept Building Illustrations

Illustration | 2 *Find the units of acceleration, force, momentum.*

Solution (a) Acceleration $= \dfrac{\text{velocity}}{\text{time}}$

Unit of acceleration $= \dfrac{\text{unit of velocity}}{\text{unit of time}}$

$= \dfrac{m/s}{s} = m/s^2$ or ms^{-2}

*In actual pound is the unit of weight.

(b) Force = mass × acceleration

$= kg \times (ms^{-2}) = kg\text{-}ms^{-2}$ or N (newton)

(c) Momentum (p) = mass × velocity

$= kg \times (ms^{-1})$

$= kg\text{-}ms^{-1}$

Illustration | 3 *Find the units of acceleration due to gravity, kinetic energy and potential energy.*

Solution (a) Acceleration due to gravity is the acceleration of a body caused due to gravity force, so it would have same unit as acceleration *ie*, ms^{-2}.

(b) Kinetic energy $(K) = \dfrac{mv^2}{2}$

$= \dfrac{\text{mass} \times (\text{speed})^2}{2}$

$= kg \times (m/s)^2$

$= kg\text{-}m^2s^{-2}$

[Remember that constants are not having any units, like 2 in above equation.]

In SI system of units, the unit of kinetic energy is joule and $1\ J = 1\ (kg\text{-}m^2s^{-2})$

(c) Gravitational potential energy (U)

$= mgh$

= mass × acceleration due to gravity × height

$= kg \times (m/s^2) \times (m)$

$= kg\text{-}m^2s^{-2}$

It is clear from (b) and (c) that units of KE and PE both are the same and it is quiet obvious also because both are different forms of energy.

Illustration | 4 *Determine the units of work and power.*

Solution (a) Work, W = force × displacement

$= (kg\text{-}m/s^2) \times (m)$

$= kg\text{-}m^2s^{-2}$

SI unit of work is Joule.

(b) Power $= \dfrac{\text{work}}{\text{time}}$

$= \dfrac{kg\text{-}m^2s^{-2}}{s}$

$= kg\text{-}m^2s^{-3}$

SI unit of power is Watt (W).

$1\ \text{Watt} = 1\ kg\text{-}m^2s^{-3}$

Interconversion of Units

While substituting the values of different physical quantities in the mathematical expression of physical law, it has to be kept in mind that all values (of different physical quantities) should be in same system of units. For example, if a body of mass 3 kg is moving with a speed of 50 cms^{-1}, then what would be its KE? In this case, mass is given in kg (SI system) while speed is given in cms^{-1} (CGS system). To compute KE either both should be converted to SI or CGS.

In SI system,

$$K = \frac{mv^2}{2}$$

$$= \frac{3\ kg \times (0.5\ ms^{-1})^2}{2} \quad [\because 1\ ms^{-1} = 100\ cms^{-1}]$$

$$= \frac{3}{8}\ J$$

In CGS system,

$$K = \frac{mv^2}{2} = \frac{(3 \times 1000\ g) \times (50\ cms^{-1})^2}{2} \quad [\because 1\ kg = 1000\ g]$$

$$= 37.5 \times 10^5\ g\text{-}cm^2s^{-2}$$

$$= 37.5 \times 10^5\ erg$$

In CGS system, the unit of energy is also called erg.

C-BIs

Concept Building Illustrations

Illustration | 5 *Convert 1 kmh^{-1} into ms^{-1}.*

Solution $1\ \text{km/h}^{-1} = \dfrac{\mathbf{1\ km}}{\mathbf{1\ h}}$

$$= \frac{(10^3)\ \text{m}}{(3600)\ \text{s}} = \frac{5}{18}\ \text{ms}^{-1}$$

So, $1\ \text{kmh}^{-1} = \dfrac{5}{18}\ \text{ms}^{-1}$

Illustration | 6 *Convert 100 W into CGS system of units.*

Solution We know $1\ \text{W} = 1\ \dfrac{\text{kg-m}^2}{\text{s}^3}$

So, $100\ \text{W} = 100 \times \dfrac{(1\ \text{kg}) \times (1\ \text{m})^2}{(1\ \text{s})^3}$

$$= 100 \times \frac{(10^3\ \text{g}) \times (100\ \text{cm})^2}{(1\ \text{s})^3}$$

$$= 100 \times 10^7\ \frac{\text{g-cm}^2}{\text{s}^3} = 10^9\ (\text{g-cm}^2\text{s}^{-3})$$

Illustration | 7 *Convert 36 kmh^{-1} into ms^{-1}*

.

Solution We know from illustration 5, that $1\ \text{kmh}^{-1} = \dfrac{5}{18}\ \text{ms}^{-1}$

So, $36\ \text{kmh}^{-1} = 36 \times 1\ \text{kmh}^{-1}$

$$= 36 \times \frac{5}{18}\ \text{ms}^{-1} = 10\ \text{ms}^{-1}$$

Illustration | 8 *Convert 13.6 gcm^{-3} into MKS system of units.*

Solution $13.6\ \text{gcm}^{-3} = 13.6 \times \dfrac{1\ \text{g}}{(1\ \text{cm})^3}$

$$= 13.6 \times \frac{(10^{-3}\ \text{kg})}{(10^{-2}\ \text{m})^3}$$

$$= 13.6 \times 10^{-3} \times 10^6 = 13600\ \text{kgm}^{-3}$$

Illustration | 9 *Convert 5.4 N into CGS unit.*

Solution $1\ \text{N} = 1\ \text{kg-ms}^{-2}$

So, $5.4\ \text{N} = 5.4\ \dfrac{\text{kg-m}}{\text{s}^2}$

$$= 5.4 \times \frac{(1\ \text{kg}) \times (1\ \text{m})}{(1\ \text{s})^2}$$

$$= 5.4 \times \frac{(10^3\ \text{g}) \times (10^2\ \text{cm})}{(1\ \text{s})^2}$$

$$= 5.4 \times 10^5\ \text{g-cms}^{-2}$$

Dimensional Analysis

We have seen that all physical quantities could be expressed in terms of seven fundamental quantities. These seven fundamental quantities are termed as seven dimensions of the physical world. Any physical quantity can be written as a product of fundamental quantities. For example, take density. We know that density is mass divided by volume. Volume is cube of length. So,

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$= \frac{\text{mass}}{(\text{length})^3}$$

$$= \text{mass} \times (\text{length})^{-3}$$

The powers of the fundamental quantities that come into the expression of a physical quantity, is termed as dimensions of the quantity in corresponding fundamental quantity. Like in above expression for density, it has 1 dimension in mass and –3 dimensions in length. The dimensions in all other fundamental quantities are zero.

If we use symbols M, L, T, A, K, mol and cd for fundamental quantities mass, length, time, current, temperature, amount of substance and luminous intensity respectively, then above relation for density can be written as

$$\text{Density} = ML^{-3}$$

If we enclose above expression in square bracket then it is termed as dimensional formula. Thus dimensional formula of density is $[ML^{-3}]$.

With the help of dimensional formula we can write the unit of a physical quantity directly in any system of units. For example, a physical quantity is having the dimensional formula $[MLT^{-1}]$, then the unit of this physical quantity in MKS system of units is kg-ms^{-1}.

C-BIs

Concept Building Illustrations

Illustration | 10 *Determine the dimensional formula of force.*

Solution Force = mass × acceleration

$$= \text{mass} \times \frac{\text{velocity}}{\text{time}}$$

$$= \text{mass} \times \frac{\text{length / time}}{\text{time}}$$

$$= \text{mass} \times \frac{\text{length}}{\text{time}^2}$$

$$[\text{Force}] = [MLT^{-2}]$$

Illustration | 11 *Determine the dimensional formula of KE.*

Solution $KE = \frac{mv^2}{2}$

$$= \frac{\text{mass} \times (\text{velocity})^2}{2}$$

$$= \text{mass} \times \frac{(\text{length / time})^2}{2}$$

2 is a constant and is having no dimensions.

So, $[KE] = [ML^2T^{-2}]$.

Illustration | 12 *Determine the dimensional formula of universal gravity constant G.*

Solution We know $F = G\frac{m_1m_2}{r^2}$

$$\Rightarrow \quad G = \frac{Fr^2}{m_1m_2}$$

$$= \frac{\text{force} \times (\text{distance})^2}{\text{mass} \times \text{mass}}$$

$$[G] = \frac{[MLT^{-2}]\,[L^2]}{[M]\,[M]} = [M^{-1}L^3T^{-2}]$$

Illustration | 13 *Determine the dimensional formula for*

(a) ut *(b) at^2*

(c) $\frac{at^2}{2}$

where u is velocity, a is acceleration and t is time.

Solution (a) ut = velocity × time

$$[ut] = [LT^{-1}] \times [T] = [L]$$

(b) at^2 = acceleration × (time)^2

$$= \frac{\text{velocity}}{\text{time}} \times (\text{time})^2$$

$$[at^2] = \frac{[LT^{-1}]}{[T]} \times [T^2] = [L]$$

(c) $\frac{at^2}{2} = \frac{1}{2} \times \text{acceleration} \times (\text{time})^2$

$$= \frac{1}{2} \times \frac{\text{velocity}}{\text{time}} \times (\text{time})^2$$

$$\left[\frac{at^2}{2}\right] = \frac{[LT^{-1}]\,[T^2]}{[T]} \qquad [2 \text{ is a constant}]$$

$$= [L]$$

Scalar and Vector Physical Quantities

If someone asks you the location of your school, then to explain the location of school clearly you may answer : It is approximately 4 km from here towards the Sports Stadium. Here you are mentioning two things—One that how much far away your school is and second is that in which direction it is located. Let us consider another question—At what time you will leave for school ? Then you may answer—At 7:30 am. Here you have to tell only time.

In physics we are going to deal with two types of physical quantities one which doesn't require any direction and the other which requires as direction for their complete specification. On this basis all the physical quantities are classified as :

(a) Scalar quantities, and

(b) Vector quantities

(a) **Scalar Physical Quantities** Those physical quantities which are completely described by magnitude only. For example, mass of a body is 3 kg, the duration of the lecture is 40 min, a block is moving with a speed of 5 ms^{-1}. Here in all these examples the physical quantity is completely specified by a numerical value along with unit which together is termed as magnitude. For these physical quantities we don't need any direction like if we say mass of a body is 3 kg towards east, then it is meaningless.

Magnitude of a physical quantity consists of two parts—one a numerical value, and the other is a unit. The actual value of the magnitude of a physical quantity remains same in all systems of unit. For example, If 3 kg is the mass of body, then 3 kg is magnitude of the physical quantity mass. 3 is numerical value of magnitude of physical quantity mass and kg is unit of magnitude of physical quantity mass.

Thus magnitude = numerical value (n) $\times$ unit (u)

$$= nu = \text{constant}$$

For above example—

$$3 \text{ kg} = (x) \text{ g}$$

$$\Rightarrow \quad 3 \times 10^3 \text{ g} = x \text{ g}$$

$$\Rightarrow \quad x = 3000$$

Thus, $\quad 3 \text{ kg} = 3000 \text{ g}$

Examples of the scalar quantities are mass, speed, distance, time, power, work etc.

To deal with scalar physical quantities we will make use of rules of ordinary algebra.

(b) **Vector Physical Quantities** Those quantities which have both magnitude and direction are termed as vector physical quantities. Velocity, displacement, acceleration, force etc are some of the vector physical quantities. We have to use vector algebra while dealing with vector physical quantities.

While dealing with physical quantities whether scalar or vector it has to be kept in mind that only same physical quantities can be added or substracted. For example, if we say what is 2 kg + 2 m equal to, then surely you will say that question is meaningless. What we want to say here is that displacement can be added/substracted to displacement only not with velocity or any other physical quantity.

Towards Proficiency Problems

A. Subjective Discussions

1. How many experiments do you think are needed to be conducted critically argue your answer to prove a theory ?
2. The quantity $\pi = \frac{22}{7}$ is a number with no dimensions, since it is a ratio of two lengths. Describe two or three other geometrical physical quantities which are dimensionless.
3. List the characteristics that must be possessed by a universal standard.
4. Why there are no SI base units for area or volume?
5. Can length be measured along a curved line ? If yes, then explain.
6. A man seeing a lightning starts counting seconds until he hears thunder. He then claims to have found an approximate but simple rule that if the count of second is divided by an integer, the result directly gives, in km, the distance of the lightning source. What is the integer if velocity of sound is 330 ms^{-1} ? If you are able to conceptualize this situation, then analyse the various possibilities in the situation.
7. Mention one scalar and one vector physical quantity having same dimensional formula.
8. Identify three physical quantities which are having same dimensions.
9. What is the basic difference between SI system of units and the MKS system of units ?
10. On what basis the physical quantities are classified as fundamental or derived quantities ?

B. Numerical Answer Types

1. Convert 1228 kmh^{-1} into ms^{-1}.
2. Volume of largest cut diamond is 1.84 cubic inche. What is its volume in cubic centimetre and cubic metre. [1 inch = 2.54 cm]
3. Density of a material is equal to its mass divided by its volume. What is the density (in kgm^{-3}) of a rock of mass 1.8 kg and volume 6×10^{-4} m^3?
4. Convert 0.473 L into cubic inche. [1 L = 1000 cm^3]
5. Write down the units and dimensional formulae of following physical quantities :

(a) Length (b) Area (c) Volume (d) Distance

(e) Displacement (f) Speed (g) Velocity (h) Acceleration $= \frac{\text{velocity}}{\text{time}}$

(i) Force = mass × acceleration (j) Momentum = mass × velocity.

(k) Work = force × distance (l) $\text{KE} = \frac{\text{mass} \times (\text{speed})^2}{2}$

(m) PE = mgh (n) Power $= \frac{\text{work}}{\text{time}}$ (o) Spring Constant $= \frac{\text{force}}{\text{elongation}}$

(p) Elastic PE $= \frac{(\text{Spring constant}) \times (\text{elongation})^2}{2}$ (q) Impulse = force × time

(r) Density (s) Torque = force × length (t) Pressure $= \frac{\text{force}}{\text{area}}$

(u) Stress $= \frac{\text{force}}{\text{area}}$ (v) Strain $= \frac{\text{elongation}}{\text{actual length}}$ (w) Young's modulus $= \frac{\text{stress}}{\text{strain}}$

(x) Universal gravitational constant $G, F = \frac{Gm_1m_2}{r^2}$ where symbols have their usual meanings.

(y) Coefficient of viscosity η, $F = 6\pi\eta rv$ where $F \rightarrow$ force, $r \rightarrow$ radius of ball, $v \rightarrow$ velocity of ball.

6. A physical quantity X is expressed as, $X = \frac{Wb^3}{F}$ where W is work, b is length and F is force, then determine the dimensional formula for X.
7. Energy density is defined as energy per unit volume. Determine the dimensional formula for energy density.
8. Convert 100 W into CGS unit.
9. Convert 15.6 N into CGS unit.
10. SI unit of energy is joule while its CGS unit is erg. Determine the relation between joule and erg.
11. If value of G (universal gravitational constant) is 6.67×10^{-11} in SI unit, then determine its value in CGS system.
12. Value of acceleration due to gravity near to earth's surface is 9.81 ms^{-2}. Determine its value in fts^{-2}. (1 ft = 12 inch, 1 inch = 2.54 cm)
13. Kilowatt-hour is the unit of energy, 1 kW-h = 1 kW × 1 h. Determine how may joules equal to 1 kWh?
14. A car is moving with 1.81 mile h^{-1}. Determine its speed in ms^{-1}. (1 mile = 1.62 km)
15. Determine the dimensional formula for X in the followings.

    (a) $X = \frac{G \times m_1m_2}{r}$ (b) $X = F \times v$ (c) $X = \frac{at^3}{3} \times F^2$

    where $G \rightarrow$ Gravitational constant, $m_1, m_2 \rightarrow$ masses, $r \rightarrow$ distance, $F \rightarrow$ force, $v \rightarrow$ velocity, $a \rightarrow$ acceleration, $t \rightarrow$ time.

C. Fill in the Blanks

1. If impulse = force × time, the SI unit of impulse is ......
2. In a given system of unit, the ratio of the units of volume to that of area gives the unit of ......
3. The dimensional formula of acceleration is ......
4. $1\ m^2 = ...\ cm^2$.
5. $1\ cm^3 = ...\ m^3$.
6. Dimensions of time in power is ......
7. Kilowatt-hour is the unit of ......
8. Dimensional formula for PE is ......
9. If $(\text{velocity})^x = (\text{pressure})^{3/2} \times (\text{density})^{-3/2}$ then x would be ....
10. Torque and work are having ...... dimensional formula.

Answers

Towards Proficiency Problems

B. Numerical Answer Types

1. 341.1 ms^{-1} 2. $30.16 \times 10^{-6}\ m^3$ 3. $0.3 \times 10^4\ kgm^{-3}$ 4. 28.86 $inch^3$

5. (a) m, [L] (b) m^2, [L^2] (c) m^3, [L^3] (d) m, [L] (e) m, [L]
(f) m/s, [LT^{-1}] (g) m/s, [LT^{-1}] (h) m/s^2, [LT^{-2}] (i) kg·m/s^2; [MLT^{-2}]

(j) kg-m/s, $[MLT^{-1}]$ (k) kg-m^2/s^2, $[ML^2T^{-2}]$ (l) J, $[ML^2T^{-2}]$
(m) J, $[ML^2T^{-2}]$ (n) W, $[ML^2T^{-3}]$ (o) N/m, $[MT^{-2}]$ (p) J, $[ML^2T^{-2}]$
(q) N-s, $[MLT^{-1}]$ (r) kg/m^3, $[ML^{-3}]$ (s) N-m, $[ML^2T^{-2}]$ (t) N/m^2, $[ML^{-1}T^{-2}]$
(u) N/m^2, $[ML^{-1}T^{-2}]$ (v) No unit, No dimension (w) N/m^2, $[ML^{-1}T^{-2}]$
(x) N- m^2/kg^2, $[M^{-1}L^3T^{-2}]$ (y) kg/m-s, $[ML^{-1}T^{-1}]$

6. $[L^4]$ **7.** $[ML^{-1}T^{-2}]$ **8.** 100×10^7 **9.** 15.6×10^7
10. 10^5 **11.** 6.67×10^{-8} cm^{3}g^{-1}s^2 **12.** 32.18 ft/s^2
13. 3.6×10^5 J **14.** 0.8145 m/s **15.** (a) $[ML^2T^{-2}]$, (b) $[ML^2T^{-3}]$, (c) $[M^2L^3T^{-3}]$

C. Fill in the Blanks

1. N-s **2.** Length **3.** $[LT^{-2}]$ **4.** 10^{+4} **5.** 10^{-6}
6. –3 **7.** Energy **8.** $[ML^2T^{-2}]$ **9.** 3 **10.** Same

Explanations

Towards Proficiency Problems

1. $1228\,\frac{\text{km}}{\text{h}}=\frac{1228\times(1\text{ km})}{1\text{ h}}$

$=\frac{1228\times(1000\text{ m})}{(3600\text{ s})}=341.1\text{ ms}^{-1}$

2. $V=1.84\,(\text{inche})^3$

$=1.84\,(2.54\text{ cm})^3$
$=1.84\times(2.54)^3\text{ cm}^3$
$=30.16\text{ cm}^3$
$=30.16\times(10^{-2}\text{ m})^3$
$=30.16\times10^{-6}\text{ m}^3.$

3. $\rho=\frac{\text{mass}}{\text{volume}}=\frac{1.8}{6\times10^{-4}}\text{ kgm}^{-3}$

$=0.3\times10^4\text{ kgm}^{-3}$

6. $[X]=\frac{[ML^2T^{-2}][L]^3}{[MLT^{-2}]}=[L^4].$

7. $u=\frac{\text{energy}}{\text{volume}}$

$[u]=\frac{[ML^2T^{-2}]}{[L^3]}=[ML^{-1}T^{-2}].$

10. $1\text{ J}=1\text{ kg-ms}^{-2}$

$=\frac{1\text{ kg}\times1\text{ m}}{1\text{ s}^2}=\frac{10^3\text{ g}\times10^2\text{ cm}}{(1\text{ s})^2}$

$=10^5\,\frac{\text{g-cm}}{\text{s}^2}=10^5\text{ erg.}$

11. $G=6.67\times10^{-11}\,\frac{\text{N-m}^2}{\text{kg}^2}$

$=6.67\times10^{-11}\times\frac{(1\text{ kg-ms}^{-2})\times\text{m}^2}{\text{kg}^2}$

$=6.67\times10^{-11}\left[\frac{\text{m}^3}{\text{kg-s}^2}\right]$

$=6.67\times10^{-11}\left[\frac{10^6\text{ cm}^3}{10^3\text{ g-s}^2}\right]$

$=6.67\times10^{-8}\,\frac{\text{cm}^3}{\text{g-s}^2}$

13. $1\text{ kWh}=(10^3\text{ W})\times(3600)\text{ s}$

$=\left(10^3\,\frac{\text{J}}{\text{s}}\right)\times(3600\text{ s})=3.6\times10^5\text{ J}$

15. (a) $X=\frac{Gm_1m_2}{r}=\frac{Gm_1m_2}{r^2}\times r=Fr$

$[X]=[MLT^{-2}][L]=[ML^2T^{-2}].$

(b) $X=Fv$

$[X]=[MLT^{-2}][LT^{-1}]=[ML^2T^{-3}].$

(c) $X=\frac{at^3}{3}\times F^2$

$[X]=[LT^{-2}][T]^3[MLT^{-2}]^2=[M^2L^3T^{-3}].$

Chapter 3

Motion in Straight Line

The First Steps' Learning

- Rest & Motion
- The Particle Model
- Frame of Reference
- Position of an Object
- Distance and Displacement
- Average Speed and Average Velocity
- Instantaneous Velocity
- Acceleration
- Motion with Constant Acceleration
- Freely Falling Bodies
- Quantitative Description of a Freely Falling Body
- Graphical Representation of Straight Line Motion

Traditionally, there has been a good reason for the students of science and engineering to start their journey of physics with the understanding of mechanics. ***Mechanics is the cornerstone of pure and applied sciences and is the basic pre-requisite to understand any branch of physics.*** *The same tradition we are following in present text for you—"The budding IIT JEE aspirants." Mechanics, unlike some advanced branches of physics can be taught effectively in a step-to-step technique where the presentation of the concepts is in the form of a series of sequential interlinked concepts rather than as a collection of a number of concepts in few pages and then their applications. The receptive mind of the student is the only requirement for understanding physics through a step-to-step method.*

Motion is one of the most prominent features of the universe. Galaxies are moving wrt other galaxies, planets are moving around the sun, events involving motion in our daily life, like walking, running, automobiles in motion etc, and at microscopic level atoms (from which all matter is made up of) are moving about their equilibrium position. And mechanics is the "branch of physics which deals with the causes and effects of motion". There are two branches of mechanics—Kinematics and Dynamics. ***Kinematics*** *deals with the concepts that are needed to describe the motion, without going into the details of the cause of motion, while* ***Dynamics*** *deals with the cause of the motion ie,* ***the force.***

The present chapter deals with the concepts needed to describe motion in one dimension ie, motion along a straight line.

Rest and Motion

When do we say that a particular object is at rest or in motion ? Is the notebook on which you are writing at rest ? Is the pen, with the help of which you are writing in motion ? Is the chair on which you are sitting, the table on which you have kept your notebook, the floor, the walls of your room at rest ? The answer to all these questions is that these "questions are incomplete/irrevalent". You might ask why ? The rationale is that, rest or motion of any object is to be specified *wrt* some observer. For example, you are sitting on a chair in a classroom and writing in your notebook, then *wrt* you, your notebook is at rest but *wrt* the teacher who is walking on the podium in the class, your notebook is in motion. Something seems to be quite interesting and confusing, but it is absolutely true that the same object at the same time is in motion as well as at rest *wrt* two different observers.

An object is said to be in **motion** *wrt* an observer, when the position of the object **changes** with time *wrt* same observer and when the position of the object is **not changing** with time *wrt* a observer then the object is said to be **at rest** *wrt* the same observer.

For example, a car moves from location X to Y as shown in the figure. With respect to observer A, the position of car is changing with time so car is in motion *wrt* A. But if we consider the things *wrt* driver of car then *wrt* him the position of car is not changing with time and hence the car is at rest *wrt* driver.

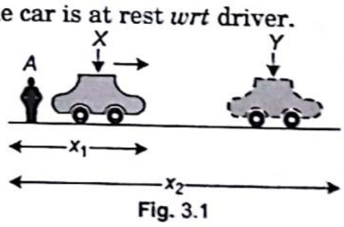


Fig. 3.1

Rest and Motion are Relative Terms

Consider once again the above example, your notebook is at rest *wrt* you but is in motion *wrt* professor who is in a sweet lecture walk in the class. Let us consider that the professor now stands at a particular place, then *wrt* him also your notebook would be at rest, but *wrt* a bell

boy who is just crossing the class, your notebook would be in motion, let us consider the bell-boy also stops and stand at a particular place then *wrt* him also your notebook is at rest. But *wrt* some other object (outside the class) *wrt* which the position of your notebook changes with time, your notebook is in motion. If we consider that all the things on the earth are stationary then *wrt* an observer from outside the earth your notebook is in motion as earth is rotating about its own axis. So it means we can never say that a particular object is at absolute rest *ie*, the terms like absolute rest and absolute motion have no meaning as rest and motion are relative terms.

Remember !

- Rest and motion has to be specified *wrt* some frame of reference.
- Absolute rest and absolute motion have no meaning or relevance.
- Rest and motion are relative terms.

The Particle Model

As we are going step-by-step to underst-and the physics, to simplify our discussion of motion we are starting with objects, whose position could be described by locating one point *ie*, size of object is very small as compared to the distance for which its motion is considered and its internal and rotational motions are of no interest to us. Such an object we call as **particle or point object.**

Remember that there is no limit on the size of an object to be treated as particle, it totally depends on the situation whether an object can be treated as a particle or not. For example, even earth can be treated as particle when we are interested in its nearly circular motion around the sun, while it can't be treated as a particle when we are interested in its rotational motion about its own axis.

Frame of Reference

In earliar sections we frequently used the phrase "with respect to observer A". In somewhat more simple manner this can be read as "as seen by A". For example, *wrt* you, your notebook is at rest means as seen by you, your notebook is at rest. In the same manner we would say, ***wrt* your frame of reference, your notebook is at rest.**

Any local surroundings *wrt* which the position of other objects has been considered can be defined as the *frame of reference*. In other words, we can say that frame of reference is some array (collection) of physical objects that remain at rest relative to each other, or may be even a single object. Within, any such frame we set up a co-ordinate system of some kind so that we can have an idea about the motion of objects *wrt* this frame of reference. To specify the position of any object *wrt* a choosen frame of reference, we consider a reference point on frame of reference called **origin** and all the distances that we measure are measured from origin. *We are free to choose any point as origin according to our convenience.*

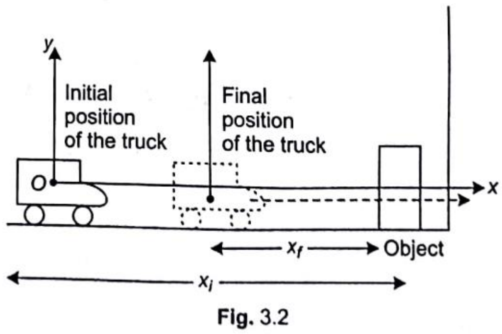


Fig. 3.2

Consider an object placed on ground as shown in the figure, and this object is observed by the driver of an approaching truck. Our aim is to locate the object *wrt* truck frame of reference, so we set up a coordinate system within the truck frame of reference as shown. [Two mutual perpendicular lines constitute our coordinate system, and these two lines are termed as x and y axes. The intersection point of these lines is taken as the reference point *ie*, origin]. For initial position of truck the object is at a distance of x_i from origin and for the final position of truck the object distance is x_f. Final position of truck is shown dotted in diagram and only for the sake of convenience the x-axis in initial and final positions are shown separately otherwise they would be along the same line.

Position of an Object

When the particle is moving along a straight line then its motion is said to be *rectilinear motion or motion in one dimension.* As in this chapter we are concerned only with rectilinear motion here we are explaining you how to locate the position of a particle in the rectilinear motion.

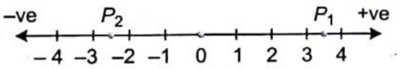


Fig. 3.3

Consider a particle moving along a straight line, say along X-axis. Then to describe its position we choose the origin O on line (X-axis) as per our convenience and consider one side of origin as positive and other is negative as shown in figure. We have the full freedom to choose positive and negative sides.

To describe the position of a particle at any time, we have to specify that, at how much distance is it from the origin and on which side—whether positive or negative side/ direction.

Let us consider two particles P_1 and P_2 which at any time t are at the positions shown in the diagram. Then

Position of P_1 is, $x_1 = 3.5$ units.

Position of P_2 is, $x_2 = -2.5$ units.

$x_2 = -2.5$ units means particle P_2 is at a distance of 2.5 units distance from origin on negative side.

Position of a particle is always *wrt* origin of the chosen frame of reference. With respect to different frame of references position of the same object at same time can be different, as origin of different frames of reference may be different.

Remember !

- Origin and direction can be choosen as per our own convenience.
- Position is completely specified by distance from origin and direction.
- With respect to different frames of reference, the position of the same particle at the same time may be different.

Distance and Displacement

Let us consider that a particle moves from position x_i to x_f, then the displacement of particle is $\Delta x = x_f - x_i$ = change in position. [x_f and x_i have to be substituted with sign]. Here Δx is read as "delta x" or "change in x", or we can say that displacement of the particle is change in its position. [*Remember that change in any variable is always the final value minus the initial value*]. So we can define displacement as **"A vector that points from an object's initial position towards its final position and has a magnitude equal to the shortest distance between the two positions".**

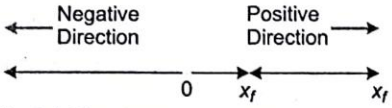


Fig. 3.4 Direction is associated with a vector

Displacement is a vector quantity and its SI unit is metre.

Distance is the actual path length traversed by the particle or in other words, the total path length traversed by particle is the distance travelled by particle. Distance is a scalar quantity and its SI unit is metre.

Let us consider a particle starts from point B *ie*, from position $x_1 = +2$ m and moves along positive direction reaches C *ie*, position $x_2 = +4$ m and then reverses its direction of motion to reach point D *ie*, at position $x_3 = -3$ m. Then for this journey of motion of particle, the distance travelled by particle is,

distance

$$= BC + CO + OD = 2 + 4 + 3 = 9 \text{ m},$$

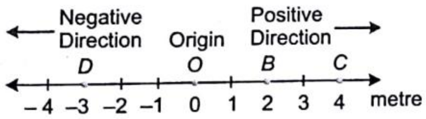


Fig. 3.5

while displacement of particle is BD,

displacement $= x_3 - x_1 = -3 - (2) = -5$ m.
where negative sign tells that the displacement vector points in negative X-direction.

Distinction between Distance and Displacement

1. Distance is a scalar quantity while displacement is vector quantity.
2. Distance would be always positive while displacement can be positive, negative or zero. For example, in above case, for motion of particle from B to C both displacement and distance are positive while from B to O *via* C displacement is negative while distance is positive.
3. Between two given positions, displacement would be single-valued function while distance can be many valued function. For example, in above case the particle can reach from B to D *via* many paths like directly from B to D or *via C or via* some other path, in all cases the distance travelled is different but displacement would be same in all cases.
4. Magnitude of displacement can decrease with time but distance can never decrease as the time passes. For example, in above case as the particle moves back from C to B after reversing its direction, the magnitude of its displacement is decreasing, but distance is still increasing as it is the actual path length.
5. For any time interval magnitude of distance is always equal to or greater than magnitude of displacement. You can analyse this for various portions of the above discussed situation.
6. Distance is equal to magnitude of displacement for all time interval, only and only if particle is moving along a straight line without any change in direction. For example, in above case for the time interval during which particle moves from B to C, distance and magnitude of displacement are always same, but then onwards magnitude of displacement is less than distance travelled by particle.

If the particle is moving in a plane or in 3-dimensional space, then some of the facts discussed about distance and displacement may not be applicable, we will make you aware about them in chapters that follow.

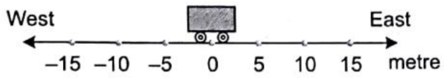
Remember !

- The displacement vector is independent of the choice of origin.

C-BIs

Concept Building Illustrations

Illustration | 1 *Consider a toy car which is moving along a straight line (East-West direction).*



Initially at $t = 0$ s, the car is at choosen origin as shown in figure and then it moves with following observational : (The car would be considered as particle ie, size of car can be neglected).

First it moves towards west for 60 s and at the end of 60th second it reaches $x = -10$ m, there it stops for a moment and then starts moving along east for 100 s and at the end of this time interval it is at $x = +5$ m. Then it stops there for 40 s and again starts moving towards east and at the end of 250th second (measured from $t = 0$) it is at $x = +20$ m.

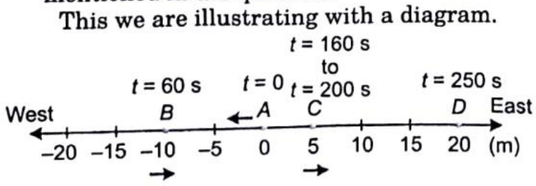
For this motion answer the following questions :

1. *What is the position of particle at $t = 60$ s ?*
2. *What is the position of particle at $t = 160$ s, $t = 180$ s, $t = 200$ s, $t = 250$ s?*
3. *What are the distance and displacement of the particle for*
   (a) $0 < t < 60$ s; $0 < t < 160$ s; $0 < t < 180$ s; $0 < t < 250$ s.
   (b) $60 < t < 160$ s; $60 < t < 180$ s; $60 < t < 250$s;
   (c) $200 < t < 250$ s.

[$t_1 < t < t_2$ means t is lying between t_1 to t_2]
Take east as positive direction.

Solution First of all locate the position of the particle (toy car) at various instants mentioned in the question.

This we are illustrating with a diagram.



The toy car starts from origin towards west at $t = 0$, we have marked it as A. Direction of arrow represents the direction in which the toy car moves at that instant.

At $t = 60$ s, the toy car reaches $x = -10$ m after travelling for 10 m.

At $t = 60$ s itself it reverses its direction of motion and after travelling for 100 s *ie*, at $t = 160$ s it is at $x = +5$ m. Then for 40 s it remains at rest at $x = +5$ m *ie*, for $t = 160$ s to $t = 200$ s the toy car is at $x = +5$ m. At $t = 200$ s it again starts moving towards east and at $t = 250$ s it is at $x = +20$ m.

1. At $t = 60$ s, the toy car is at $x = -10$ m.
2. At $t = 160$ s, the toy car is at $x = +5$ m.
   At $t = 180$ s, the toy car is at $x = +5$ m.
   At $t = 200$ s, the toy car is at $x = +5$ m.
   At $t = 250$ s, the toy car is at $x = +20$ m.
3. (a) *For* $0 < t < 60$ *s*

   Distance and magnitude of displacement would be same as direction of motion is not changing for this time interval.

   Distance = 10 m

   Displacement = $-10 - 0 = -10$ m

   For $0 < t < 160$ *s* :

   Distance = $AB + BA + AC$

   $= 10 + 10 + 5 = 25$ m

   Displacement = $5 - 0 = 5$ m

   For $0 < t < 180$ *s* :

   Same as for $0 < t < 160$ s, as toy car is at rest for $t = 160$ s to $t = 200$ s

   For $0 < t < 250$ *s* :

   Distance = $AB + BA + BD$

   $= 10 + 10 + 20 = 40$ m

   Displacement = $+20 - 0 = +20$ m

   (b) *For* $60 < t < 160$ *s* :

   Distance = $BA + AC$

   $= 10 + 5 = 15$ m

   Displacement = $+5 - (-10)$

   $= +15$ m

As direction of motion is not changing for $t > 60$ s, distance and magnitude of displacement would be same for time $t > 60$ s.

For $60 < t < 180\ s$

Same as for $60 < t < 160$ s, as particle is at rest for $t = 160$ s to $t = 200$ s.

For $60 < t < 250$ s

Distance $= BA + AD = 10 + 20 = 30$ m

Displacement $= +20 - (-10) = +30$ m

(c) *For* $200 < t < 250$ s.

Distance $= CD = 15$ m

Displacement $= +20 - (+5) = +15$ m

Average Speed and Average Velocity

The quantities *speed* and *velocity* describe just how fast the position of an object changes. Most of us are familiar about the speed, if we travel by a bus, car or train sometimes we say to others that now train is running fast or running slow, but in specialists' ways what we mean by **running fast or speed is more** is that vehicle traverse a given distance in less time and **speed is less** means that the vehicle traverses a given distance in more time. Now, here we are refining our concept of speed.

Average speed for a particular time interval of motion is defined as the total distance travelled divided by the time interval taken to travel the distance.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Time interval}}$$

Speed is a scalar quantity and its SI unit is ms^{-1}. As distance travelled is always positive and so is time, the average speed can never be negative, it would be always positive although it can decrease with time. (Think it!)

Average speed tells us about how fast the object is moving in a particular time interval, but it doesn't give any information about the direction of motion of object and to have this information we have another physical quantity, *average velocity.* Unlike average speed, average velocity refers only to how fast the displacement changes and not the total distance travelled.

Average velocity for a particular time interval of motion is defined as the displacement of particle divided by time interval for the displacement.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

Let us consider a particle moving along a straight line, which at time t_i is at position x_i and at some later time t_f it is at x_f, then from definition of average velocity,

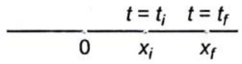


Fig. 3.6

$$\vec{\mathbf{v}}_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average velocity is a vector quantity whose direction is same as that of displacement, and its SI unit is ms^{-1}. Just as the displacement in straight line motion, we use positive and negative signs for two possible directions of average velocity. If displacement points in positive direction then average velocity also points in positive direction and *vice-versa.*

Remember !

- Average speed and average velocity are defined for some time interval.
- Average speed can never be negative but can decrease with time, while average velocity can be positive, negative or zero.
- Direction of average velocity and displacement would be same for a particular time interval.
- Magnitude of average velocity and average speed measured for same time interval are not necessarily the same, they would be equal only when direction of motion is not changing.

C-BIs

Concept Building Illustrations

Illustration | 2 *What is the distance travelled by a particle in 150 s, if its average speed for the given duration is 10 ms^{-1} ?*

Solution We know, average speed

$$=\frac{\text{Total distance travelled}}{\text{Time interval}}$$

$\Rightarrow$ Total distance travelled

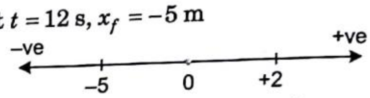
$$= \text{Average speed} \times \text{Time interval}$$

$$= (10\ \text{ms}^{-1}) \times 150\ \text{s} = 1500\ \text{m}$$

Illustration | 3 *A particle is moving along a straight line (along X-axis). At $t = 0$ it starts from $x = +2$ m and reaches $x = -5$ m at $t = 12$ s. Find the average velocity of the particle for 12 s interval starting from $t = 0$.*

Solution At $t = 0, x_i = +2$ m

At $t = 12$ s, $x_f = -5$ m



So from, $\vec{v}_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{12}$

$$= \frac{-5-2}{12} = -\frac{7}{12}\ \text{ms}^{-1}$$

Negative sign of $\vec{v}_{av}$ tells that average velocity points towards negative X-axis.

Instantaneous Velocity

Average velocity provides us with only limited information. For example, if average velocity for an hour trip of a car is 50 kmh^{-1} towards east then it tells us that in concerned 1 h duration, the displacement is 50 km towards east but it would not tell anything about how fast the car is moving at any instant in this 1 h. Has the car stopped at any instant in between? It may be possible that car is initially moving faster and later on goes slow, it may be also possible that the car initially moves towards west and then changes its direction of motion to reach final destination—and many more such possibilities.

But average velocity over 1 h duration doesn't give any information about these possibilities. To have more detailed information about the motion we now introduce the term *instantaneous velocity*.

Instantaneous velocity of a particle indicates that how fast the particle is moving and in which direction it is moving at each instant of time. Before knowing about the instantaneous velocity it is better to know the meaning of an instant.

In daily life, instant means a particular point of time which has no duration, but in physics instant means a time interval of very very small duration, which is just tending to zero-length.

Let us consider that a particle displaces by $\Delta \vec{x}$ in a time interval Δt, then average velocity for this time interval is, $\vec{v}_{av} = \frac{\Delta \vec{x}}{\Delta t}$. If Δt is very small *ie*, Δt tends to zero, then Δx also tends to zero but the *ratio* $\frac{\Delta x}{\Delta t}$ *is finite*. We then say that we are taking the limit as Δt approaches zero, symbolically written as $\Delta t \to 0$, this limit refers to a particular time t and ratio $\frac{\Delta x}{\Delta t}$ gives us the average velocity over a very small time interval Δt which tends tending to zero (or around time t). This is defined as instantaneous velocity. We can say that instantaneous velocity at time t is defined as the average velocity for a very small time interval Δt around t.

ie, $\vec{\mathbf{v}} = \dfrac{\Delta \vec{\mathbf{x}}}{\Delta t}$ as Δt is approaches zero.

As Δt is very small and hence displacement (Δx), for this small time, interval, so in this small duration we can say that displacement (*magnitude*) and distance are same hence instantaneous speed is equal to magnitude of instantaneous velocity,

$$v = |\vec{\mathbf{v}}|$$

An automobile speedometer shows you the instantaneous speed. Now onwards, we will use the word velocity for instantaneous velocity and speed for instantaneous speed while for average speed and average velocity we use the word average specifically prefixed.

Velocity is said to be uniform (constant) if neither its magnitude nor direction changes. If we say that a particle is moving with constant velocity of 10 ms^{-1} along east then it means the particle's velocity during its course of motion is not changing and it displaces by equal amount in equal intervals of time. Such a motion is said to be uniform.

Remember !

- Instantaneous velocity and speed are defined at an instant.
- $\vec{v}$ and v together provide a detailed information about the motion.
- If particle is moving with constant velocity the particle traverses equal distances in equal intervals of time.

Acceleration

The velocity of a particle may change in a number of ways, it can increase or decrease or may remain constant. The change in velocity can be for any duration, may be for first 1 min it increases faster, then for subsequent time-intervals it increases slowly and finally may stop by decreasing the velocity. To describe the change in velocity in a given time-interval we introduce the concept of acceleration.

If a particle is having velocity $\vec{\mathbf{v}}_i$ at time t_i and $\vec{\mathbf{v}}_f$ at time t_f, then we define the average acceleration $\vec{\mathbf{a}}_{av}$ for time-interval $\Delta t = t_f - t_i$ as,

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$

$$= \frac{\text{change in velocity vector}}{\text{time - interval}}$$

From the above expression we can conclude that average acceleration is providing a measure of how much the velocity changes in a given time interval.

Average acceleration is a vector quantity whose direction is same as that of change in velocity vector [Note that acceleration and velocity need not to be parallel vectors although in some situations they may be so], and its SI unit is ms^{-2}.

Just like velocity, the quantity that is more important in physics is instantaneous acceleration. The instantaneous acceleration $\vec{\mathbf{a}}$ can be defined in the same way as we define instantaneous velocity *ie*,

$$\vec{\mathbf{a}} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t} \text{ as } \Delta t \text{ approaches zero.}$$

Now onwards if we say acceleration it means we are talking about instantaneous acceleration.

Using same convention as for displacement and velocity, positive and negative signs of acceleration indicate the two possible directions of acceleration for a particle moving along a straight line.

When the particle's velocity and acceleration are in same direction, then the speed of particle increases with time and the particle's motion is said to be accelerated. When particle's velocity and acceleration are in opposite directions, then the speed of particle

decreases with time and the particle is said to be decelerated or retarded and the acceleration in this case is termed as deceleration or retardation. *It is absolutely wrong to say that negative acceleration is deceleration, as when acceleration is negative, velocity may be also negative and in this situation the particle is accelerating in negative direction even though the acceleration is negative.*

Remember !

- Direction of acceleration is same as that of change in velocity vector.
- Negative acceleration doesn't mean deceleration.

C-BIs

Concept Building Illustrations

Illustration | 4 *A particle is moving with velocity of 10 ms^{-1} along east and in a time interval of 3 s its velocity changes to 5 ms^{-1} along west. Determine the acceleration of particle over this 3 s interval.*

Solution Let east direction is considered as positive, then west would be negative.

Initial velocity, $\vec{v}_i = +10 \text{ ms}^{-1}$

Final velocity, $\vec{v}_f = -5 \text{ ms}^{-1}$

From the definition of average acceleration,

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$= \frac{-5-(10)}{3} = -5 \text{ ms}^{-2}$$

Negative sign tells that average acceleration for given 3 s duration is towards west.

Illustration | 5 *A particle is moving along a straight line with constant acceleration of 5 ms^{-2}. If the particle starts from rest at $t = 0$, then determine the velocity of particle at the end of 6th sec.*

Solution As the acceleration of particle is constant, the average acceleration for any time interval is equal to instantaneous acceleration.

$$\vec{a} = \vec{a}_{av} = \frac{\vec{v} - 0}{6 - 0}$$

where $\vec{v}$ is the velocity of particle at the end of 6th sec.

$$\Rightarrow \quad 5 = \frac{v}{6}$$

$$\Rightarrow \quad v = 30 \text{ ms}^{-1}$$

along the direction of acceleration.

Motion with Constant Acceleration

In this section we are going to discuss the motion of a particle which is moving with constant acceleration along a straight line. To describe the motion of a particle with constant acceleration, we can make use of equations of kinematics for constant acceleration, these equations don't tell anything new but using these equations we can analyse the motion very easily.

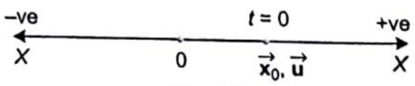


Fig. 3.7

Let us consider a particle moving along X-axis with constant acceleration $\vec{a}$, let us assume that the particle is located at $\vec{x}_0 = +x_0$ and having a velocity $\vec{u}$ at $t = 0$.

Let us say at any time t, the velocity of the particle is $\vec{\mathbf{v}}$ and its location is $\vec{\mathbf{x}}_1$, then from the definition of acceleration.

$$\vec{\mathbf{a}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{u}}}{t-0} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{u}}}{t}$$

[∵ As particle is moving with constant acceleration, average acceleration over any interval is equal to instantaneous acceleration].

$$\Rightarrow \qquad \vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\, t$$

As already explained, for one dimensional motion on one side of origin we consider positive and on other side as negative, so simply we substitute values of $\vec{\mathbf{u}}$ and $\vec{\mathbf{a}}$ with sign and unknown quantity $\vec{\mathbf{v}}$ comes out with sign. If v comes out to be positive, it means it is along positive direction and if v comes out to be negative then it means that it is along negative X-direction.

As the acceleration of particle is constant, the velocity increases at a constant rate and hence for time interval 0 to t, the average velocity of particle is

$$\vec{\mathbf{v}}_{av} = \frac{\vec{\mathbf{v}} + \vec{\mathbf{u}}}{2}$$

And from the definition of average velocity

$$\vec{\mathbf{v}}_{av} = \frac{\vec{\mathbf{x}}_1 - \vec{\mathbf{x}}_0}{t}$$

$$\Rightarrow \qquad \vec{\mathbf{x}}_1 = \vec{\mathbf{x}}_0 + (\vec{\mathbf{v}}_{av})\, t$$

$$\text{So,} \qquad \vec{\mathbf{x}}_1 - \vec{\mathbf{x}}_0 = \left(\frac{\vec{\mathbf{v}} + \vec{\mathbf{u}}}{2}\right) t = \vec{\mathbf{u}}\, t + \frac{\vec{\mathbf{a}}\, t^2}{2}$$

where $\vec{\mathbf{x}}_1 - \vec{\mathbf{x}}_0 = \vec{\mathbf{x}}$ is the displacement of particle in time t.

$$\text{So,} \qquad \vec{\mathbf{x}} = \vec{\mathbf{u}}\, t + \frac{1}{2}\vec{\mathbf{a}}\, t^2$$

Solving above two boxed equations, after eliminating t, we get

$$v^2 = u^2 + 2ax$$

The three equations $\vec{\mathbf{v}} = \vec{\mathbf{u}} + \vec{\mathbf{a}}\, t$

$$\vec{\mathbf{x}} = \vec{\mathbf{u}}\, t + \frac{1}{2}\vec{\mathbf{a}}\, t^2$$

and $v^2 = u^2 + 2ax$ are together known as equations of motion.

In somewhat more usual presentation, we can write equations of motion as : $v = u + at$; $x = ut + \frac{1}{2}at^2$ and $v^2 = u^2 + 2ax$, but here you keep in mind that velocity, acceleration and displacement are vector quantities and when you substitute the values of known quantities in above equation substitute them with sign as mentioned earlier.

C-BIs

Concept Building Illustrations

Illustration | 6 *A particle starts from rest and is moving with constant acceleration of 5 ms^{-2} along east. After a certain time t(s) it acquires a velocity of 25 ms^{-1} along east. Determine the displacement of the particle in t sec.*

Solution As the particle starts from rest its initial velocity is zero. Let the displacement of particle in t sec is x, then from equation of motion

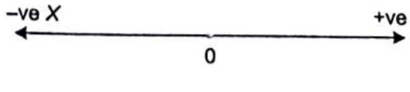
$$v^2 = u^2 + 2\,ax$$

$$\Rightarrow \qquad (+25)^2 = 0 + 2(+5)x$$

[we have considered east as positive direction]

$$\Rightarrow \qquad x = +\frac{125}{2}\ \text{m} = 62.5\ \text{m along east.}$$

Illustration | 7 *A particle is moving along X-axis with a constant acceleration of 3 ms^{-2} towards negative X-axis. The particle is having velocity of 6 ms^{-1} towards positive X-axis at $t = 0$. Determine the velocity of particle at the end of 4th sec.*



Solution Here the direction of initial velocity and acceleration are opposite to each other, so the motion is retarding one for sometime.

Using $v = u + at,$

here $v = ?;$

$u = +6 \text{ ms}^{-1};$

$a = -3 \text{ ms}^{-2};$

$t = 4 \text{ s}.$

$v = 6 - 3 \times 4 = -6 \text{ ms}^{-1}$

So, velocity of the particle at the end of 4th sec is 6 ms^{-1} along negative X-axis.

Remember that the unknown quantity comes out with sign as in above example.

Keep in mind that equations of motion are valid only for constant acceleration, they can't be used when the acceleration is varying either with time or position or with velocity. Another point to keep in mind is that equations of motion are vector equations and in equation $x = ut + \frac{1}{2}at^2$ *and* $v^2 = u^2 + 2ax$, *x is displacement and not distance.*

Now here we are classifying the types of rectilinear motion (motion in a straight line) with constant acceleration, as such this type of classification is not general but to make your grip better on the topic we need to classify it.

(A) **When acceleration is zero** : When acceleration of a particle is zero then the particle moves with constant velocity and motion is said to be uniform. Some of the features of this type of motion are :

1. Acceleration is zero, and hence velocity is constant.
2. As velocity is constant, average velocity over any interval of time is same as that of instantaneous velocity.
3. Motion is uniform as velocity is uniform (constant).
4. The only expression needed to answer any question is,
   displacement = velocity × time.
5. As particle is moving with constant velocity, distance and displacement are always equal and the particle follows a straight line motion without any change in direction.

(B) **When direction of acceleration and initial velocity are in same direction :** In this situation as the direction of acceleration and initial velocity are in same direction, the particle is accelerating *ie*, its speed is increasing.

1. Acceleration is constant and is parallel to initial velocity.
2. Velocity of the particle changes, as time passes magnitude of velocity increases but the direction remains constant.
3. As acceleration is constant, so average velocity over any time interval is simply average of two velocities at the two ends of time-interval. [As an exercise prove this]
4. Motion is said to be non-uniform or uniformly accelerated motion.
5. Distance and displacement are always equal as the direction of motion of particle is not changing.
6. Equations of motion are valid.
   If initial velocity is zero, then it is a special case of above classification.

(C) **When direction of acceleration and initial velocity are opposite :** In this situation the particle is decelerated for sometime and then accelerates in a direction opposite to that of initial velocity. Basic features of this type of motion are as follows :

1. Acceleration is constant and is anti-parallel to initial velocity.
2. Velocity of the particle first decreases, reaches zero and then increases in opposite direction. It means direction of motion is first along the initial velocity and then it reverses its direction.
3. As acceleration is constant, so average velocity over any time interval is simply average of two velocities at the two ends of time-interval. [Remember when you substitute values of velocities you have to do so with appropriate signs.]

4. Motion is said to be uniform.
5. Up to the instant when velocity of particle becomes zero, distance and magnitude of displacement are equal and then onwards they are different.
6. Equations of motion are valid. In this situation be careful about the fact that displacement is used in equations of motion, and not distance.
7. In this case initial velocity can't be zero.

C-BIs

Concept Building Illustrations

Illustration | 8 *A particle is moving along a straight line (along X-axis) with an initial velocity of 5 ms^{-1} towards positive X-axis. A constant acceleration of 2.5 ms^{-2} towards negative X-axis starts acting on particle at $t = 0$.*

Determine the

(a) *time at which its velocity gets zero.*
(b) *time at which particle reverses its direction of motion.*
(c) *velocity of particle at $t = 2$ s.*
(d) *distance travelled by particle in $t = 1$ s, $t = 2$ s, $t = 3$ s, $t = 4$ s, $t = 6$ s.*
(e) *displacement of the particle in $t = 1$ s, $t = 2$ s, $t = 3$ s, $t = 4$ s and $t = 6$ s.*

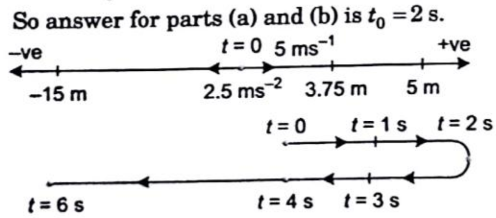
Solution Let us assume that the particle is at origin at $t = 0$. The particle will reverse its direction of motion at the instant when its velocity becomes zero and up to this instant the distance and displacement would be the same. Let us say velocity gets zero at $t = t_0$, then from $v = u + at$

$\Rightarrow \quad 0 = 5 - 2.5t_0$ as $v = 0$; $u = 5\ \text{ms}^{-1}$;

$a = -2.5\ \text{ms}^{-2}$; $t = t_0$

$\Rightarrow \quad t_0 = 2$ s

So answer for parts (a) and (b) is $t_0 = 2$ s.



At $t = 2$ s, the velocity of particle is zero as found above.

Displacement of the particle in time t is given by

$$x(t) = ut + \frac{1}{2}at^2$$

So, $\quad t = 1$ s,

$$x(1) = 5 \times 1 - \frac{1}{2} \times 2.5 \times 1^2 = 3.75\ \text{m}$$

Similarly, $x(t = 2\ \text{s}) = 5 \times 2 - \frac{1}{2} \times 2.5 \times 2^2$

$$= 5\ \text{m}$$

$$x(t = 3\ \text{s}) = 5 \times 3 - \frac{1}{2} \times 2.5 \times 3^2 = +3.75\ \text{m}$$

$$x(t = 4\ \text{s}) = 5 \times 4 - \frac{1}{2} \times 2.5 \times 4^2 = 0\ \text{m}$$

$$x(t = 6\ \text{s}) = 5 \times 6 - \frac{1}{2} \times 2.5 \times 6^2 = -15\ \text{m}$$

The path traced by the particle is shown in the figure. Upto $t = t_0 = 2$ s, the distance travelled and magnitude of displacement are same as direction of motion of particle is not changing but for $t > 2$ s distance $\neq$ magnitude of displacement.

From diagram, distance travelled in 1 s = 3.75 m.

Distance travelled in 2 s = 5 m.

Distance travelled in 3 s

$= 5 + (5 - 3.75) = 6.25$ m

Distance travelled in 4 s $= 5 + 5 - 0 = 10$ m

Distance travelled in 6 s $= 5 + 5 + 15 = 25$ m

In these types of questions you can find distance travelled in any time for $t > t_0$ by using the expression :

Distance travelled in t sec = 2 [displacement in time $t = t_0$] – | displacement in time t |

Both the terms on RHS have to be substituted with appropriate signs.

When acceleration is not constant *ie*, acceleration is varying, then the equations of motion can't be used and to solve questions based on varying acceleration, one has to make use of calculas.*

*** Calculus is a vast field in mathematics about which you will learn in your intermediate classes.**

Remember !

- Equations of motion are valid only for constant acceleration.
- In equations of motion, x is displacement and not distance.
- Be careful about the direction of velocity and acceleration.
- When acceleration and initial velocity are in opposite directions then, particle reverses its direction of motion at some instant.
- All the physical quantities in equations of motion are measured *wrt* the same frame of reference.

Problem Solving Tips

For solving the questions in which the particle is moving with constant acceleration, keep in mind the following points :

1. Make a diagramatical representation of the given situation.
2. Decide which direction you will consider as the positive and which one as the negative according to your convenience with respect to the chosen origin.
3. In this way write down the values of given variables with appropriate signs (+ or –) and units.
4. Then look for the things you have to find and/or prove.
5. First of all see whether the initial velocity is zero or non-zero. If it is zero then the particle will move along the direction of acceleration without any change in direction of its motion. In this situation distance and magnitude of displacement would be always equal.
6. If the initial velocity is not zero and is in same direction as that of acceleration, then in this case also the particle will move along its initial direction of motion without any change in direction and hence distance and magnitude of displacement would always be the same.
7. If initial velocity is not zero and its direction is opposite to that of acceleration, then it is advisable first to determine the time when velocity gets zero. Upto this instant distance and displacement would be same, afterwards distance and displacement would be different. Remember equations of motion give you the displacement and not distance.
8. There may be two possible answers for a kinematics question you have, so look for the interpretation of these two answers it may be that one answer is irrevalent.

Freely Falling Bodies

One of the most common examples of motion along a straight line is of the bodies falling under gravity of earth. It is a known fact that earth attracts every object towards its centre, and the acceleration which the body acquires due to gravitational infulence of earth is called *acceleration due to gravity* denoted by $\vec{g}$.

Acceleration due to gravity is a vector quantity and its direction is towards the centre of earth. If the height from the surface of earth is small compared to the radius of earth, then acceleration due to gravity can be considered as constant at all places up to the height considered and $\vec{g}$ can be taken in vertical downward direction. It has been also found that in the absence of air friction, all bodies fall vertically downwards with the same acceleration. The idealized motion, in which air resistance (force exerted by air on objects) is neglected and acceleration due to gravity is nearly constant, is known as *free fall motion.* Since the acceleration is constant in free fall, the equations of motion can be used. The value of acceleration due to gravity near to surface of earth is, $g = 9.8\ \text{ms}^{-2}$.

It may be a well observed phenomenon by most of you, that when you drop a stone and a paper (or feather or leaf) simultaneously from the top of a building, the stone falls faster than paper. The effect of air resistance* is responsible for the slower fall of the paper. If we repeat the experiment by **wadding** the paper (to make it ball type structure by squeezing it) we can easily observe that falling times for

paper and stone are almost equal, the reason is as we squeeze the paper the effect of air resistance is decreased as air resistance depends on area of an object too and we decreased the area by squeezing the paper.

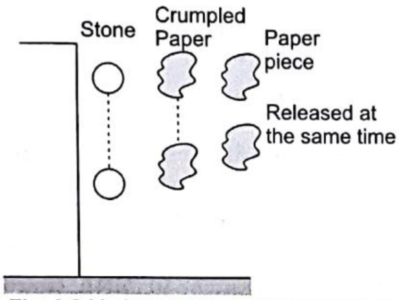


Fig. 3.8 Motion under gravity in presence of air friction

If we repeat the experiment in a chamber where no air is present *ie*, there is vacuum in the chamber, then both stone and paper fall at same rate and both exhibit free fall. Free fall is closely approximated for objects falling near the surface of the moon, where there is no air to retard the motion. In 1971, astronaut David Scott performed a classic experiment on moon's surface to demonstrate free fall—he released a hammer and a feather simultaneously on the moon, and they dropped together to the moon's surface.

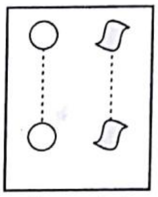


Fig. 3.9 Motion under gravity when air friction is neglected.

Quantitative Description of a Freely Falling Body

To solve the questions based on free fall, it is easiest to set up a coordinate system in a direction perpendicular to earth's surface *ie*, in vertical direction, as motion is along vertical direction or we can say in vertical plane. It is also reasonable to use the symbol y for the displacement instead of x since the motion occurs in the vertical direction or assumed y direction. Thus, when we use equations of motion for free fall, we will simply replace x with y. As such, there is no significance to this change as the equations of motion have same algebraic form provided acceleration is constant during the motion, which is the case in free fall motion.

Let us consider that a particle is dropped* from the top of a high tower as shown in the figure. Let us consider vertical direction as the Y-axis, we have two choices for positive Y-axis. Along vertical upward or along vertical downward. If we take upward as positive Y-axis, then $a = -g$ and if downward is taken as positive Y-axis, then $a = +g$, [This is because of the fact that acceleration due to gravity is always pointing towards centre of earth *ie*, in vertical downward direction]. We can go for either convention *ie*, either upward or downward as positive Y-axis as per our convenience and wish according to situation but in this book we will choose the downward direction as negative Y-axis. So for the above considered case the equations of motion would be

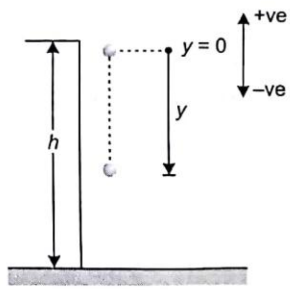


Fig. 3.10 Motion of a dropped body under gravity.

$$v = 0 - gt \quad \Rightarrow \quad v = -gt \qquad \ldots\text{(i)}$$

* Dropped means the velocity of particle at the instant of dropping is zero.

$$y = 0 - \frac{1}{2}gt^2 \quad \Rightarrow \quad y = -\frac{gt^2}{2} \qquad \text{...(ii)}$$

$$v^2 = 0 - 2g(-y) \quad \Rightarrow \quad v^2 = 2gy \qquad \text{...(iii)}$$

where v is the velocity of particle after time t and y is the displacement of particle in time t.

If h is the height of tower, then the time taken by the particle to reach the ground is given by

$$-h = -\frac{gt^2}{2}$$

$$\Rightarrow \qquad t = \sqrt{\frac{2h}{g}}$$

Distance travelled by the particle in n^{th} second in given by

$$y = y_n - y_{n-1}$$

where y_n is the distance (displacement) travelled in n sec and y_{n-1} is the distance travelled in $(n-1)$ second. [Here distance and magnitude of displacement would be always same].

$y_n = \frac{1}{2}g(n)^2$, here we are concerned with magnitude of displacement hence we have not considered the negative sign.

$$y_{n-1} = \frac{1}{2}g(n-1)^2$$

$$= \frac{g}{2}(n^2 + 1 - 2n)$$

So, $$y = y_n - y_{n-1} = \frac{g}{2}(2n-1)$$

To find out the distance travelled in n^{th} sec in general we apply the above derived expression, but some very good questions can be framed in such a way that particle reaches the ground before n second. In such cases it is advisable first to locate the position of object at $(n-1)^{th}$ sec and finding total time for which particle is in motion. To make this situation clear let us have a look in following example.

C-BIs

Concept Building Illustrations

Illustration | 9 *A particle is dropped from the top of a high building of height 180 m. Determine the distance travelled by the particle in 6th second and in 7th second. [Take $g = 9.8\ ms^{-2}$]*

Solution In these types of questions, it is better to find first the total time which the particle takes to reach the ground.

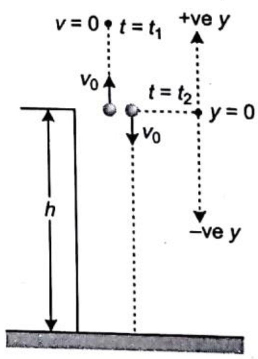
Using, $t = \sqrt{\frac{2h}{g}}$

where $h = 180$ m;

$g = 9.8\ \text{ms}^{-2}$

We get, $\qquad t = 6.061$ s.

As the total time of motion is less than 7 s and greater than 6 s, we can't apply the standard formula for distance travelled in nth sec for 7th sec but we can apply it for 6th sec. So, distance travelled in 6th second



$$= \frac{g}{2}(2 \times 6 - 1)$$

$$= \frac{9.8}{2}(11) = 53.9 \text{ m}$$

Let the particle is at $y = y_0$ at the end of 6th sec, then $y_0 = -\frac{g \times 6^2}{2} = -176.4$ m.

So, distance travelled in 7th sec is, $180 - 176.4 = 3.6$ m. Here it has been assumed that the particle comes to rest after colliding with the ground.

If the particle is not dropped *ie*, initial velocity is not zero, but projected downward with some initial velocity v_0, then equations of motion would be

$$v = -v_0 - gt \quad ...(i)$$

$$y = -v_0 - \frac{gt^2}{2} \quad ...(ii)$$

$$v^2 = v_0^2 + 2(-g)(-y)$$

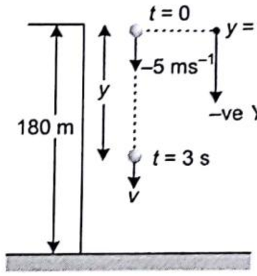
$$\Rightarrow \quad v^2 = v_0^2 + 2gy \quad ...(iii)$$

Again we are emphasizing that unknown quantities are mentioned with appropriate sign and we don't have to keep the sign with them.

Illustration | 10 *A stone is thrown downwards with a speed of* $5\ ms^{-1}$*, from the top of a building of height* 180 *m. Take point of projection as the origin and vertical downward direction as negative Y-axis. Determine*

(*a*) *the velocity of stone at the end of* 3 *s.*
(*b*) *the displacement of stone at the end of* 3 *s.*
(*c*) *the time taken by the stone to traverse a distance of* 150 *m.*
[*Take* $g = 10\ ms^{-2}$]

Solution The situation is shown clearly in the diagram.



Let v be the velocity of stone at $t = 3$ s and y be displacement of particle in 3 s. Using $v = u + at$ *ie*, 1st equation of motion

$$\Rightarrow \quad v = -5 + (-10) \times 3 \quad [\because u = -5\ \text{ms}^{-1},\ a = -g = -10\ \text{ms}^{-2},\ t = 3\ \text{s}]$$

$$\Rightarrow \quad v = -35\ \text{ms}^{-1}$$

Negative sign tells that velocity v is in downward direction.

From $\quad y = ut + \dfrac{at^2}{2}$

$$\Rightarrow \quad y = -5 \times 3 + \frac{(-10) \times 3^2}{2} \quad [\because u = -5\ \text{ms}^{-1};\ t = 3\ \text{s};\ a = -g = -10\ \text{ms}^{-2}]$$

$$\Rightarrow \quad y = -60\ \text{m}$$

Let t be the time taken by stone to travel 150 m, then

$$-150 = -5t - \frac{1}{2} \times 10t^2 \quad [\text{using } y = ut + \frac{at^2}{2}]$$

$$\Rightarrow \quad t^2 + t - 30 = 0$$

$$\Rightarrow \quad t = 5\ \text{s and } -6\ \text{s}$$

Negative time is not possible, so the required time is 5 s.

If the particle is projected with velocity v_0, in vertical upward direction from the top of a building of height h as shown in figure, then the path of the motion of the particle is as shown dotted in figure. Here as initial velocity and acceleration are in opposite direction, so distance and magnitude of displacement are same only for some time. In this case first the particle slows down and at maximum height its velocity becomes zero and afterwards it falls down and its velocity increases with time. For this case the equations of motion would be

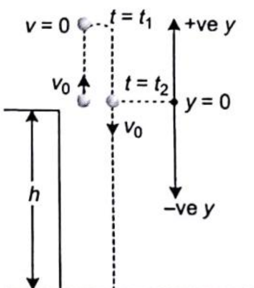


Fig. 3.11 A body projected upward from some height.

$$v = v_0 - gt \quad ...(i)$$

$$y = v_0 t - \frac{gt^2}{2} \quad \text{...(ii)}$$

$$v^2 = v_0^2 - 2g \times y \quad \text{...(iii)}$$

The time taken by particle to reach the maximum height is given by

$$0 = v_0 - gt_1 \Rightarrow t_1 = \frac{v_0}{g}$$

The time taken by particle to come back to its initial position can be found by using 2nd equation of motion.

As the particle comes back to its starting position, so $y = 0$, which gives

$$0 = v_0 t - \frac{gt^2}{2}$$

$$\Rightarrow \quad t = 0 \text{ and } \frac{2v_0}{g}$$

$t = 0$ corresponds to the instant when particle is projected up, so $t = \frac{2v_0}{g}$ is the time in which particle reaches to its initial position. So $t = t_2 = \frac{2v_0}{g}$ is the required time. It is clear from above calculations that $t_2 - t_1 = t_1$ *ie*, time taken by the particle to reach maximum height and the time taken by particle to come to initial position from maximum height are the same.

Velocity of the particle when it is crossing initial position can be found from Eq. (iii) by substituting $y = 0$ in this equation, then we get $v = v_0$, so it means that the particle crosses its initial position with same speed but direction of motion is reversed.

The total time taken by particle to reach ground can be found by using Eq. (ii). Here $y = -h$, so

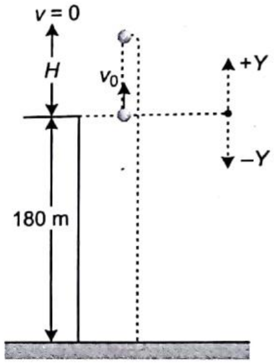
$-h = v_0 t - \frac{gt^2}{2}$, after solving these equations, we get two values of t out of which one would be inapplicable. Here we have put $y = -h$, because the displacement of particle is along negative Y direction and its magnitude is h.

Illustration | 11 *A particle is projected in vertical upward direction with velocity $5\ ms^{-1}$ from the top of a building of height 180 m. Determine the time taken by particle to reach maximum height and corresponding maximum height wrt ground level. [Take $g = 10\ ms^{-2}$].*

Solution Let the time taken for the particle to reach maximum height is t_0 and at $t = t_0$ the particle's velocity is zero, so

$$0 = 5 - 10\, t_0$$

$$\Rightarrow \quad t_0 = \frac{1}{2}\text{ s}$$



The displacement of the particle in time t_0 is

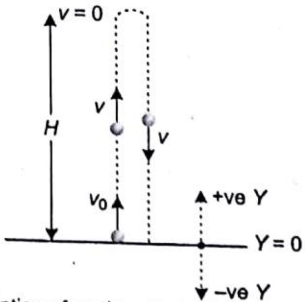
$$H = y = v_0 t_0 - \frac{gt_0^2}{2}$$

$$= 5 \times \frac{1}{2} - \frac{10}{2} \times \left(\frac{1}{2}\right)^2$$

$$H = \frac{5}{4}\text{ m}$$

So, maximum height from ground level is, $h + H = 180 + \frac{5}{4} = 181.25$ m.

Let us consider that a particle is projected in vertical upward direction from ground with velocity v_0, then as velocity and acceleration are in opposite direction initially, the speed of particle decreases and becomes zero, at this instant the particle reaches its maximum height. As gravity is still working on the particle, it will start coming down with an increasing velocity.



Description of motion of a body from ground in vertical direction.

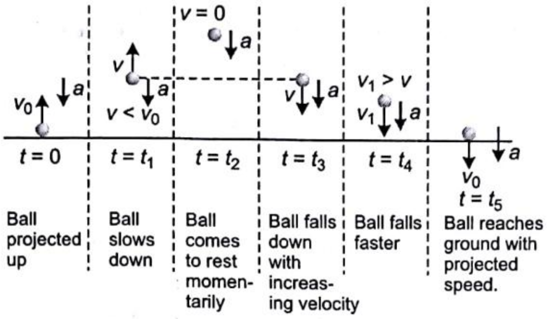


Diagram illustrating the motion of a ball projected vertically up in free fall. Although position and velocity change with time, the acceleration is constant.

For this case, the equations of motion become

$$v = v_0 - gt \quad \text{...(i)}$$

$$y = v_0 t - \frac{gt^2}{2} \quad \text{...(ii)}$$

$$v^2 = v_0^2 - 2gy \quad \text{...(iii)}$$

Just like in previous situation, we can compute the time taken by particle to reach maximum height, maximum height attained by particle etc.

Let t_a be the time taken by particle to reach maximum height, this we also call as time of ascent as for this time particle is moving up. This can be computed as follows :

At the highest point the particle comes to rest momentarity, so

Using, $v = v_0 - gt, \quad 0 = v_0 - gt_a$

$\Rightarrow \quad t_a = \dfrac{v_0}{g}$

The distance travelled by particle in this time is termed as maximum height reached by particle, which can be computed by substituting the value of t_a in Eq. (ii).

$$H = v_0 t_a - \frac{1}{2} g t_a^2$$

$$H = \frac{v_0^2}{g} - \frac{v_0^2}{2g} = \frac{v_0^2}{2g}$$

So, $$H = \frac{v_0^2}{2g}$$

To compute the time for which it will remain in air, we can make use of Eq. (ii). As the displacement y of the particle is zero when it reaches back to its initial position, so

$0 = v_0 T - \frac{1}{2} g T^2$, where T is the total time for which it will remain in air.

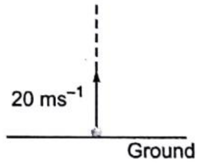
So, possible solutions of T are $\frac{2v_0}{g}$, 0.

$T = 0$ corresponds to launching time *ie*, the instant when particle is projected. So the total time for which the particle remains in motion is $\frac{2v_0}{g}$.

The time for which the particle falls down is, $T - t_a = \frac{v_0}{g}$ which is same as time of ascent, *ie*, time of ascent = *time of descent. It shows that the motion of an object that is thrown upwards and eventually returns to earth contains a symmetry. This symmetry is not limited to time only but in speed also, it exists.*

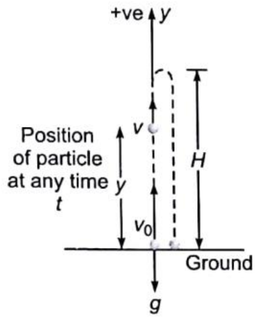
Let us say for displacement $y = +y_0$, 3rd equation of motion gives two values of v having same magnitude and opposite sign, one with positive sign corresponds to upward journey while other with negative sign corresponds to downward journey at same location.

Illustration | 12 *A particle is projected from ground in vertical upward direction with velocity 20 ms^{-1} as shown in figure. Consider $t = 0$ as the instant when particle is thrown. For this situation determine*



(a) *the time when particle stops momentarily.*
(b) *the maximum height attained by particle.*
(c) *the total time for which particle remains in motion.*
(d) *the position of the particle at $t = 1.5$ s.*
(e) *the velocity of the particle when it is 10 m above the ground level.*
[Take $g = 10\, m/s^2$]

Solution Here we are considering the point of projection as origin and vertical upward direction as +ve.



So here, initial velocity $v_0 = +20 \text{ ms}^{-1}$

acceleration $a = -10 \text{ ms}^{-2}$

(a) Using, $v = v_0 + at$

For particle to stop momentarily, $v = 0$

So, $\quad 0 = 20 - 10t$

$\Rightarrow \quad t = 2\text{s}$

(b) The maximum height is attained by particle in 2s,

so, by using $y = v_0 t + \frac{1}{2} at^2$, we get

$$y = 20 \times 2 - \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$$

(c) Let the particle comes back to ground in time t, in this much time the displacement of particle is zero, so by using

$$y = v_0 t + \frac{1}{2} at^2$$

$$\Rightarrow \quad 0 = 20 \times t - \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow \quad t = 4\text{s}$$

So, time for which particle remains in motion is 4 s.

(d) The position of the particle at any time t is given by $y = v_0 t + \frac{1}{2} at^2$

So, substituting $t = 1.5\text{s}$ in above equation, we get

$$y = 20 \times 1.5 - \frac{1}{2} \times 10 \times 1.5^2$$

$$= 18.75 \text{ m above ground level.}$$

(e) To find the velocity of particle for a given displacement we can make use of equation $v^2 = v_0^2 + 2\,ay$.

$$\Rightarrow \quad v^2 = (20)^2 - 2 \times 10 \times 10 = 200$$

$$\Rightarrow \quad v = \pm\sqrt{200} \text{ ms}^{-1}$$

$$= +10\sqrt{2} \text{ ms}^{-1},$$

and $\quad -10\sqrt{2} \text{ ms}^{-1}$

Two answers are coming because the particle acquires same position twice during its course of motion one while going up and the other while coming down.

Similarly we would have two reasonable values of t for a given displacement from equation $y = v_0 t + \frac{1}{2} at^2$.

Remember !

- In free fall motion, the air resistance* is neglected, even though in practical situation it can't be neglected.
- Value of g can be considered constant only when the height of the particle from earth's surface is neglected as compared to radius of earth.
- The expression "freely falling" doesn't necessarily mean an object is falling down. A freely falling object is any object moving either upward or downward under the influence of gravity alone.
- We have the liberty to choose vertical upward or vertical downward as positive Y-axis, according to our convenience we can choose any direction as positive Y-axis.
- During free fall motion, the velocity of particle changes from time to time but its acceleration doesn't change.
- A symmetry occurs in free fall motion.

* You can easily experience the effect of air resistance if you drive a bicycle. When wind is flowing and you are moving against the wind, then it is difficult to ride the bicycle because of air resistance.

Graphical Representation of Straight Line Motion

Graphical analysis of any event is always helpful to predict the result directly without undergoing calculation part. Like for example, you want to analyse the success rate (percentage of selected students) of any institute in last 7 yr from the graph shown below. From the graph you can easily predict that in last 5 yr there is an increase in percentage of selected students and only in 1 yr 2004 the result is not very favourable for the institute. In the same way certain graphs like displacement-time graph or velocity-time graph helps in predicting the information about motion. If we plot a graph between A *versus* B then quantity A comes on Y-axis and quantity B comes on X-axis.

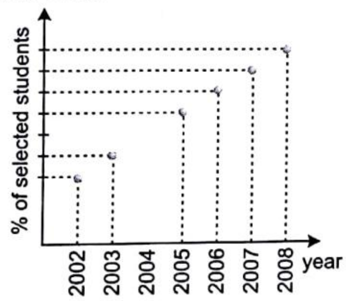


Fig. 3.12

Here first we will discuss about displacement- time graph and then about velocity-time graph.

Displacement-Time Graph

(a) If the particle is, moving with constant/ uniform velocity then displacement-time graph would be a straight line as constant velocity means particle travels by same amount in equal intervals of which means slope of displacement *versus* time graph is constant. In this case the particle moves along a straight line without changing its direction of motion and hence displacement-time plot and distance time plot would be identical.

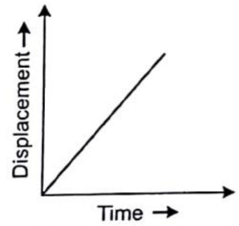


Fig. 3.13 Displacement-time graph for uniform motion *ie*, for constant velocity.

The slope of displacement-time plot at any instant gives the velocity of particle at this instant. If the slope of displacement-time graph is zero then it means particle's velocity is zero *ie*, it is at rest.

(b) When the particle is moving with constant velocity average velocity is same as velocity over any time interval and so slope of displacement-time graph also gives us average velocity in this case.

(c) If the particle is moving with changing/ varying speed (velocity) [here, direction of velocity is kept constant and only magnitude changes] *ie*, particle moves by unequal displacement in equal interval of time *ie*, when motion is non-uniform, then displacement-time graph would be a curved one (non- linear). Slope of the tangent drawn to curve at any point gives the velocity at that corresponding instant. The numerical value of slope of tangent drawn to curve gives the speed (because speed = magnitude of velocity). Seeper is the slope at any point more is the magnitude of velocity at that point. For example, in above diagram speed at P_1 is more than that at P.

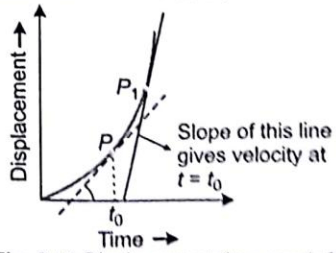


Fig. 3.14 Displacement-time graph for non-uniform motion

(d) If the particle is moving with non-uniform velocity, then average velocity and velocity are different in this case. The average velocity for a particular time-interval can be computed from displacement-time plot. The slope of the chord joining any two points on the curve gives the average velocity for corresponding time-interval. For example, in given situation slope of line PQ gives average velocity for $\Delta t = t_2 - t_1$.

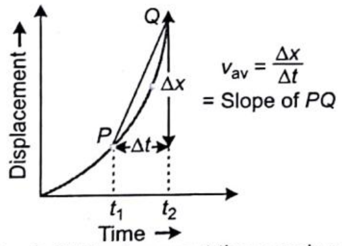


Fig. 3.15 Displacement-time graph can provide information about average velocity

(e) If displacement-time plot is concave up, then it means acceleration is positive and if it is concave down then it means acceleration is negative.

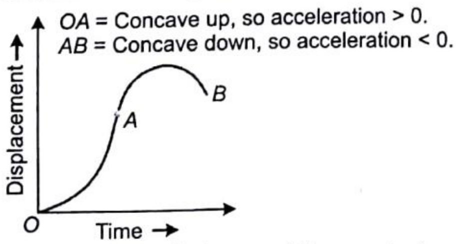


Fig. 3.16 From displacement-time graph sign of acceleration can be found.

(f) From displacement-time plot we can find whether the particle is acceleraing (speeding up) or decelerating (slowing down), as we can determine the sign of acceleration and velocity from displacement-time plot for any time-interval.

(g) From displacement-time plot we can find when the particle is changing its direction of motion, this happens when slope of displacement-time graph becomes zero *ie*, when its velocity becomes zero.

(h) From displacement-time plot we can see whether the initial velocity is zero or non-zero.

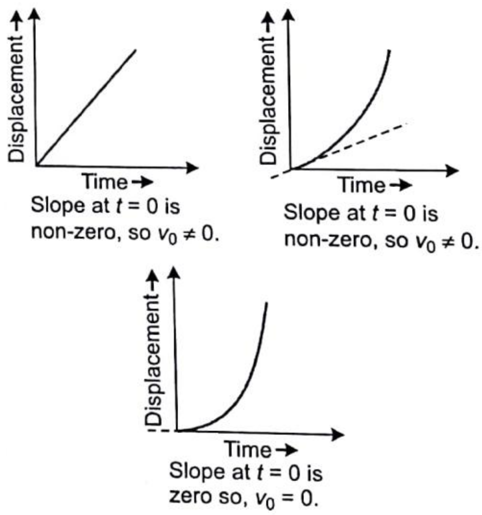
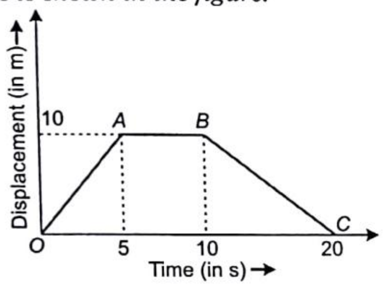


Fig. 3.17

C-BIs

Concept Building Illustrations

Illustration | 13 *The displacement-time plot for a particle moving along a straight line is shown in the figure.*



From this graph deduce the following information :

(a) *For what time interval the particle is moving fastest ?*

(b) *For what time interval the particle is moving slowest ?*

(c) *For what time interval the particle is at rest ?*

(d) *Find the displacement of the particle in* 5 *s*, 10 *s and* 20 *s*.

(e) *Find the distance travelled by the particle in* 5 *s*, 10 *s and* 20 *s*.

Solution As for OA the displacement-time graph is a straight line having non-zero slope, so it means particle is moving with constant non-zero velocity for 0 to 5 s.

For O to A, velocity of particle is

$$v = \frac{10\ \text{m}}{5\ \text{s}} = \left(\frac{\text{Displacement}}{\text{Time}}\right) = +2\ \text{ms}^{-1}$$

For A to B, slope of displacement-time graph is zero, it means particle is at rest from 5 s to 10 s.

For B to C, slope of displacement-time graph is negative it means particle changes its direction of motion at $t = 10$ s. It is clear from the graph also as magnitude of displacement starts decreasing from $t = 10$ s.

Velocity for B to C is, $v = -\dfrac{10}{10} = -1\ \text{ms}^{-1}$.

It means particle is moving fastest for 0 to 5 s, moving (not at rest) slowest for 10 s to 20 s and is at rest for 5 s to 10 s. Displacement of the particle at any instant can be deduced directly from graph.

Displacement of particle in 5 s = 10 m.

Displacement of particle in 10 s = 10 m

[as particle is at rest for 5 s to 10 s]

Displacement of particle in 20 s = 0 [as at $t = 10$ s particle reverses its direction of motion and comes to initial position at $t = 20$ s].

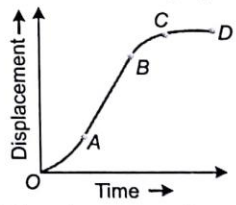
For distance travelled by particle—Up to $t = 10$ s distance and magnitude of displacement are same as particle is not changing its direction of motion up to 10 s.

Distance travelled by particle in 20 s

$$= 10\ \text{m} + 0 + 10\ \text{m} = 20\ \text{m}.$$

[You can go along displacement axis for this time interval to compute distance].

Illustration | 14 *Displacement-time graph for a particle moving along a straight line is as shown in the figure. Answer the following questions based on this graph.*



(a) *For OA the particle is (speeding up / slowing down).*

(b) *The initial velocity of particle is (zero / non-zero).*

(c) *For AB the speed of particle is (constant / varying).*

Solution For OA : Displacement-time plot is concave up, so acceleration > 0 and velocity is also positive as slope of tangent drawn to displacement-time graph is positive, so the particle is speeding up.

Slope of the tangent drawn to curve at $t = 0$ is non-zero so initial velocity is non-zero.

AB is a straight line so it means velocity is constant for this section of curve, and hence speed is also constant.

Velocity-Time Graph

(a) For uniform motion *ie*, when a particle is moving with constant velocity, the velocity-time graph would be a straight line parallel to time axis. In the shown graph, the velocity of particle is constant and equals to v_0.

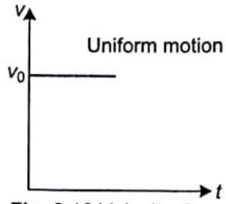


Fig. 3.18 Velocity-time graph for uniform motion

If the particle is at rest, then velocity-time graph of the particle is a line coinciding with time axis.

(b) If the particle is moving with constant non-zero acceleration, then velocity-time graph of particle is a straight line making some non-zero angle with time axis. The slope of the line gives the acceleration. For given graph $a = \tan\theta$, as θ is acute and hence $\tan\theta > 0$ *ie*, acceleration is positive. It is clear from the graph that velocity is also positive *ie*, acceleration and velocity both are in the same direction and hence particle is speeding up.

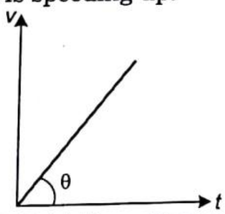


Fig. 3.19 (a) Non-uniform motion with positive constant acceleration

For the graph shown, the slope of velocity-time graph is negative, and hence the acceleration. But velocity is positive so for this situation the particle is slowing down.

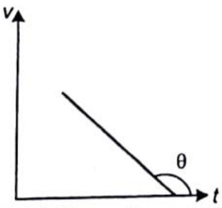


Fig. 3.19 (b) Non-uniform motion with negative constant acceleration

(c) If the particle is moving with varying acceleration then velocity-time graph of the particle is a curve whose nature depends upon the variation of acceleration. The slope of the tangent drawn to velocity-time graph at any point gives the acceleration of the particle at that point.

Fig. 3.20 Non-uniform motion with varying acceleration

(d) From velocity-time graph we can find that at what instant particle is changing its direction of motion. This instant can be found by using the fact that the particle comes to rest momentarily when it reverses its direction of motion. In the shown graph the particle changes it direction of motion at $t = t_0$.

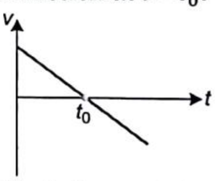


Fig. 3.21 Velocity-time graph illustrating that particle changes its direction of motion at $t = t_0$

(e) From velocity-time graph we can determine the displacement and the distance travelled by particle in any given time interval. Area under velocity-time graph gives us the displacement of particle. For the particle whose velocity-time graph is as shown in figure, the displacement of the particle for various intervals is

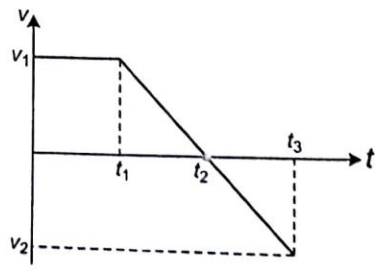


Fig. 3.22

For $0 < t < t_1 \rightarrow x = v_1 \times t_1$

For $0 < t < t_2 \rightarrow x = v_1 t_1 + \dfrac{v_1(t_2 - t_1)}{2}$

For $0 < t < t_3 \rightarrow$

$$x = v_1 t_1 + \frac{v_1(t_2 - t_1)}{2} - \frac{1}{2} \times v_2 \times (t_3 - t_2)$$

More accurately we can say algebraic area bounded by velocity-time graph gives the displacement.

To determine the distance from velocity-time graph we will use the fact that distance and magnitude of displacement are same until direction of motion of particle is not changing. And thereafter the magnitude of displacement should be added in distance.

For above shown graph upto $t = t_2$, distance and magnitude of displacement are the same. For $t > t_2$ distance and displacement would be different.

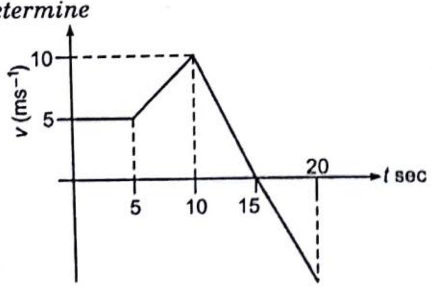
Distance travelled by the particle in t_3 sec is,

$$s = v_1 t_1 + \frac{v_1(t_2 - t_1)}{2} + \frac{v_2(t_3 - t_2)}{2}$$

ie, distance travelled by a particle is equal to sum of area of velocity-time graph irrespective of sign.

(f) We can find average speed and average velocity also from velocity-time graph, once we have the distance travelled and displacement in any given interval.

Illustration | 15 *The velocity-time graph of a particle moving along a straight line is as shown in the figure. From the given graph, determine*



(*a*) *the velocity of particle at $t = 0$.*

(*b*) *the acceleration of particle at $t = 7$ s.*

(*c*) *the acceleration of particle at $t = 14$ s.*

(*d*) *the acceleration of particle at $t = 18$ s.*

(*e*) *the instant when particle reverses its direction of motion.*

(*f*) *the displacement of particle in 10 s, 20 s.*

(*g*) *the distance travelled by particle in 10 s, 20 s.*

Solution

(a) Directly from graph, $v(t = 0) = 5$ m/s

(b) Acceleration of the particle at any instant is equal to slope of tangent drawn to velocity-time graph at that instant.

$$a(t = 7\text{ s}) = \frac{10 - 5}{10 - 5} = 1 \text{ m/s}^2$$

(c) $a(t = 14\text{ s}) = \dfrac{0 - 10}{15 - 10} = -2 \text{ m/s}^2$

(d) $a(t = 18\text{ s}) = -2 \text{ m/s}^2$

(e) At $t = 15$ s, the particle reverses its direction of motion.

(f) Area under velociy-time graph gives the displacement.

Displacement in 10 s

$$= \left[5 \times 5 + \frac{1}{2}(5 + 10) \times 5\right]$$

$$= 62.5 \text{ m}$$

Displacement in 20 s

$$= \left[5 \times 5 + \frac{1}{2} \times (5 + 10) \times 5 + \frac{1}{2}\right.$$
$$\left. \times 10 \times 5 - \frac{1}{2} + 10 \times 5\right]$$

$$= 62.5 \text{ m}$$

(g) Distance travelled in 10 s = 62.5 m

Distance travelled in 20 s

$$= \left[5 \times 5 + \frac{1}{2} \times (5 + 10) \times 5 + \frac{1}{2}\right.$$
$$\left. \times 10 \times 5 + \frac{1}{2} \times 10 \times 5\right]$$

$$= 112.5 \text{ m}$$

Towards Proficiency Problems
Exercise 1

A. Subjective Discussions

1. A particle moving in a straight line with constant acceleration is slowing down. When is the particle's velocity always opposite to acceleration ? Discuss.
2. We generally say "Tree is at rest or a pole is at rest". When absolute rest and absolute motion have no meanings, then is it correct to say like this ?
3. A car is travelling along a straight road and is decelerating. Does the car's acceleration necessarily have a negative value ?
4. Sometimes there are two possible answers to a kinematic problem, each one pertaining to different situations. Discuss one such case.
5. Often, traffic lights of a series of intersections are timed so that if you travel at a certain constant speed, you can avoid all the red lights throughout. Discuss, how the timings of the different coloured lights are determined, considering that the distance between them varies from one crossing to next ?
6. If a body, restricted to move along a straight line has a positive initial velocity, and if the acceleration is always negative, can the velocity always remain positive ?
7. Two cars are moving in the same direction along two parallel straight lines, along a highway. At a particular instant, the velocity of car A exceeds the velocity of car B. Does this mean that the acceleration of A is greater than that of B ? Explain.
8. What would happen if a moving car comes to rest with constant acceleration and the acceleration were to remain constant after the car comes to rest ?
9. Can the instantaneous velocity of a particle at a particular instant be greater in magnitude than the average velocity over a time-interval containing that instant ? Can it ever be less ?
10. A child throws a stone into the air with some initial speed. Another child drops a stone at the same instant. Compare the acceleration of the two stones while they are in motion.
11. A plant growing rapidly next to a building doubles in height every week. At the end of the 25th day, the plant reaches the height of the building. At what time, was the plant one-fourth of the height of the building ?
12. A student at the top of a building of height h throws one ball upward with an initial speed v_0 and then throws a second ball downward with the same initial speed. How were the final velocities of the balls to compare when they reach the ground ?

B. Numerical Answer Types

1. A particle moves from $x = 5$ m to $x = -3$ m, find displacement of the particle for this motion.
2. A particle first moves 4 m due east and then 3 m due south. Determine the displacement and distance travelled by the particle.

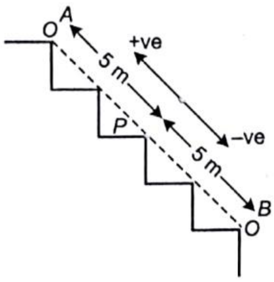
3. A particle moving along x-axis starts moving from $x = +2.5$ m at $t = 0$, after travelling for 60 s it reaches $x = -15$ m. Then, what is the position of particle at

(a) $t = 60$ s ? (b) $t = 30$ s ?

4. A particle moves along a circle of radius 10 m. Determine the distance and magnitude of displacement after

(a) 0.5 revolution (b) 2 revolutions (c) 2.5 revolutions

5. Two particles A and B are situated on stairs as shown in the figure. Determine the position of two particles *wrt* person P. Are these two positions identical ?



6. A particle travels a distance of 100 m in 5 s, then its average speed for 5 s duration is ....

7. In a journey of 100 km, a car travels first 25 km with a constant velocity of 50 kmh^{-1}, then it covers next 50 km in 15 min and remaining 25 km it travels with a speed of 100 kmh^{-1}. Determine the average speed of the car's journey.

8. A motorist drives north for 35 min at 85 kmh^{-1} and then stops for 15 min. He next continues north, travelling 130 km in 2 h.

(a) What is his total displacement ?

(b) What is his average velocity ?

9. In illustrative problem 1 find the average speed and average velocity for the time interval

(a) 0 s to 160 s (b) 60 s to 180 s (c) 60 s to 250 s

10. A person first walks at a constant speed v_1 along a straight line from A to B, and then back along the line from B to A with constant speed v_2. What are (a) the average speed for the entire trip, and (b) the average velocity over the entire trip ?

11. A drunkard walking in a narrow lane takes 5 steps forward and 3 step backward, followed again by 5 steps forward and 3 steps backward and so on. Each step is 1 m long and requires 1 s. How long the drunkard takes to fall in a pit 13 m away from the start ?

12. A car travels at a velocity of 80 kmh^{-1} during the first half of its running time and at 40 kmh^{-1} during the other half. Find the average velocity of the car over the entire trip.

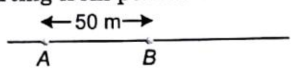
13. A car covers half a distance at a velocity of 80 kmh^{-1} and the other half at 40 kmh^{-1}. What is the average velocity of the car over the entire trip ?

14. A motor car moving uniformly at a velocity of 12 ms^{-1} covers the same distance in 10 s as another car does in 15 s. What is the constant velocity of the second car ?

15. A person started from a point lying 2 km to the east and 1 km to the north of a crossing, took an hour to walk $5\sqrt{2}$ km along a line making an angle of 135° with east. Determine the final position of the person. Take crossing as origin, east-west line as x-axis and north-south line as y-axis.

16. Two automobiles, 150 km apart, are approaching each other. One automobile is moving at 60 kmh^{-1} while the other is moving at 40 kmh^{-1}. Determine the time in which they will meet.

17. Two bodies start to move in the same direction simultaneously from two points A and B separated by 50 m, as shown in the figure. The body starting from point A has a constant velocity of 5 ms^{-1}, while the body starting from point B has a velocity of 2 ms^{-1}.



How long will it take for the first body to catch up with the second ? What will be the displacement of each body in this time? Solve the problem analytically and graphically.

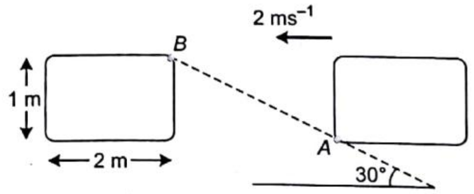
18. After 45 laps each of 10 km at an average speed of 130.7 kmh^{-1}, a speedway racer believes that he has beaten the track record of 126.3 kmh^{-1}, and hence he can relax for the next five laps at an average speed of 105 kmh^{-1}. Is this correct? What would be his final average speed for 50 laps ?

19. A plane is stationary on a runway, waiting for take off. On an adjacent parallel runway, another plane of length 36 m lands and passes the stationary plane at a constant speed of 45 ms^{-1}. A passenger in stationary plane, peeping out from a window (very narrow) sees the moving plane. For how much time the moving plane is visible to the passenger ?

20. A person being chased by a lion is running in a straight line towards his car at a constant speed of 4 ms^{-1}. The car is at a distance of d metres away from the person.The lion is 26 m behind the person and running at constant speed of 6 ms^{-1}. The person reaches the car safely. What is the maximum possible value of d ?

21. You are on a morning walk with your dog for a 4 km trip. You runs at constant velocity of 2.5 ms^{-1}, while your dog runs back and forth between you and destination at 11.5 ms^{-1} until you reaches the destination. What is the total distance travelled by dog ? What is the average velocity of the dog for the entire trip ?

22. You are in a train that is travelling at 2 ms^{-1} along a level straight track. Very near and parallel to the track is a wall that slopes upward at an angle of 30° with the horizontal. As you see outward through a window of size (1 m high, 2 m wide), the top edge of the wall first appears at the window corner A and eventually disappears at window corner B. How much time passes between appearance and disappearance of the upper edge of the wall ?



23. A person moves 3 km towards east, then 4 km towards north and then 5 km towards north-east. What would be its displacement from initial position ?

24. A car changes its velocity from 10 ms^{-1} towards east to 10 ms^{-1} towards north in a duration of 2 s. Find the magnitude and direction of average acceleration for this 2 s duration.

25. A car, initially at rest, travels first 20 m in 4 s along a straight line with constant acceleration. Determine the acceleration of car in ms^{-2}. Consider car as a particle.

26. Starting from rest, a particle moving along a straight line reaches a speed of 2 ms^{-1} in 1.5 s. What is the particle's speed after an additional 3 s has elapsed assuming that particle is moving with constant acceleration ?

27. A car is travelling with a velocity of +30 ms^{-1} along a straight road when its engine is switched off. For the next 10 s, the car slows down and its average acceleration is $-a_1$, for the next 5 s, the car slows down further and its average acceleration is $-a_2$. The velocity of the car at the end of

15 s duration is +20 ms^{-1}. The ratio of average acceleration $\frac{a_1}{a_2}$ is 1.5. Find the velocity of car at the end of 10 s interval.

28. Two particles are travelling due east with different velocities. However, after 4 s they have the same velocities. During this 4 s interval, average acceleration of 1st particle is 2 ms^{-2} due east while that of 2nd particle is 4 ms^{-2} due east. Determine the difference in their speeds at the beginning of 4 s duration and also find which one is moving faster initially.

29. An object moving with constant acceleration has a velocity of 12 ms^{-1} in the positive x-direction when its x-coordinate is +3 m. If after 2 s its x-coordinate is –5 m, then what would be its acceleration ?

30. A truck on a straight road starts from rest, accelerating at 2 ms^{-2}, until it reaches a speed of 20 ms^{-1}. Then the truck travels at constant speed for 20 s until the brakes are applied causing the truck to decelerate uniformly to stop it in additional 5 s.

(a) For how much time the truck is in motion ?

(b) What is the average speed of the truck for its entire motion ?

31. A particle starts from rest and accelerates at 0.5 ms^{-2} (constant) while moving down an inclined plane, 9 m long. When particle reaches the bottom, the particle moves up another incline, on which after moving for 15 m it comes to rest.

(a) What is the speed of particle at the bottom of 1st incline ?

(b) How much time the particle takes to come down to bottom of 1st incline ?

(c) What is the acceleration of particle (assume constant) when it moves on 2nd incline ?

(d) What is the particle's speed when it travels 8 m along 2nd incline ?

Assume that speed of particle is not changing as it comes from 1st incline to 2nd incline.

32. A bullet is fired through a fixed 10 cm thick board, in such a manner that the bullet's line of motion is perpendicular to the face of the board. If the initial speed of the bullet is 400 ms^{-1} and it emerges from other side of the board with a speed of 300 ms^{-1}, then

(a) find the deceleration of bullet [assume constant] as it passes the board.

(b) find the total time the bullet is in contact with the board.

33. Two cars A and B start for a race, but car A leaves the starting line 1 s before car B. Car A moves with a constant acceleration of 3.5 ms^{-2} while driver of car B maintains his constant acceleration of 4.9 ms^{-2}. Find

(a) the time in which car B overtakes car A.

(b) the distance car B travels before catching up car A.

(c) the speeds of both the cars at the instant when car B overtakes the car A.

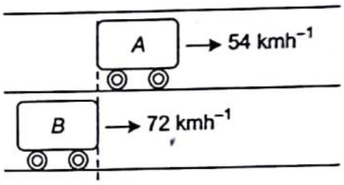
Assume the car size to be very small.

34. A car accelerates from rest at 2 ms^{-2} for 5 s, travels at a constant speed of for 7 s, accelerates at 1 ms^{-2} for 10 s and then decelerates to rest at 3 ms^{-2}. Determine the average speed of car for entire journey.

35. Your bus is leaving the stop, accelerating at a constant rate of 1 ms^{-2}. When the bus leaves the stop with zero velocity you are 20 m behind it. With what minimum constant speed you must run to catch the bus ? If you run at a speed more than the minimum speed, then analyse the situation.

36. A particle is moving along x-axis with a constant acceleration of –2 ms^{-2}. The particle leaves the origin with a velocity of +5 ms^{-1}. Determine the distance travelled by the particle during the 3rd second of its motion.

37. Two cars A and B are moving on a two-lane road with constant velocities as shown in the figure. The length of each car is 5 m. Determine



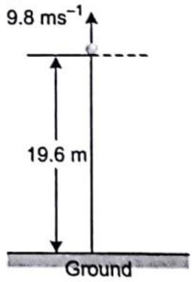
In the position shown, the overtaking process is said to start.

(a) the time in which car B overtakes car A.
(b) the total road distance used in the overtaking process.

38. On a two lane road, car A is travelling with a speed of 36 kmh^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 kmh^{-1} each. At a certain instant when the distance AB is equal to AC both being 1 km, car B decides to overtake car A before car C does. What minimum acceleration of car B is required to avoid an accident ?

39. Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do these buses move on the road ?

40. A ball is projected in vertical upward direction from the top of a building of height 19.6 m with a speed of 9.8 ms^{-1} as shown in the figure.



Determine
(a) the time when the ball reverses its direction of motion.
(b) the time taken by ball to reach the ground.
(c) the velocity of the ball (magnitude and direction) after 3 s of its motion.
(d) the distance travelled by ball in 2 s.
(e) the velocity with which ball strikes the ground.

41. A body falls freely from a height of 490 m. Determine the displacement of the body during the last second of its descent.

42. A ball thrown vertically upwards from the ground falls back to ground in 3 s. Determine the velocity of projection of ball and maximum height attained by it.

43. A body is thrown vertically upwards at a velocity of 4.9 ms^{-1}. Another body is thrown vertically downwards at the same initial speed simultaneously from the maximum height that can be achieved by the first body. Determine when and where the two bodies will meet.

44. A body falls from a height of 2000 m. How long will it take to traverse the last 100 m ? [Take $g = 10$ ms^{-2}]

45. During the last second of its free fall a body covers half of the total distance travelled. Find
(a) the height h from which the body falls.
(b) the duration of falling.

46. A body A is thrown vertically upward with an initial velocity v_0, and at the same instant another body B starts falling down with zero initial velocity from height h along same line. Find how the distance between the bodies x depends upon the time t? Consider the critical points.

47. A stone is thrown upward at an initial speed of 5 ms^{-1} from a height of 1 m. How much later on, must a second stone be dropped from same initial height so that the two stones reach the ground simultaneously?

48. A ball is thrown upwards from the ground. It passes a small window 20 m above the ground and is seen to descend past the window 5 s after it came up. It reaches the ground 6.4 s after it was thrown. From this information compute the value of g.

49. A hot air balloon is ascending straight up at a constant speed of 7 ms^{-1}. When the balloon is 12 m above the ground, a gun fires a bullet straight up from ground level with an initial speed of 30 ms^{-1}. Along the paths of the balloon and the bullet, there are two places where each of them has the same altitude at the same time. Locate these positions.

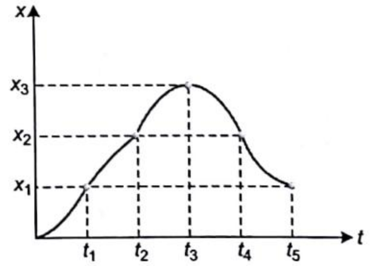
50. An object falls from rest from the top of a building. An observer inside the building notices that it takes 0.2 s for the object to pass his window, whose height is 1.6 m. How far above the top of this window is the place from where object falls? [Take $g = 9.8\ ms^{-2}$]

51. A particle is moving along a straight line whose position-time graph is as shown in figure. For the motion of particle answer the following questions :

(a) What is the initial velocity of particle—zero or non-zero?

(b) For what time interval the particle is not accelerating?

(c) For what time interval the particle is accelerating *ie*, speeding up?

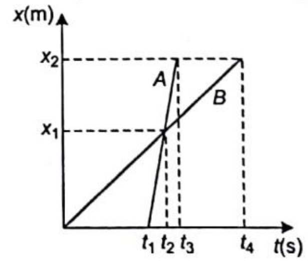


(d) At what instant the particle comes to rest momentarily?

(e) For which time-interval particle is decelerating (slowing down)?

(f) What is the distance travelled in $0 < t < t_5$ s?

52. Displacement-time graphs of two particles A and B, both moving along x-axis are as shown in figure. For this situation described, answer the following questions :



(a) Which one is moving faster?

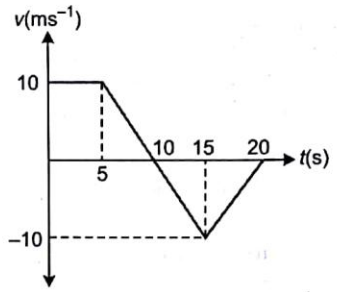
(b) Which one is accelerating?

(c) After how much time does A start its motion than B?

(d) At what time they reached at the same location?

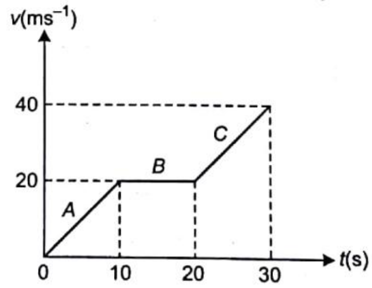
(e) Who reaches first to its final destination $- x = x_2$?

53. The velocity-time graph of a particle moving along a straight line is as shown in the figure. Find
(a) the displacement of particle in 20 s duration.
(b) the distance travelled by particle in 20 s duration.
(c) At which instant the particle changes its direction of motion, if it does ?
(d) For what interval the particle is moving with constant velocity.
(e) For what interval the particle is accelerating ?



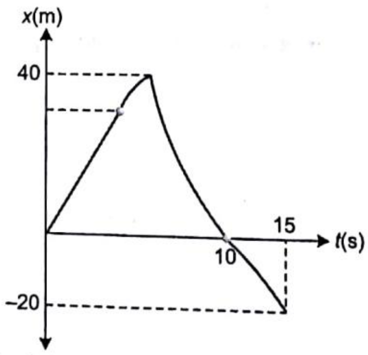
(f) For what interval, the particle is decelerating ?
(g) Upto which instant, the distance and magnitude of displacement are the same ?

54. A particle is moving along a straight line whose velocity-time plot is as given in the figure.



(a) Determine the average acceleration for sections A, B and C.
(b) Determine the average velocity for sections A, B and C.
(c) Determine the distance travelled by particle in 30 s.

55. The displacement-time graph of a particle moving along a straight line is as shown in figure. Find



(a) the average velocity for 0-10 s interval.
(b) the average velocity for 0-15 s interval.
(c) the average velocity for 10-15 s interval.
(d) the average speed for 0-15 s interval.

C. Fill in the Blanks

1. Rest and motion are .... terms.
2. If a particle travels unequal distances in equal intervals of time, then its motion is said to be .... motion.
3. A particle is continuously moving along a circle of radius R with constant speed v, then its average velocity over a long interval of time is ....
4. A player throws a ball in vertical upwards direction. The direction of acceleration during upward motion of the ball is ....
5. At highest point in above question, the direction of acceleration is .... and magnitude is ....
6. A car moving along a straight line with an initial velocity of 25 ms^{-1} north has a constant acceleration of 5 ms^{-2} south. After 7 s the velocity of the car will be ....
7. A ball is in free fall. Upwards is taken to be the positive direction. The vertical component of the displacement of the ball during a short time interval is .... during ascent, and .... during descent. [positive/negative]
8. As a rocket is accelerating verticaly upward at 9.8 ms^{-2} near earth's surface, it releases a projectile. Immediately after release the acceleration (in ms^{-2}) of the projectile is ....
9. A particle is thrown in vertical upward direction with a given initial velocity. It reaches a maximum height of 200 m. If the projection speed is doubled then maximum height attained would be ....
10. The coordinate time graph of a particle moving along a straight line is a straight line with positive slope. The particle is moving with ....

D. True/False

1. If a particle is moving along a straight line, then it is compulsory to consider direction of initial velocity as positive direction.
2. For a particle undergoing straight line motion, distance and magnitude of displacement mean the same thing.
3. You wake up at 5 : 30 am, then after taking bath etc, you go to the school, come back, then play in evening, study in night and go to bed, and again wake up at 5 : 30 am the next morning. The displacement of your in this 24 h duration is zero.
4. For a particle moving with constant acceleration in a straight line, the speed and magnitude of velocity are always equal.
5. For a particle moving with constant acceleration in a straight line, the average speed and magnitude of average velocity are always equal.
6. The speed of a particle moving along a straight line increases as time passes and the acceleration is negative, then it means body is under deceleration.
7. The above situation is not possible in practice.
8. Negative acceleration means deceleration.
9. It is possible that acceleration of particle is non- zero and velocity is zero at same instant.
10. A body can have varying velocity and constant acceleration.
11. When the acceleration of particle is positive and is increasing, then its velocity must be positive.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. We say that the displacement of a particle is a vector quantity. Our best justification for this assertion is
   (a) displacement can be specified by a magnitude and a direction
   (b) operating with displacements according to the rules for manipulating vectors leads to results in agreement with experiments
   (c) a displacement is obviously not a scalar
   (d) a displacement is associated with motion

2. A particle moves along the x-axis from x_i to x_f. Of the following values of x_i and x_f, which results in the displacement with the largest magnitude ?
   (a) $x_i = 4$ m and $x_f = 6$ m
   (b) $x_i = -4$ m and $x_f = -8$ m
   (c) $x_i = -4$ m and $x_f = 6$ m
   (d) $x_i = 4$ m and $x_f = -8$ m

3. In the above question in which of the cases the displacement is along negative x-axis ?
   (a) A and B (b) B and C
   (c) A and C (d) B and D

4. The average speed of a moving object during a given interval of time is always
   (a) equal to the magnitude of its average velocity over the interval
   (b) equal to the distance covered during the time-interval divided by the time-interval
   (c) one-half of its speed at the ends of the interval
   (d) its acceleration multiplied by the time-interval

5. A car travels 40 km at an average speed of 80 kmh^{-1} and then travels 40 km at an average speed of 40 kmh^{-1}. The average speed of the car for this 80 km trip is
   (a) 40 kmh^{-1} (b) 45 kmh^{-1}
   (c) 53 kmh^{-1} (d) 56 kmh^{-1}

6. A particle is moving with velocity of 4 ms^{-1} along positive x-direction, an acceleration of 1 ms^{-2} is acted upon the particle along negative x-direction. Find the distance travelled by the particle in 10 s.
   (a) 10 m (b) 26 m
   (c) 16 m (d) 8 m

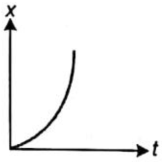
7. At a distance of 500 m from the traffic light, brakes are applied to an automobile moving at a velocity of 20 ms^{-1}. The position of automobile *wrt* traffic light 50 s after applying the brakes, if its acceleration is −0.5 ms^{-2}, is
   (a) 125 m (b) 375 m
   (c) 400 m (d) 100 m

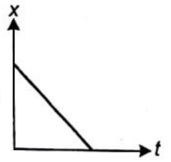
8. At a stop light, a truck travelling at 15 ms^{-1} passes a car as it starts from rest. The truck travels at constant velocity and the car accelerates at 3 ms^{-2}. The vehicles move along parallel straight lines. How many seconds will it take for the car to catch up the truck ?
   (a) 5 s (b) 10 s
   (c) 15 s (d) 25 s

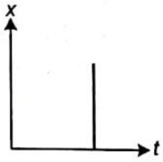
9. A ball is in free fall. Its acceleration is
   (a) downward during ascent and upward during descent
   (b) downward during descent and upward during ascent
   (c) downward during both ascent and descent
   (d) upward during both ascent and descent

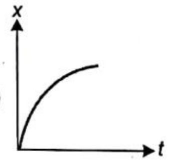
10. An object moving along a straight line has a constant acceleration of 3 ms^{-2}. The coordinate *versus* time graph for this object has a slope
    (a) that increases with time
    (b) that decreases with time
    (c) that is constant
    (d) None of the above

11. Which of the following displacement-time graphs represent the motion of an object moving along a straight line with a constant non-zero speed ?

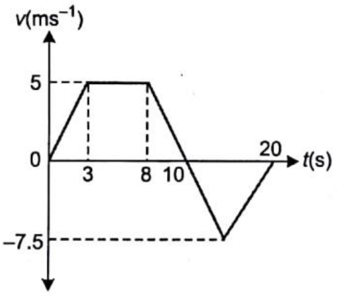
(a) 

(b) 

(c) 

(d) 

12. The velocity of an object starting from rest and moving along a straight line is shown as a function of time. For this situation mark out the correct statement(s).



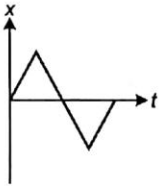
(a) Average speed of the particle is zero for 0–20 s.

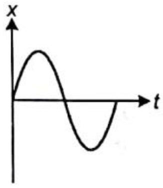
(b) Average velocity of the particle is zero for 0–20 s.

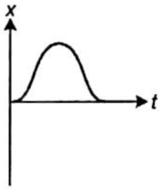
(c) Average velocity of the particle is zero for 0–10 s.

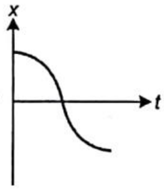
(d) Average speed of the particle is zero for 0–10 s.

13. A car accelerates from rest on a straight road. After some time, the car decelerates to come to rest and then returns to its initial position in a similar manner. Which of the following position-time graphs best describes its motion ?

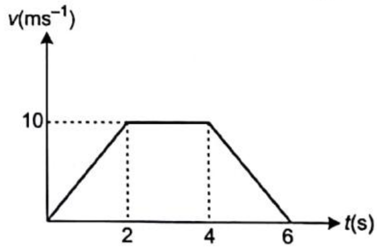
(a) 

(b) 

(c) 

(d) 

14. The diagram shows the velocity-time graph of a particle moving along a straight line. Mark out the correct statement(s).



(a) The particle first accelerates, stops and then reverses.

(b) The particle first accelerates and then decelerates to stop.

(c) The particle speed increases for first 2 s and decreases for last 2 s.

(d) None of the above

15. A particle starts from rest and moves with a constant acceleration of a_1 ms^{-2} for sometime and then decelerates to come to rest at constant acceleration of a_2 ms^{-2}. If the time for which particle is in motion is t, then the total distance travelled by the particle is

(a) $\dfrac{a_1 a_2 t^2}{a_1 + a_2}$ (b) $\dfrac{a_1 a_2 t^2}{2(a_1 + a_2)}$

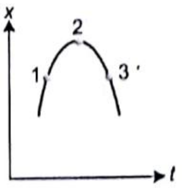
(c) zero (d) $\dfrac{(a_1 + a_2)t^2}{2}$

16. In above question, find the time for which the particle accelerates with acceleration a_1.

(a) $\dfrac{a_2}{a_1 + a_2} \times t$ (b) $\dfrac{a_1}{a_2} \times t$

(c) $\dfrac{a_1}{a_1 + a_2} \times t$ (d) $\dfrac{a_2}{a_1} \times t$

17. The displacement-time curve of a particle moving along a straight line is as shown in figure. At which of the point(s) marked the object is speeding up ?



(a) 1 (b) 2
(c) 3 (d) None of these

18. Two cars travel along a level straight highway. It is observed that the separation between the cars is increasing. Which one of the following statement(s) concerning this situation is necessarily true ?
(a) Velocity of both the cars is increasing.
(b) The front car has greater acceleration.
(c) Both the cars may have same acceleration.
(d) The rear car has smaller acceleration.

19. A ball is thrown in vertical upward direction from the top of high tower with a speed of 20 ms^{-1}. The distance travelled by ball in 6 s and velocity of ball at the end of 6th second of its flight are respectively [Take $g = 10\ ms^{-2}$]
(a) 60 m, 40 ms^{-1} down
(b) 100 m, 40 ms^{-1} down
(c) 60 m, 40 ms^{-1} up
(d) 100 m, 40 ms^{-1} up

20. A juggler performs an act with two balls, he throws one ball when the other is at its maximum height of 9 ft. (Balls are thrown in vertical upward direction). When the balls are passing each other, how far are they above the juggler's hand ? [Take $g = 32\ ft\ s^{-2}$]
(a) 4.5 ft (b) 8 ft
(c) 5 ft (d) 6.75 ft

21. A ball is thrown straight up. The magnitude of acceleration at the top point on its path is
(a) g (b) $\frac{g}{2}$
(c) zero (d) $\frac{2g}{3}$

22. A ball is released from a height of 20 m and after striking the floor it rebounds to a height of 5 m. If the ball remains in contact with the floor for 0.02 s, then the magnitude and direction of the average acceleration during this interval would be [Take $g = 10\ ms^{-2}$]
(a) 10 ms^{-2} down (b) 10 ms^{-2} up
(c) 1500 ms^{-2} up (d) 30 ms^{-2} up

B. More Than One Options Correct

1. In which of the following cases the distance and magnitude of displacement are always equal for any time ?
(a) A block moving down a long incline
(b) A ball is released from rest from the top of high building (assume that the ball comes to rest after striking the ground)
(c) A ball projected up from the ground
(d) A particle moving along a straight line with initial velocity zero and constant acceleration

2. In above question in which of the cases the magnitude of average velocity and average speed are same for any time interval ?
(a) A block moving down a long incline
(b) A ball is released from rest from the top of high building (assume that ball comes to rest after striking the ground)
(c) A ball projected up from the ground
(d) A particle moving along a straight line with initial velocity zero and constant acceleration

3. Throughout a time interval, while the speed of a particle increases as it moves along the x-axis, its velocity and acceleration might be
(a) positive and negative, respectively
(b) positive and positive, respectively
(c) negative and negative, respectively
(d) negative and positive, respectively

4. For a particle moving along x-axis, mark out the correct statement(s).
(a) If x is positive and is increasing with time, then the average velocity of particle is positive.
(b) If x is negative initially and becomes positive after sometime, then velocity of the particle is always positive.

(c) If x is negative and becoming less negative as time passes, then average velocity of the particle is positive.

(d) If x is positive and is increasing with time, then velocity of the particle is always positive.

5. Mark the correct option(s).

(a) If the velocity of a particle is constant, then the average velocity for different intervals would be different.

(b) If the velocity of a particle is constant, then the average velocity for different intervals would be the same.

(c) If average velocity of a moving particle is zero for a time-interval, then the instantaneous velocity at all instants within this interval may be zero.

(d) If average velocity of a moving particle is zero for a time-interval, then instantaneous velocity at some instants within this interval may be zero.

6. Mark out the correct statement(s).

(a) The instantaneous velocity vector is always in the direction of motion.

(b) The instantaneous acceleration vector is always in the direction of motion.

(c) The instantaneous acceleration vector is always in the direction of instantaneous velocity vector.

(d) The instantaneous velocity vector and the acceleration vector may be in opposite directions.

7. A particle is moving with constant speed along a straight line. Mark out the correct statement(s) for this situation.

(a) Average speed is always equal to magnitude of average velocity measured for a particular time-interval.

(b) Particle is having zero acceleration.

(c) Velocity of the particle is constant.

(d) Acceleration of the particle is constant and having non-zero value.

8. Starting from rest, a particle is imparted a constant acceleration of 5 ms^{-2}. For this situation, mark out the correct statement(s).

(a) Velocity-time graph is a straight line passing through origin and having slope of 5 units.

(b) Position-time graph is not a straight line.

(c) Position-time graph is a straight line.

(d) Slope of position-time graph increases as the time increases.

9. Of the following situations which are possible in practice ?

(a) Zero velocity and non-zero acceleration

(b) Constant velocity and variable acceleration

(c) Variable velocity and constant acceleration

(d) Non-zero velocity and zero acceleration

C. Assertion & Reason

Directions (Q. Nos. 1 to 6) *Each question contains two statements : **Statement I (Assertion)** and **Statement II (Reason)**. Each question has 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**

(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**

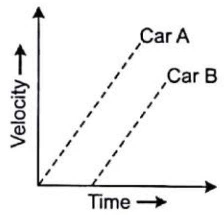
(c) **Statement I** is True, **Statement II** is False

(d) **Statement I** is False, **Statement II** is True

1. **Statement I** For a particle moving in a straight line, if velocity is positive and acceleration is negative then it means particle is decelerating.

Statement II Negative acceleration and deceleration are the same.

2. **Statement I** Average speed of a continuously moving particle can never decrease with time.
   Statement II Distance travelled by a continuously moving particle is always increasing.
3. **Statement I** For a particle moving along a straight line, the displacement in a particular time interval is independent of choice of origin.
   Statement II The displacement is a vector that points from an object's initial position to object's final position and its magnitude is equal to the shortest distance between the two positions.
4. **Statement I** For a particle moving along a straight line, the average velocity vector over a time interval would be equal to instantaneous velocity vector at the end of the interval only if the particle is moving with constant velocity.
   Statement II If a particle is moving with constant velocity, then average velocity for any time interval is equal to the velocity of particle.
5. **Statement I** The average velocity of a moving particle may be zero for some time interval, although the average velocity for a shorter time interval included in the first interval is not zero.
   Statement II Displacement of a moving particle can decrease as the time passes.
6. **Statement I** The graph below shows velocity-time relationship for two cars. From the graph you can conclude that acceleration of both the cars is same.

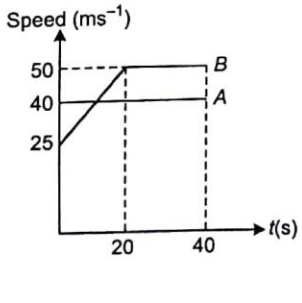


   Statement II The slope of velocity-time graph gives the acceleration.

D. Comprehend the Passage Questions

Passage I

Figure shows the speed-time graph of two cars A and B which are travelling in the same direction, and both are moving along same direction over a period of 40 s. Car A travelling at a constant speed of 40 ms^{-1}, overtakes car B at $t = 0$. In order to catch up with car A, car B immediately accelerates uniformly for 20 s.

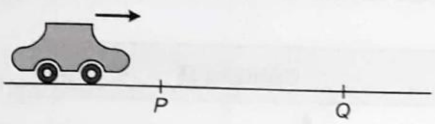


Based on above information, answer the following questions :

1. Distance travelled by car B in 20 s is
   (a) 800 m (b) 750 m
   (c) 1000 m (d) 500 m
2. What is acceleration of car B for first 20 s ?
   (a) 1.25 ms^{-2} (b) 3.75 ms^{-2}
   (c) 2.5 ms^{-2} (d) 5 ms^{-2}
3. At what time car B overtakes car A ?
   (a) 12 s (b) 50 s
   (c) 18 s (d) 25 s
4. What is the distance travelled by car A before car B overtakes it ?
   (a) 480 m (b) 1000 m
   (c) 800 m (d) 1200 m
5. What is the maximum separation between two cars during 40 s interval ?
   (a) 90 m (b) 480 m
   (c) 390 m (d) None of these

Passage II

A car of length 6 m accelerates from rest along a straight level road as shown in the figure. Some information is given about the motion of car :



The car takes 2.0 s to cross the point P, 10 s later the car has just crossed the point Q. The car takes 0.4 s to cross the point Q.

Based on above information, answer the following questions.

6. From P to Q, the car is

(a) speeding up
(b) slowing down
(c) moving with constant velocity
(d) None of the above

7. Average speed of the car as it passes Q is

(a) 3 ms^{-1} (b) 15 ms^{-1}
(c) 0.6 ms^{-1} (d) 6 ms^{-1}

8. Average acceleration of the car between P and Q

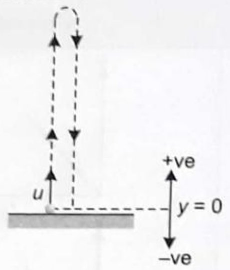
(a) 1.2 ms^{-2}
(b) 1.8 ms^{-2}
(c) 1.5 ms^{-2}
(d) 0.3 ms^{-2}

9. Assuming that the car moves with constant acceleration, determine the distance PQ.

(a) 120 m (b) 90 m
(c) 16 m (d) 180 m

Passage III

A ball is thrown up with initial velocity $u = 50\ ms^{-1}$ as shown in the figure. The origin and positive and negative directions are also indicated in the figure. Neglect air resistance and take $g = 10\ ms^{-2}$.



Based on above information, answer the following questions.

10. How much time the ball takes to reach to a height which is half of the maximum height ?

(a) 1.46 s (b) 2.5 s
(c) 3 s (d) 1.82 s

11. After how much time will it again cross the same position ?

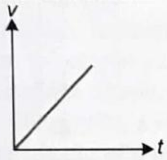
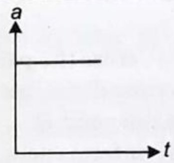
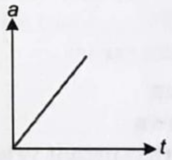
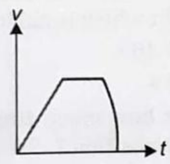
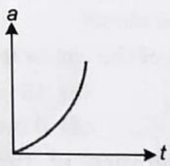
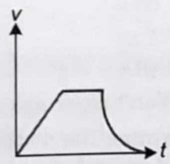
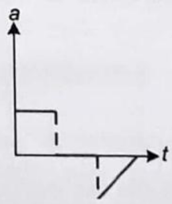
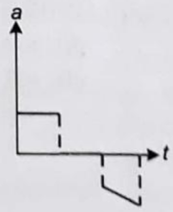
(a) 7.07 s
(b) 7.5 s
(c) 8.18 s
(d) Won't cross again

12. Determine the distance travelled by the ball in 3rd second of its motion.

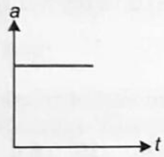
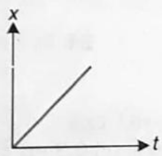
(a) Zero
(b) 45 m
(c) 90 m
(d) 80 m

E. Match the Columns

1. In Column I velocity-time graph of a particle moving along a straight line are given and in Column II the corresponding acceleration-time graph. Match the entries of Column I with the entries of Column II.

Column I	Column II
(A) 	(P) 
(B) 	(Q) 
(C) 	(R) 
(D) 	(S) 
	(T) 

2.

Column I		Column II
(A)		(P) Constant velocity
(B)		(Q) Varying velocity
(C)		(R) Constant acceleration
(D)		(S) Varying acceleration

3. For a particle moving along a straight line, match the entries of Column I with the entries of Column II.

Column I	Column II
(A) Distance	(P) Can decrease with time
(B) Average speed	(Q) Can increase with time
(C) Average velocity	(R) Can be zero
(D) Magnitude of displacement	

4. In Column I some statements related to various physical quantities are given while in Column II information about motion of a particle moving along a straight line are given. Match the entries of Column I with the entries of Column II.

Column I	Column II
(A) Distance is always equal to magnitude of displacement.	(P) Initial velocity zero and constant acceleration
(B) Velocity = average velocity over any interval	(Q) Direction of motion is not changing.
(C) Average speed and magnitude of average velocity over same time-interval are always equal.	(R) Acceleration is constant and no information about initial velocity.
(D) Velocity is continuously increasing.	(S) Initial velocity and acceleration are in the same direction.

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. – 8 m **2.** 5 m at 37° S of E, 7 m
3. (a) – 15 m, (b) can't be determined **4.** (a) 10π m, 20 m, (b) 40π m, 0, (c) 50π m, 20 m
5. Position of A is + 5 m and position of B is – 5 m **6.** 20 ms^{-1} **7.** 100 km/h
8. (a) 180 km, (b) 63.4 km/h
9. (a) 0.15625 ms^{-1}, 0.03125 ms^{-1}, (b) 0.125 ms^{-1}, 0.125 ms^{-1}, (c) 0.16 ms^{-1}, 0.16 ms^{-1}
10. (a) $\frac{2v_1v_2}{v_1+v_2}$, (b) zero **11.** 37 s **12.** 60 km/h
13. $\frac{160}{3}$ km/h **14.** 8 ms^{-1} **15.** 6 km south, 3 km west of crossing **16.** 1.5 h
17. $\frac{50}{3}$ s, $\frac{250}{3}$ m of A; $\frac{100}{3}$ m of B **18.** Yes, 127.56 km/h **19.** 0.8 s **20.** 52 m
21. 18.4 km, zero **22.** 1.866 s **23.** $\sqrt{50+\frac{70}{\sqrt{2}}}$ m **24.** $5\sqrt{2}$ ms^{-2} along N-W
25. 5 ms^{-2} **26.** 6 ms^{-1} **27.** 22.5 ms^{-1}
28. 8 ms^{-1}, one with smaller acceleration **29.** 16 ms^{-2} along –ve X-axis
30. (a) 35 s, (b) 15.7 ms^{-1} **31.** (a) 3 ms^{-1}, (b) 6 s, (c) 0.3 ms^{-2}, (d) $\sqrt{4.2}$ ms^{-1}
32. (a) 35×10^4 ms^{-2}, (b) 2.86×10^{-4} s, (c) v_A = 22.75 ms^{-1}, v_B = 26.75 ms^{-1}
33. (a) 5.5 s, (b) 72.9 m **34.** 10.87 ms^{-1} **35.** $\sqrt{40}$ ms^{-1} **36.** 0.5 m
37. (a) 2 s, (b) 45 m **38.** 1 ms^{-2} **39.** 9 min, 40 kph
40. (a) 1 s, (b) $(1+\sqrt{5})$ s, (c) 19.6 ms^{-1}, down, (d) 9.8 m, (e) $9.8\sqrt{5}$ ms^{-1} **41.** 93.1 m
42. 14.7 ms^{-1}, 11.025 m, **43.** $\frac{1}{8}$ s, 0.54 m from ground level **44.** 0.5 s
45. (a) $g(\sqrt{2}+1)^2$, (b) $(2+\sqrt{2})$ s **46.** $x_{rel} = h - v_0t,\ 0 < t < \frac{h}{v_0}$
47. 0.72 s **48.** 10.025 ms^{-2} **49.** 16.2 m and 40 m above the ground **50.** 2.52 m
51. (a) non-zero, (b) t_1 to t_2, (c) 0 to t_1 and t_3 to t_4, (d) t_3, (e) t_2 to t_3 and t_4 to t_5, (f) $2x_3 - x_1$
52. (a) A, (b) none, (c) t_1s, (d) t_2 s, (e) A
53. (a) 25 m, (b) 125 m, (c) 10 s, (d) 0 to 5 s, (e) 10 to 15 s, (f) 5 to 10 s and 15 to 20 s, (g) 10 s
54. (a) 2 ms^{-2}, 0, 2 ms^{-2}, (b) 10 ms^{-1}, 20 ms^{-1}, 30 ms^{-1}, (c) 600 m
55. (a) 0, (b) $-\frac{4}{3}$ ms^{-1}, (c) – 4 ms^{-1}, (d) $\frac{20}{3}$ ms^{-1}

C. Fill in the Blanks

1. Relative **2.** Non-uniform **3.** Zero **4.** Vertical downward
5. Vertical downward, 9.81 ms^{-2} **6.** – 10 ms^{-1} towards south
7. Positive, Negative **8.** g ms^{-2} **9.** 800 m **10.** Constant velocity

D. True/False

1. F **2.** F **3.** T **4.** F **5.** F **6.** F **7.** F **8.** F
9. T **10.** T **11.** F

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (b)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (a)
11. (b)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (d)
21. (a)	**22.** (c)								

B. More Than One Options Correct

1. (a, b, d)	**2.** (a, b, d)	**3.** (b, c)	**4.** (a, c, d)	
5. (b, d)	**6.** (a, d)	**7.** (a, b, c)	**8.** (a, b, d)	**9.** (a, c, d)

C. Assertion & Reason

1. (c)	**2.** (d)	**3.** (a)	**4.** (d)	**5.** (b)
6. (a)				

D. Comprehend the Passage Questions

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (b)	**10.** (a)
11. (a)	**12.** (c)								

E. Match the Columns

1. A → P;	B → Q, R;	C → T;	D → S
2. A → Q, R;	B → Q, S;	C → Q, R;	D → P
3. A → Q, R;	B → P, Q, R;	C → P, Q, R;	D → P, Q, R
4. A → P, Q, S;	B → P, Q, S;	C → P, Q, S;	D → P, Q, S

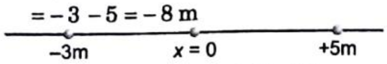
Explanations

Towards Proficiency Problems
Exercise 1

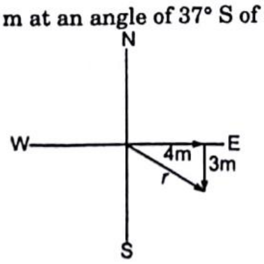
Numerical Answer Types

1. $\Delta\vec{x} = x_f - x_i$

$= -3 - 5 = -8$ m



2. $r = 5$ m at an angle of 37° S of E.



3. Directly from question at $t = 60$ s the particle is at $x = -15$ m. For any instant in between 0 and 60 s no information about the motion of particle is mentioned so we can't determine position of particle at $t = 30$ s.

4. (a) Distance $= \pi R = 10\pi$ m,

Displacement $= 2R = 20$ m

(b) Distance $= 2 \times 2\pi R = 40\pi$ m,

Displacement $= 0$

(c) Distance $= 2.5 \times 2\pi R = 50\pi$ m,

Displacement $= 2R = 20$ m.

5. $x_A = +5\text{m}$ and $x_B = -5\text{ m}$

These positions are different as they are on different sides.

6. Average speed

$$= \frac{\text{Total distance}}{\text{Time taken}} = \frac{100}{5} = 20\text{ ms}^{-1}$$

7. Let t_1, t_2 and t_3 be the time taken by the car for three different parts of the journey.

$s_1 = 25\text{ km}$, $s_2 = 50\text{ km}$ and $s_3 = 25\text{ km}$

$$t_1 = \frac{1}{2}\text{ h},$$

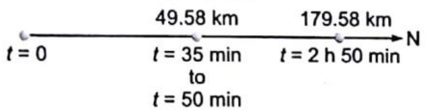
$$t_2 = \frac{1}{4}\text{ h}$$

and $$t_3 = \frac{1}{4}\text{ h}$$

$$v_{av} = \frac{100\text{ km}}{1\text{ h}} = 100\text{ km/h}$$

8. In 35 min, $s_1 = (85\text{ km/h}) \times \left(\frac{35}{60}\right)\text{h}$

$$= 49.58\text{ km}$$



$$s = s_1 + 130 = 179.58\text{ km} \approx 180\text{ km}$$

$$v_{av} = \frac{180}{2\text{ h }50\text{ min}} = \frac{180}{2.83}\text{ km/h}$$

$$= 63.4\text{ km/h}$$

10. $$v_{av} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$$

$\vec{v}_{av} = 0$

11. In each cycle of forward and backward stepping the person advances by net 2 m in forward direction and takes 8 s.
So at the end of 32 s the person is at 5 m from the pit and as he steps forward he will fall in the pit and as he steps forward he will fall in pit, *ie*, in 37 s he will fall in pit.

12. $$v_{av} = \frac{80t + 40t}{2t} = 60\text{ km/h}$$

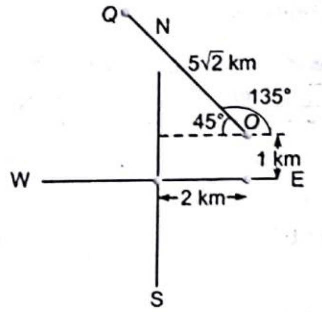
13. $$v_{av} = \frac{2s}{\frac{s}{80} + \frac{s}{40}} = \frac{160}{3}\text{ km/h}$$

14. Let v be the required velocity then

$$12 \times 10 = v \times 15$$

$$\Rightarrow \quad v = 8\text{ ms}^{-1}$$

15. The situation is clearly as shown in the figure.

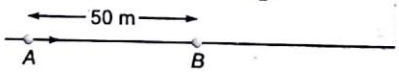


O is the initial position,
Q is the final position.
Final position is $(5 - 2 = 3\text{ km})$ towards West and $(5 + 1 = 6\text{ km})$ towards North of crossing.
IInd answer → Its answer can also be (3 km West and 4 km South) of crossing.

16. Let t be the required time in which the two automobiles will meet, $t = \frac{150\text{ km}}{(40 + 60)\text{ km/h}}$

$$= 1.5\text{ h}$$

17. Both bodies move towards right, and consider initial position of A as origin.



The position of A at any time t is, $x_A = 5t$
The position of B at any time t is,

$$x_B = 50 + 2t$$

Let 1st body catches 2nd in time t, then

$$5t = 50 + 2t$$

$$\Rightarrow \quad t = \frac{50}{3}\text{ s}$$

At $$t = \frac{50}{3}\text{ s}, x_A = \frac{250}{3}\text{ m}$$

and $$x_B = \frac{100}{3}\text{ m}$$

18. Total distance travelled in 50 laps is,

$$s = 50 \times 10 = 500\text{ km}$$

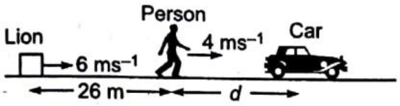
Total time taken for 50 laps is,

$$T = 45 \times \frac{10}{130.7}\text{ h} + \frac{5 \times 10}{105}\text{ h}$$

$$v_{av} = \frac{s}{T} = 127.56\text{ km/h}$$

19. $t = \dfrac{36\text{ m}}{45\text{ m/s}} = 0.8\text{ s}$

20. For safety of man,



$$26 + d = 6t \quad \text{and} \quad d = 4t$$

$\Rightarrow$ $d = 52\text{ m}$

21. Time taken by you, $t = \dfrac{4000}{\frac{5}{2}} = 1600\text{ s}$

Total distance travelled by dog

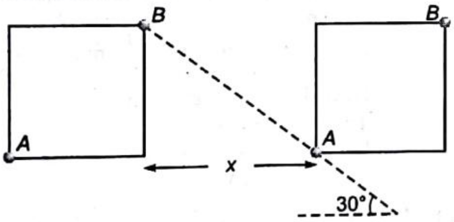
$= 11.5 \times 1600\text{ m}$

$= 18.4\text{ km}$

As displacement of dog after the entire trip is 0, so

$$\vec{v}_{av} = 0$$

22. Let t be the required time, then $x + 2 = vt = 2t$ and $x \tan 30° = 1\text{ m}$

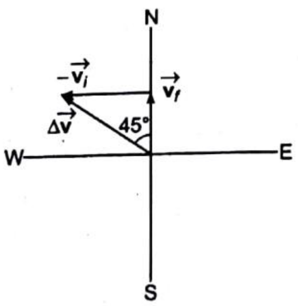


$\Rightarrow$ $x = \sqrt{3}\text{ m}$

and $t = \dfrac{3.732}{2} = 1.866\text{ s}$

24. $\vec{v}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$= \dfrac{10\sqrt{2}}{2} = 5\sqrt{2}\text{ ms}^{-2}$ at 45° W of N.



25. $s = ut + \dfrac{1}{2}at^2$

$\Rightarrow$ $20 = 0 + \dfrac{1}{2} \times a \times 4^2$

$\Rightarrow$ $a = 5\text{ ms}^{-2}$

26. Let a be the acceleration of particle, then

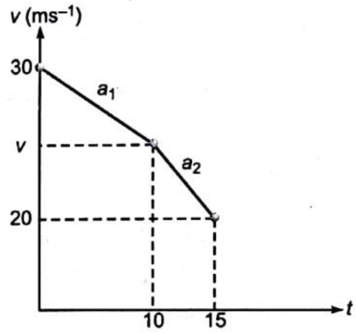
$2 = 0 + a \times 1.5$

$\Rightarrow$ $a = \dfrac{4}{3}\text{ ms}^{-2}$

Let v be the required speed, then

$v = 0 + a \times 4.5 = 6\text{ ms}^{-1}$

27. Let v be the required speed and v-t graph for given situation is as shown in the figure.



$$\frac{v - 30}{10} = -a_1$$

and $$\frac{20 - v}{5} = -a_2$$

$\dfrac{a_1}{a_2} = 1.5$ gives

$v = 22.5\text{ ms}^{-1}$

28. For 1st particle, $v_1 = u_1 + a_1 t = u_1 + 2 \times 4$

For 2nd particle, $v_2 = u_2 + a_2 t = u_2 + 4 \times 4$

As $v_1 = v_2$,

So, $u_1 - u_2 = 2 \times 4 = 8$,

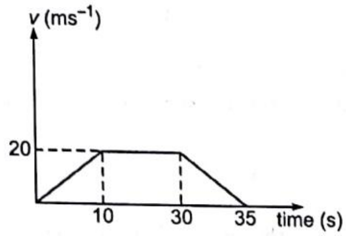
So $u_1 > u_2$

29. Using $\vec{s} = \vec{u}t + \dfrac{1}{2}\vec{a}t^2$

$\Rightarrow$ $-8 = 12 \times 2 + \dfrac{1}{2} a \times 2^2$

$\Rightarrow$ $a = -16\text{ ms}^{-2}$

30. The velocity-time graph for the truck is as shown in the figure.



$$v_{av} = \frac{\frac{1}{2}[35 + 20] \times 20}{35} = 15.7 \text{ ms}^{-1}$$

31. (a) $v^2 = 0 + 2 \times \frac{1}{2} \times 9 \Rightarrow v = 3 \text{ ms}^{-1}$

(b) $v = 0 + \frac{1}{2} t \Rightarrow t = 2v = 6 \text{ s}$

(c) $0 = v^2 - 2a_2 \times 15 \Rightarrow a_2 = 0.3 \text{ ms}^{-2}$

(d) $v_1^2 = 3^2 - 2 \times 0.3 \times 8$

$\Rightarrow v_1 = \sqrt{4.2} \text{ ms}^{-1}$

32. (a) Using $v^2 = u^2 - 2as$,

$$(300)^2 = (400)^2 - 2a \times 0.1$$

$$\Rightarrow \quad a = 35 \times 10^4 \text{ ms}^{-2}$$

(b) $300 = 400 - (35 \times 10^4)t$

$$\Rightarrow \quad t = 2.86 \times 10^{-4} \text{ s}$$

33. (a) At overtaking the position of both cars would be same.

$$0 + \frac{1}{2} \times 3.5(t+1)^2 = 0 + \frac{1}{2} \times 4.9t^2$$

(b) $s = \frac{1}{2} \times 4.9 t^2 = 72.9 \text{ m}$

(c) $v_A = 3.5(t+1)$ and $v_A = 4.9t$

35. Let v be the required speed, then

$$vt = 20 + \frac{1}{2} \times 1t^2$$

$$\Rightarrow \quad t^2 - 2vt + 40 = 0$$

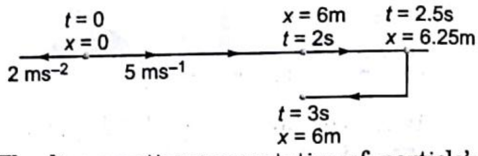
For t to be real, the roots of above equation must be real, *ie*,

$$\sqrt{(2v)^2 - 4 \times 40} = 0$$

$$\Rightarrow \quad v_{min} = 40 \text{ ms}^{-1}$$

36. Here, the particle comes to rest instantaneously *ie*, changing its direction of motion at $t = 2.5$ s, so distance travelled in 3rd sec can't be found by using

$$s_n = u + \frac{a}{2}(2n - 1)$$



The diagramatic representation of, particle's motion is shown above along with the values.

37. (a) As the two vehicles are moving in the same direction their relative velocity

$$= 72 - 54 = 18 \text{ km/h} = 5 \text{ m/s}$$

Now, as relative velocity = 5 m/s
and distance = 5 m + 5 m = 10 m
$\therefore$ time in which car B overtakes car A

$$= \frac{\text{relative distance}}{\text{relative velocity}} = \frac{10}{5} = 2\text{s}$$

(b) The total road distance used in overtaking process will be equal to the distance covered by the car B

Speed of car $B = 72 \times \frac{5}{18}$ m/s = 20 m/s

Now distance = $20 \times 2 = 40$ m

Total distance = distance covered + the length of the car

$$= 40 + 5 = 45 \text{ m}$$

39. Let $\vec{v}_B$ and $\vec{v}_C$ represent the velocities of the bus and the cyclist respectively.

If $\vec{v}_{BC}$ is the relative velocity of the bus *wrt* the cyclist, then

$$\vec{v}_{BC} = \vec{v}_B - \vec{v}_C \quad \text{...(i)}$$

Here, $\vec{v}_C = 20$ km/h (from the town A to B)

When the bus goes past the cyclist in the direction from the town A to B:

Since, $\vec{v}_B$, $\vec{v}_C$ and $\vec{v}_{BC}$ all are in same direction, from the Eq. (i), we have

$$\vec{v}_{BC} = \vec{v}_B - \vec{v}_C$$

As the bus plying from the town A to B goes past the cyclist after every 18 min, *ie*, $\frac{18}{60}$ h, the distance covered by a bus during this time interval

$$= (v_B - v_C) \times \frac{18}{60}$$

Since, a bus leaves in the either direction after every T min *ie*, $\frac{T}{60}$ h, we have

$$(v_B - v_C) \times \frac{18}{60} = v_B \times \frac{T}{60}$$

or $$v_B - v_C = \frac{v_B \times T}{18} \quad \text{...(ii)}$$

When the bus goes past the cyclist in the direction from town B to A.

Since, $\vec{v}_B$ and $\vec{v}_C$ are in opposite direction, from the Eq. (i), we have

$$v_{BC} = v_B - (-v_C) = v_B + v_C$$

As in this case, a bus goes past the cyclist after every 6 min *ie*, $\frac{6}{60}$ h, we have

$$(v_B + v_C) \times \frac{6}{60} = v_B \times \frac{T}{60}$$

or $$v_B + v_C = \frac{v_B \times T}{6} \quad \text{...(iii)}$$

Dividing the Eq. (iii) by Eq. (ii), we have

$$\frac{v_B + v_C}{v_B - v_C} = \frac{v_B \times T}{6} \times \frac{18}{v_B \times T} = 3$$

or $$v_B + v_C = 3v_B - 3v_C$$

or $$v_B = 2v_C = 2 \times 20 = 40 \text{ km/h}$$

Substituting for v_B and v_C in Eq. (ii), we have

$$40 - 20 = \frac{40 \times T}{18}$$

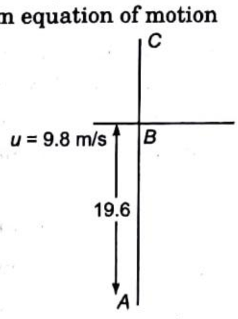
or $$T = \frac{20 \times 18}{40} = 9 \text{ min}$$

40. (a) Here, for upward motion

$$u = 9.8 \text{ ms}^{-1}$$

$v = 0$ (after which the ball reverses its direction)

From equation of motion



$$v = u + at$$

or $$0 = 9.8 - 9.8\,t$$

$\Rightarrow$ $$t = 1 \text{ s}$$

(b) Distance $AC = AB + BC$

$$BC = s_1 = ut_1 + \frac{1}{2}at_1^2$$

$$= 9.8 \times 1 + \frac{1}{2} \times (-9.8)(1)^2$$

$$= 9.8 - 4.9$$

$$= 4.9$$

$\therefore$ Total distance

$$AC = 4.9 + 19.6 = 24.5 \text{ m}$$

Now, again from

$$s = ut + \frac{1}{2}at^2$$

we get, $$24.5 = 0 + \frac{1}{2} \times 9.8\,t^2$$

$\Rightarrow$ $$t = \sqrt{5} \text{ s}$$

(c) Velocity of the ball after 3 s of its motion will be equal to the velocity of the ball after 2 s from position C as if covers distance from B to C in 1 s

$\therefore$ from $\quad v = u + at$

we get, $\quad v = 0 - 9.8 \times 2 = 19.6$ m/s

(d) The distance travelled by ball in 2 s = distance travelled by ball in 1 s after it has reached C + distance BC (s_1)

$\therefore$ $$s_2 = ut_2 + \frac{1}{2}at_2^2$$

$$= 0 + \frac{1}{2} \times 9.8\,(1)^2 = 4.9$$

Now, $s_1 + s_2 = 4.9 + 4.9 = 9.8$ m

(e) The final velocity $v = u + at$

$$= 0 + 9.8 \times \sqrt{5}$$

$$= 9.8 \times \sqrt{5} \text{ m/s}$$

43. The maximum height that the body thrown upwards be y.

Then according to equation of motion

$$v^2 - u^2 = 2\,as$$

$$0 - (4.9)^2 = -2 \times 9.8 \times s$$

$\Rightarrow$ $$s = 1.2 \text{ m}$$

Now, the relative velocity of two bodies will be $4.9 + 4.9 = 9.8$ m/s.

$\therefore$ Time after which they will meet $= \frac{1.2}{9.8} = \frac{1}{8}$ s

The distance covered by the body which is thrown upward in $\frac{1}{8}$ s $= 4.9 \times \frac{1}{8} - \frac{1}{2} \times 9.8 \left(\frac{1}{2}\right)^2$

Hence, the two bodies will meet 0.54 m from ground level.

44. Let velocity at B be v

Then for BC

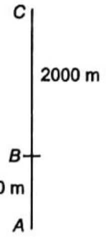
$$v^2 - (0)^2 = 2 \times 10 \times 1900$$

$\Rightarrow$ $$v = 195 \text{ m/s}$$

Now, for AB

$$100 = (195)\,t + \frac{1}{2} \times 10 \times t^2$$

$\Rightarrow$ $$t = 0.5 \text{ s}$$



47. If stone is thrown upwards with velocity u, then

$$h = -ut_1 + \frac{1}{2} g\, t_1^2$$

or $$1 = -5t_1 + \frac{1}{2} \times 9.8\, t_1^2$$

$\Rightarrow$ $$4.9\, t_1^2 - 5t_1 - 1 = 0$$

$\Rightarrow$ $$t_1 = 1.19 \text{ s}$$

Now, if a stone is dropped from height h, then

$$h = \frac{1}{2} gt^2$$

$\Rightarrow$ $$1 = \frac{1}{2} \times 9.8\, t^2$$

$\Rightarrow$ $$t = 0.45$$

Now, $$t_1 - t = 0.74 \text{ s}$$

50. Let the height of the building measured from the bottom of the window be s and body take time t to fall from top to bottom

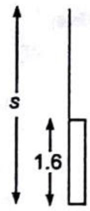
Then, $$s = \frac{1}{2} gt^2$$

In last 0.2 s body travels a distance of 1.6 m so in $(t - 0.2)$ s distance travelled $= (s - 1.6)$ m

$$(s - 1.6) = \frac{1}{2} g\,(t - 0.2)^2$$

$\Rightarrow$ $$t = 0.9 \text{ s}$$

$$s = 4.05$$



$\therefore$ Object falls from the height $(4.05 - 1.6)$ m

$= 2.45$ above the top of the window.

52. (a) As the slope or $\tan\theta$ for A is more hence its velocity will be more. Thus, A is moving faster.

(b) Both the graphs have the constant velocity. Hence, none of them are accelerating.

(c) From time axis we can see clearly that A starts its motion after t_1 s.

(d) At t_2 s the displacement (x_1) of A and B is same.

(e) The final destination is x_2 m. A reaches there first at t_3 s, B reaches at x_2 at t_4 s.

Chapter 4

Motion in a Plane

The First Steps' Learning

- Position and Displacement
- Velocity
- Acceleration
- Projectile Motion
- Projectile Motion from a Height
- Concept of Relative Velocity

In the preceding chapter we discussed about the motion of a particle moving along a straight line. However, there is an another type of motion—Motion in a Plane—which is more common viz., the motion of artificial satellites in circular and elliptical orbit around the earth, the motion of a ball thrown in the vertical plane, the motion of a simple pendulum, the motion of a particle along any two-dimensional curved path etc. The understanding of motion in two-dimensions will pave the way for understanding the motion in three dimensions. In one-dimensional motion, the vector nature of physical quantities such as displacement, velocity and acceleration was insignificant in analysis except for assigning positive and negative values to them. But, for the complete analysis of a two-dimensional motion, we require the expert working knowledge of vectors. In this chapter, we shall be extending our discussion of displacement, velocity and acceleration to two-dimensional motion followed by analysis of the projectile motion, a classic illustration of the motion in a plane, and finally we shall learn about the relative velocity concept.

Position and Displacement

If a particle moves 10 m along east and then takes a 90° left turn to move along north by another 20 m then what would be its final position *wrt* its initial location ? You can easily answer this by drawing two mutually perpendicular lines one along east-west direction and the other along north-south direction as shown in the figure.

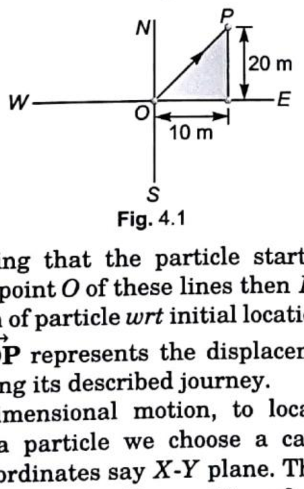


Fig. 4.1

Assuming that the particle starts from intersection point O of these lines then P is the final location of particle *wrt* initial location, and the vector $\vec{\mathbf{OP}}$ represents the displacement of particle during its described journey.

In 2-dimensional motion, to locate the position of a particle we choose a cartesian system of coordinates say X-Y plane. Then *wrt* reference point origin, the position of particle would be described. Consider any point $P(x, y)$ in the X-Y plane, the meaning of x and y are made very clear from Fig. 4.2 (a), it is something like the illustration mentioned above. X-axis could be considered as east-west line while Y-axis could be considered as north-south line, then x represents the x-component of location of particle and y represents the y-component. In vector form the position of a particle which is at point $P(x, y)$ at any time t can be expressed as, $\vec{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$ where $\vec{\mathbf{r}}$ is known as the position vector of point P and $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ are unit vectors along X and Y axes, respectively.

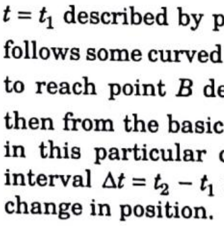


Fig. 4.2 (a)

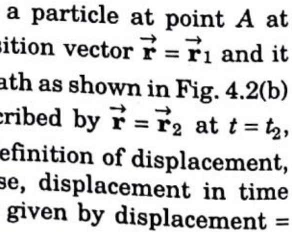


Fig. 4.2 (b)

Let us consider a particle at point A at $t = t_1$ described by position vector $\vec{\mathbf{r}} = \vec{\mathbf{r}}_1$ and it follows some curved path as shown in Fig. 4.2(b) to reach point B described by $\vec{\mathbf{r}} = \vec{\mathbf{r}}_2$ at $t = t_2$, then from the basic definition of displacement, in this particular case, displacement in time interval $\Delta t = t_2 - t_1$ is given by displacement = change in position.

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$$

In the Fig. 4.2 (b), $\Delta \vec{\mathbf{r}}$ (displacement) of particle is shown clearly. If you recall the definition of displacement which is as follows, "The vector joining the initial and final position," then you can easily correlate the things. In above case the direction of displacement is along the line AB (not BA).

The displacement in one-dimensional motion is always along the straight line, while in two-dimensional motion the displacement vector can lie anywhere in plane. However, the basic concept of displacement remains the same in any type of motion.

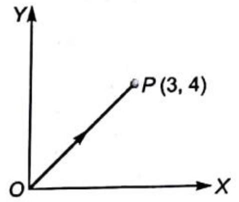
The path which the object follows *ie*, the path traced out by the particle is termed as its **trajectory**. As any curve in plane can be described by some equation, the equation describing the path traced by particle is termed as the **equation of trajectory**. In layman terms, the equation which relates x and y is termed as equation of trajectory, whereas (x, y) are the coordinates of the particle at any time.

C-BIs

Concept Building Illustrations

Illustration | 1 *A particle is at origin at $t = 0$ and it is observed at (3 m, 4 m) at $t = 3$ s in X-Y plane. Determine the position of particle wrt origin at $t = 3$ s.*

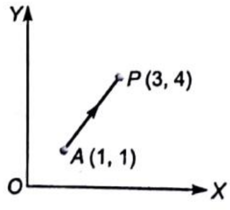
Solution The position of particle at $t = 3$ s *wrt* origin O is described by, $\vec{\mathbf{OP}} = \vec{\mathbf{r}} = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$



The situation is clearly shown in the figure.

Illustration | 2 *In above illustration, determine the position of particle at $t = 3$ s, wrt a point A whose coordinates are (1 m, 1 m).*

Solution The situation is clearly shown in figure as in the question position has been



asked *wrt* point A (1 m, 1 m) it means we have to find $\vec{\mathbf{AP}}$

$$\vec{\mathbf{AP}} = (3-1)\,\hat{\mathbf{i}} + (4-1)\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{AP}} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$$

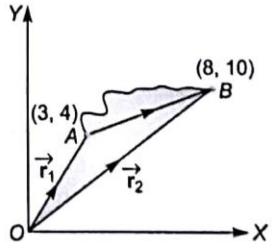
Illustration | 3 *A particle is moving along some unknown curved path in X-Y plane. Its position at $t = 0$ is described by $\vec{\mathbf{r}}_1 = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$ and at $t = 5$ s it is observed at $\vec{\mathbf{r}}_2 = 8\,\hat{\mathbf{i}} + 10\,\hat{\mathbf{j}}$.*

For this moving particle, determine

(a) the displacement of particle in 5 s interval measured from $t = 0$.

(b) the distance travelled by particle in above defined interval.

Solution Let particle moves from A to B in 5 s along some arbitrary curved path as shown in figure.



Displacement in 0 to 5 s duration is,

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\text{Displacement} = \vec{r}_2 - \vec{r}_1$$
$$= (8\hat{i} + 10\hat{j}) - (3\hat{i} + 4\hat{j})$$
$$= 5\hat{i} + 6\hat{j}$$

In this case we can't determine the distance travelled by particle in 5 s duration, as the path along which the particle is moving is not known and distance can be computed if we know the path followed by particle as distance is the actual path length.

Velocity

Just like we define average speed and average velocity for one dimensional motion, we can define the same for 2D motion also.

For the situation described in Fig. 4.2 (b), the average velocity for the time interval $\Delta t = t_2 - t_1$ is given by,

$$\vec{v}_{av} = \frac{\text{Displacement}}{\text{Time interval}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

The direction of $\vec{v}_{av}$ is along the displacement vector $\Delta \vec{r}$.

Average speed for the interval Δt is defined as,

$$v_{av} = \frac{\text{Total distance travelled}}{\text{Time interval}}$$

$$v_{av} = \frac{\text{Path length}}{\text{Time interval}}$$

In two dimensional motion, for any time interval distance is not equal to magnitude of displacement and hence average speed is not equal to magnitude of average velocity, while in one dimensional motion average speed may or may not be equal to magnitude of average velocity.

In **Chapter 3**, we define the instantaneous velocity for one dimensional motion, in similar manner, the instantaneous velocity in two dimensional motion is given by limiting value of $\frac{\Delta \vec{r}}{\Delta t}$ as Δt is approaches to zero.

Instantaneous velocity, $\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$ as $\Delta t \to 0$.

As in very small time Δt, we can safely assume that direction of motion doesn't change, so instantaneous speed is equal to magnitude of instantaneous velocity.

Let us consider that a particle moves in X-Y plane from A to B along the solid curve shown in figure. This curve can be considered as position-time curve of the particle, as at any particular instant position-vector of the particle can be known from this curve, although exactly it is the trajectory curve of the particle. Direction of average velocity for any time-interval and instantaneous velocity could be easily known from this curve.

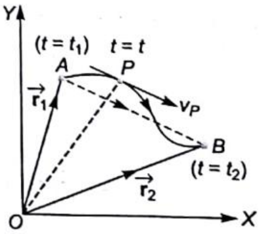


Fig. 4.3 (a) Dotted line OP represents position vector of particle at any time t

To determine the direction of average velocity for interval $t_2 - t_1$, join the two points on the curve as shown and then $\vec{v}_{av}$ would be along $\overrightarrow{AB}$ *ie*, along the displacement. The instantaneous velocity at any time t is along the tangent to the path of the particle at time t. In the diagram the direction of velocity at P is shown clearly. Here remember slope of tangent drawn to above curve at any point doesn't give the velocity at that point as here displacement and time are not along Y and X-axes, respectively. The curve described above is not exactly like position-time graph we discussed in **Chapter 3**, but is different in certain aspects. Below we mention a few of them. Let us consider above shown graph as the 1st and

position-time graph for a particle moving along a straight line as the 2nd.

Fig. 4.3 (b) Position *versus* time plot for a particle moving along a straight line.

1. In 1st, displacement vector is given by vector joining any two points on trajectory curve, while in the 2nd, displacement is computed by the projection of line joining any two points on curve on vertical axis *ie*, on position axis.
2. In 1st, the tangent to trajectory curve at any point gives the direction of velocity while in 2nd slope of tangent drawn to curve at any point gives the velocity.
3. In 1st, the direction of average velocity between any two instants is along the line joining the corresponding points on the trajectory curve while in the 2nd, slope of the line joining any two points on the curve gives the magnitude of average velocity between two corresponding instants.

For a particle moving in a plane, the velocity vector at any instant can be resolved into its components along X and Y-axes as shown in the figure.

$$\vec{\mathbf{v}} = v_x\,\hat{\mathbf{i}} + v_y\,\hat{\mathbf{j}}$$

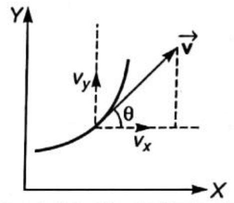


Fig. 4.4 Resolution of velocity vector particle moving in 2D.

The magnitude of velocity is given by $v = \sqrt{v_x^2 + v_y^2}$ and the angle which the velocity vector makes with the X-axis is given by

$$\tan\theta = \frac{v_y}{v_x}$$

C-BIs

Concept Building Illustrations

Illustration | 4 *A particle moves in X-Y plane along some curved path. At $t = 10$ s, the particle is at location described by position vector $\vec{\mathbf{r}}_1 = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$, after 10 s, the particle is observed at $\vec{\mathbf{r}}_2 = 15\,\hat{\mathbf{i}} - 6\,\hat{\mathbf{j}}$. Determine the average velocity of the particle for 10 to 20 s interval.*

Solution Average velocity is defined as,

$$\vec{\mathbf{v}}_{av} = \frac{\text{displacement}}{\text{time interval}}$$

For time interval, $t = 10$ s to 20 s, displacement is given by,

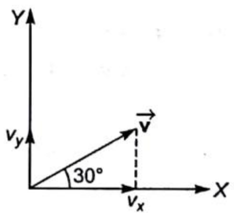
$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = (15\,\hat{\mathbf{i}} - 6\hat{\mathbf{j}}) - (3\,\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$$

$$= 12\,\hat{\mathbf{i}} - 10\hat{\mathbf{j}}$$

So, $$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

$$= \frac{12\,\hat{\mathbf{i}} - 10\hat{\mathbf{j}}}{20 - 10} = \frac{12\,\hat{\mathbf{i}} - 10\hat{\mathbf{j}}}{10} = (1.2\,\hat{\mathbf{i}} - \hat{\mathbf{j}})\ \text{ms}^{-1}$$

Illustration | 5 *For a particle moving in X-Y plane, at any instant velocity vector is making an angle of 30° with +ve X-axis. The magnitude of velocity at this instant is 10 ms^{-1}. Determine the X and Y components of its velocity at this instant.*



Solution The described situation is shown in figure.

$$v_x = v\cos 30°$$

and $$v_y = v\sin 30°$$

So, component of velocity along X-axis is,

$$v_x = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\ \text{ms}^{-1}$$

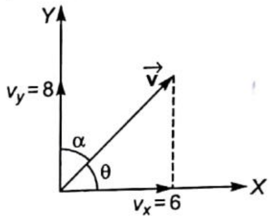
Component of velocity along Y-axis is,

$$v_y = 10 \times \frac{1}{2} = 5\ \text{ms}^{-1}$$

Illustration | 6 *The X and Y components of velocity of a particle moving in X-Y plane at any particular instant are 6 ms^{-1} and 8 ms^{-1} respectively. Determine the speed of particle and the angle which the velocity vector makes with positive direction of Y-axis at this instant.*

Solution Magnitude of velocity = speed

$$= v = \sqrt{v_x^2 + v_y^2} \Rightarrow v = \sqrt{6^2 + 8^2} = 10\ \text{ms}^{-1}$$



$$\tan\theta = \frac{v_y}{v_x} = \frac{8}{6} = \frac{4}{3}$$

So, required angle,

$$\alpha = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{4}{3}\right)$$

Acceleration

The instantaneous acceleration and average acceleration for a particle moving in a plane is defined just as it is for one dimensional motion.

$$\vec{\mathbf{a}}_{av} = \frac{\text{Change in velocity}}{\text{Time interval}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{t_2 - t_1}$$

Instantaneous acceleration, $\vec{\mathbf{a}}_{av} = \dfrac{\Delta\vec{\mathbf{v}}}{\Delta t}$ as $\Delta t \to 0$

The direction of acceleration is along the direction of change in velocity vector. Just like velocity, acceleration can also be resolved into its component along X and Y-axes.

C-BIs

Concept Building Illustrations

Illustration | 7 *If a particle is moving with velocity $\vec{\mathbf{v}}_1 = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ at $t = 2$ s and its velocity changes to $\vec{\mathbf{v}}_2 = 6\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ in a duration of 5 s ie, at $t = 7$ s the particle's velocity is $\vec{\mathbf{v}}_2$, then determine the average acceleration of particle for mentioned 5 s interval.*

Solution From $\vec{\mathbf{a}}_{av} = \dfrac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{t_2 - t_1}$

$$= \frac{(6\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) - (3\hat{\mathbf{i}} + 2\hat{\mathbf{j}})}{7 - 2}$$

$$= \frac{3\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}{5} = (0.6\hat{\mathbf{i}} - 1.2\hat{\mathbf{j}})\ \text{ms}^{-2}$$

Equations of motion in 2 dimensions

When a particle moves with constant acceleration in a plane, then its motion can be analysed by making use of concepts developed for straight line motion. Before developing the theoretical background of 2D motion, let us consider a situation in which a block is kept at origin of XY-coordinate system. Let us consider

A vector quantity is said to be constant when its direction and magnitude both remain constant *ie*, don't change with time.

only two constant*. Forces F_x and F_y are acting on the block as shown in figure. Now let us imagine that F_y is not

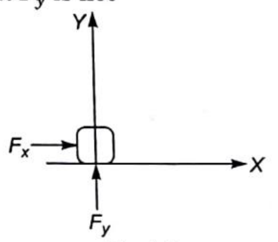


Fig. 4.5

acting on block, then what would be the motion of block ? We hope very easily you can answer, that it is going to move along X-axis with constant acceleration a_x. The force F_x causes acceleration a_x—means we can analyse the motion of the block in this situation with the help of theory developed in previous Chapter. Let x, v_x, u_x and a_x represent displacement of particle in X-direction, v_x be the X-component of velocity of particle at any time, u_x be the X-component of initial velocity and a_x be the X-component of acceleration. In present situation $x = u_x = 0$. Once we know the acceleration and initial velocity along X-axis we can analyse the X-component of motion completely by using the concepts of straight line motion.

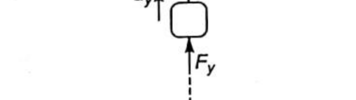


Fig. 4.6 Motion of the block under action of F_x alone. There is no motion in Y-direction as there is no acceleration in that direction.

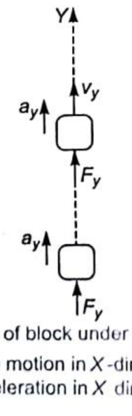


Fig. 4.7 Motion of block under the action of F_y alone. There is no motion in X-direction as there is no acceleration in X direction.

Now let us consider the motion of the block as if only F_y is acting and F_x is not acting on the block, then in this case easily you can predict that particle is going to move in a straight line along Y-axis. Just as discussed for X-axis, let the corresponding variables of Y-axis motion be y, v_y, u_y and a_y.

Now, it means we are able to analyse the motion of object along X and Y-axis independently when F_x and F_y are acting individually (as if other is not acting) in respective direction. But the actual situation which we have to analyse is, when both the forces F_x and F_y are acting simultaneously on the block. In this position, the resulting motion takes place in part along x-axis and in part along y-axis.

The force F_x causes the acceleration of block in x-direction *ie*, X-component of acceleration, while force F_y causes acceleration in Y-direction.

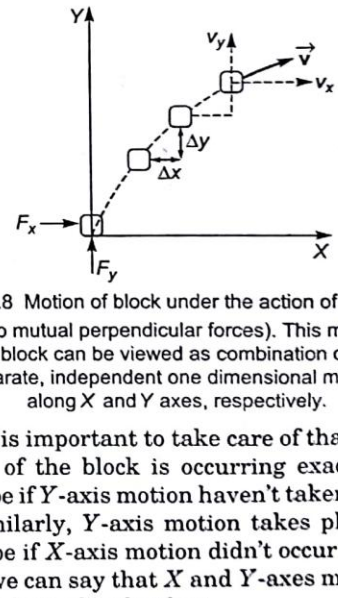


Fig. 4.8 Motion of block under the action of F_x and F_y (two mutual perpendicular forces). This motion of the block can be viewed as combination of two separate, independent one dimensional motions along X and Y axes, respectively.

It is important to take care of that, X-axis motion of the block is occurring exactly as it would be if Y-axis motion haven't taken place at all. Similarly, Y-axis motion takes place as it would be if X-axis motion didn't occur at all. In short, we can say that X and Y-axes motion are independent of each other.

So, we can conclude that motion in X-Y plane is a combination of two independent one dimensional motions—one along X-axis, and the other along Y-axis. In somewhat more general way, we can say that motion in a plane

is a combination of two independent one dimensional motions taking place in mutual perpendicular directions *ie*, two perpendicular 1D motions are independent of each other, this is because of the fact that the component of a vector in a direction perpendicular to the vector would be zero and hence motion along two mutual perpendicular directions are independent of each other.

The independence of two mutual perpendicular motions lies at the heart of 2 D motion. This fact only, allows us to resolve 2D motion, into two independent and distinct 1D motions.

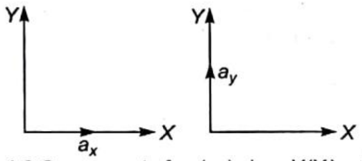


Fig. 4.9 Component of a_x (a_y) along Y(X) axis is zero, hence a_x (a_y) does not cause any motion along Y(X) axis.

How to analyse a 2D motion ?

Now the question arises, how to analyse the motion of a particle moving in a plane. To do so we will make use of above concept and here we are developing the concepts to analyse the motion in a plane. Let us consider a particle, moving with constant acceleration in XY plane, let us say the components of acceleration along X and Y axes are a_x and a_y, respectively, so that acceleration of particle can be written in component form as $\vec{a} = a_x \hat{i} + a_y \hat{j}$. Let at $t = 0$ *ie*, initially particle is at origin and its initial velocity at this instant is having magnitude u and its component along X and Y axes are u_x and u_y, respectively.

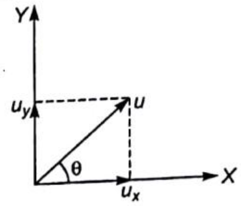


Fig. 4.10

Now we are providing you a step-by-step procedure to analyse the 2D motion.

1. Draw two perpendicular lines, call horizontal one as X-axis and the vertical one as Y-axis. The intersection point of these lines is termed as origin. Mark which direction (on X and Y-axes) has to be taken as +ve and opposite as −ve. Remember that +ve and −ve directions exist for both X and Y-axes.
2. Now write the initial parameters (velocity and position) and acceleration for X and Y directions, separately.

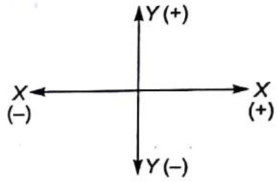


Fig. 4.11

For X-axis : Initial velocity $= u_x$; $x_0 = 0$;
acceleration $= a_x$
For Y-axis : Initial velocity $= u_y$; $y_0 = 0$;
acceleration $= a_y$
Remember that these parameters have to be written according to proper sign convention as in previous chapter.

3. Now write down the various parameters (position and velocity) for X and Y-axes as a function of time (with appropriate signs).

For X-axis motion	For Y-axis motion
$v_x = u_x + a_x \times t$	$v_y = u_y + a_y t$
$x = x_0 + u_x t + \frac{1}{2} a_x t^2$	$y = y_0 + u_y t + \frac{1}{2} a_y t^2$

Remember that the time variable t has the same value for X and Y part of the motion.

4. Now, from the known data (information given in the question) find the unknown values. Remember, the known data have to be substituted with appropriate signs and unknown physical quantities come out with their signs.

At the end of this step you are equipped with the details of X and Y motions separately,

resultant of which gives you the details of combined motion *ie*, motion in X-Y plane.

Initial velocity of particle,

$$\vec{u} = u_x \hat{i} + u_y \hat{j}$$

Initial position of particle,

$$\vec{r}_0 = x_0 \hat{i} + y_0 \hat{j}$$

Acceleration of particle,

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

Velocity of particle at any time t,

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$= (u_x + a_x t) \hat{i} + (u_y + a_y t) \hat{j}$$

$$= (u_x \hat{i} + u_y \hat{j}) + (a_x \hat{i} + a_y \hat{j}) t$$

$$\vec{v} = \vec{u} + \vec{a}\, t$$

Position of particle at any time t,

$$\vec{r} = x \hat{i} + y \hat{j}$$

$$\Rightarrow \quad \vec{r} = \vec{r}_0 + \vec{u}\, t + \frac{1}{2} \vec{a}\, t^2$$

If $\vec{r}_0 = 0$, then

$$\vec{r} = \vec{u}\, t + \frac{1}{2} \vec{a}\, t^2$$

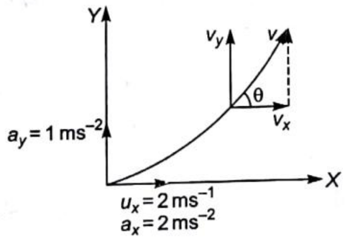
From last step it is clear the equations of motion for constant acceleration studied in previous chapter can be used for two-dimensional motion by using vectors.

C-BIs

Concept Building Illustrations

Illustration | 8 *A particle starts from origin with an initial velocity of* $2\ ms^{-1}$ *along x-axis and moves under the constant acceleration whose X-axis component is,* $a_x = 2\ ms^{-2}$ *and Y-axis component is* $a_y = 1\ ms^{-2}$*. Determine the velocity (magnitude and direction) of particle at* $t = 5$ *s.*

Solution The situation is shown clearly in the figure below :



For X-axis motion,

$$u_x = +2\ ms^{-1}; \quad a_x = 2\ ms^{-2}$$

$$v_x = u_x + a_x t$$

$$= 2 + 2 \times 5 = 12\ ms^{-1} \qquad [\because t = 5\ s]$$

For Y-axis motion,

$$u_y = 0; \quad a_y = 1\ ms^{-2}$$

$$v_y = u_y + a_y t$$

$$= 0 + 1 \times 5 = 5\ ms^{-1} \qquad [\because t = 5\ s]$$

Magnitude of velocity at $t = 5$ s is,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{144 + 25} = 13\ ms^{-1}$$

From figure, $\tan\theta = \dfrac{v_y}{v_x} = \dfrac{5}{12}$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{5}{12}\right]$$

So, at $t = 5$ s, velocity vector is making an angle of $\theta = \tan^{-1}\left[\frac{5}{12}\right]$ with X-axis and having magnitude $13\ ms^{-1}$.

Illustration | 9 *A particle starts with an initial velocity of* $2\ ms^{-1}$ *along X-axis and a constant acceleration of* $1\ ms^{-2}$ *along Y-axis. Assume that initially particle is at origin. Determine the position of particle at* $t = 3$ *s.*

Solution For X-axis motion,

$$u_x = 2\ ms^{-1}, \quad a_x = 0$$

$$x = u_x t = 2 \times 3 = 6\ m$$

For Y-axis motion,

$$u_y = 0; \quad a_y = 1\ ms^{-2}$$

$$\Rightarrow \quad y = \frac{1}{2} a_y t^2 = \frac{1}{2} \times 1 \times 3^2 = 4.5\ m$$

So, the position of particle at $t = 3$ s is,

$$\vec{r} = 6\hat{i} + 4.5\hat{j}$$

Projectile Motion

Projectile motion is one of the simplest illustrations of the motion in a plane. When a particle is projected near the surface of earth and it moves under the earth's gravity alone, then the particle's motion is said to be *projectile motion*, and the particle itself is generally referred to as projectile. In projectile motion we neglect or ignore the effect of **air friction*** and assume the acceleration due to gravity as constant [as usually the height of projectile from earth's surface is very small compared to the radius of earth].

Projectile motion can be analysed by using the theory and concepts developed in previous sections. Projectile motion can be considered as combination of two independent one dimensional motions, one along horizontal direction with constant velocity and the other along vertical direction under gravity effects.

Consider a projectile which has been projected from the ground with initial speed u at an angle θ with the horizontal as shown in figure, in this case the projectile is moving in a vertical plane under the effect of earth's gravity. The projectile motion can be analysed very easily by considering it as a combination of two one dimensional motions—one along the horizontal direction (say X-axis) with constant velocity and the other along vertical direction (say Y-axis) under the earth's gravity effect [Just like in preceding Chapter we discussed motion under gravity in vertical direction].

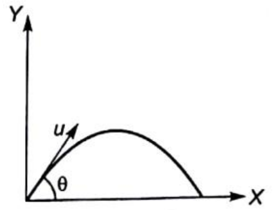


Fig. 4.12

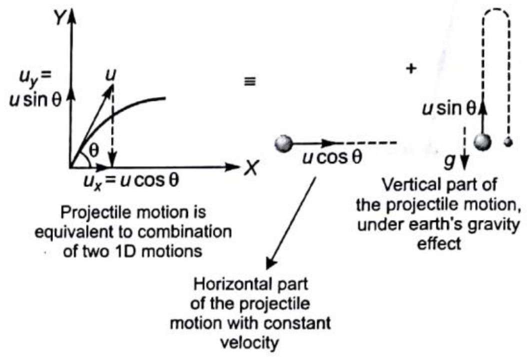


Fig. 4.13

The X and Y motion equation *ie*, horizontal and vertical motion equation can be written as

For horizontal direction motion (X-axis)

$$u_x = u\cos\theta; \quad a_x = 0$$

Component of velocity along horizontal direction at any time t is, $v_x = u_x$ *ie*, horizontal component of velocity remains constant and this is obvious as component of acceleration in horizontal direction is zero.

Displacement in horizontal direction at any time t is, $x = u_x t = u\cos\theta \times t$. So, the equations of motion for X-direction are

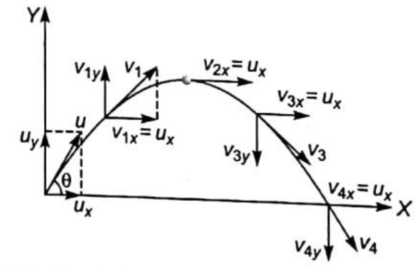


Fig. 4.14 Horizontal component of velocity during motion of a projectile is remaining constant, while vertical component is first decreases, becomes zero momentarily and then starts increasing in opposite direction.

$$v_x = u\cos\theta; \quad a_x = 0$$

and $$x = u\cos\theta \times t$$

* Air friction is the force experienced by moving objects due to air.

For vertical direction motion (*Y*-axis motion)

$$u_y = u \sin \theta; \quad a_y = -g$$

In acceleration –ve sign is there because vertical upward direction is considered as +ve and acceleration due to gravity is in vertical downward direction. Vertical component of velocity at any time t is,

$$v_y = u_y + a_y t$$

$$\Rightarrow \quad v_y = u \sin \theta - gt$$

The above expression is very similar to velocity of a particle at any time t which is thrown vertically upward and only earth's gravity effect is considered. The value of v_y first decreases and becomes zero momentarily, this is the instant when projectile is at its maximum height and then v_y starts increasing in opposite direction. Displacement in y direction at any time t is given by,

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad y = u \sin \theta t - \frac{1}{2} g t^2$$

So, the equations of motion in vertical directions are

$$u_y = u \sin \theta$$
$$a_y = -g$$
$$v_y = u \sin \theta - gt$$
$$y = u \sin \theta t - \frac{1}{2} g t^2$$

After combining these independent two one-dimensional motions, one along horizontal direction and the other along vertical direction we can have the detailed description of the projectile motion. Initially the projectile is moving up for some time until its vertical component of velocity gets zero, at this instant speed of the projectile is minimum and is equal to $u \cos \theta$ *ie*, the horizontal component of velocity for the duration for which projectile is moving up is termed as **time of ascent**. Once the projectile reaches its maximum height *ie*, at the instant when $v_y = 0$, the projectile starts coming down *ie*, it starts descending and reaches to the same level of projection, the duration for which the projectile descends is termed as **time of descent**.

Time of Flight

The total time for which the projectile is in motion is termed as **time of flight** (T). To compute the time of flight for a projectile motion, we can make use of equation $y = u \sin \theta \, t - \frac{gt^2}{2}$. When the projectile reaches the same horizontal level *ie*, when it reaches B, the vertical displacement of projectile is zero. So by substituting $y = 0$ in above equation, we will get the time when the projectile reaches B as measured from projected instant which is nothing but equal to time of flight.

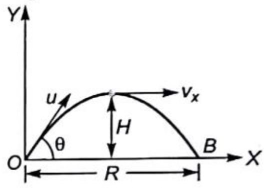


Fig. 4.15

By putting $y = 0$ in $y = u \sin \theta t - \frac{1}{2} g t^2$,

$$0 = \left(u \sin \theta - \frac{gt}{2} \right) t$$

$$\Rightarrow \quad t = 0 \text{ and } t = \frac{2u \sin \theta}{g}$$

$t = 0$, corresponds to launching time of projectile, as at this instant $y = 0$.

$t = T = \frac{2u \sin \theta}{g}$, corresponds to landing time of projectile *ie*, the time taken by projectile to reach B from O. This is equal to time of flight.

So, time of flight (T) $= \frac{2u \sin \theta}{g}$.

Be careful that in kinematics situations we can get two solutions, try to check the applicability and correctness of two solutions.

Time of Ascent

The time for which the projectile is ascending up is termed as **time of ascent** (t_a). The instant when the vertical component of projectile's velocity becomes zero is the end of ascending motion of projectile, so we can compute the value of time of ascent by substituting $v_y = 0$.

$$v_y = u\sin\theta - gt$$

$$\Rightarrow \quad 0 = u\sin\theta - gt$$

$$\Rightarrow \quad t = t_a = \frac{u\sin\theta}{g}$$

Time of ascent, $t_a = \dfrac{u\sin\theta}{g} = \dfrac{T}{2}$

At this instant *ie*, at $t = t_a$, projectile is at its maximum height from ground (level of projection).

Time of Descent

The time for which the projectile is descending down is termed as **time of descent** (t_d).

As $\quad T = t_a + t_d$

From above expressions,

$$t_d = T - t_a = T - \frac{T}{2} = \frac{T}{2} = t_a$$

ie, $\quad t_d = \dfrac{u\sin\theta}{g}$

It is clear that time of ascend is equal to time of descend and it can also easily be interpreted by symmetrical motion of projectile under earth's gravity effect.

Range

The horizontal displacement of projectile during its motion is termed as **range** (R) of the projectile. To compute the range of a projectile we can make use of equation, $x = u\cos\theta \times t$. In the mentioned equation if we substitute $t = T$, then we get the range of projectile. So,

$$R = u\cos\theta \times T = \frac{u\cos\theta \times 2u\sin\theta}{g}$$

$$R = \frac{u^2 \times 2\sin\theta\cos\theta}{g}$$

$$R = \frac{u^2\sin 2\theta}{g}$$

Maximum Height

The maximum value of vertical displacement of projectile during its course of motion is termed as the **maximum height** (H). The projectile reaches its maximum height when $v_y = 0$, so to compute the value of maximum height we have to substitute $t = t_a$ in vertical displacement equation.

So, $\quad H = y(t = t_a)$

$$= u\sin\theta \times t_a - \frac{1}{2}gt_a^2$$

$$= \left[u\sin\theta - \frac{g}{2} \times \frac{u\sin\theta}{g}\right]\frac{u\sin\theta}{g}$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

Equation of Trajectory

The equation (relation between x and y) which describes the path traced by projectile is termed as its equation of trajectory. As equation of trajectory is a relation between x and y, we can make use of equation $x = u\cos\theta \times t$ and $y = u\sin\theta \times t - \frac{1}{2}gt^2$ and by eliminating t from these equations we can get the required equation of trajectory.

From $\quad x = u\cos\theta \times t \quad \Rightarrow \quad t = \dfrac{x}{u\cos\theta}$.

Substitute this value of t in equation of y.

$$y = u\sin\theta \times \frac{x}{u\cos\theta} - \frac{g}{2}\left[\frac{x}{u\cos\theta}\right]^2$$

$$\Rightarrow \quad y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \quad \text{... equation of}$$

trajectory which represents a parabola.

Equation of trajectory can also be written as $y = x\tan\theta\left[1 - \dfrac{x}{R}\right]$ which is simply a rearrangement of

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\Rightarrow \quad y = x\tan\theta\left[1 - \frac{gx}{2u^2\sin\theta\cos\theta}\right]$$

$$y = x\tan\theta\left[1 - \frac{x}{\dfrac{u^2\sin 2\theta}{g}}\right]$$

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

Equation of trajectory is a very important tool to solve the questions in a clear and easy ways.

Now here we are providing you with some of the important facts related to the projectile motion.

Projectile Motion Fact Sheet

1. Horizontal component of the velocity of projectile always remains constant.
2. Speed of the projectile is minimum at the topmost post of projectile's flight and it is equal to $u_x = u\cos\theta$.
3. Time of ascent is equal to time of descent.
4. Range of a projectile for a given projection speed is maximum when projection angle $\theta = 45°$.

   From $R = \dfrac{u^2 \sin 2\theta}{g}$, for given u, R is depends upon θ and as maximum value of $\sin 2\theta = 1$ at $\theta = \pi/4 = 45°$, so R is maximum at $\theta = 45°$.
5. Range of projectile for angles of projection θ and $90° - \theta$ would be same, this is because of the fact that $\sin(180° - 2\theta) = \sin 2\theta$.
6. Acceleration of projectile, during its motion always remains constant.
7. Speed of the projectile first decreases, reaches a minimum value and then again increases to acquire the same value as of the projection speed.
8. When projectile reaches the ground back (*ie*, to same horizontal level), then projectile's velocity makes the same angle θ with the horizontal, as with which it has been projected as shown below.

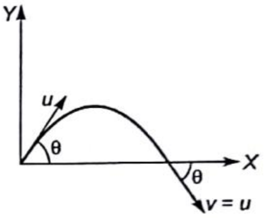


Fig. 4.16

9. The range of a projectile are same for angles of projection, $45° - \alpha$ and $45° + \alpha$.

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Illustration | 10 *A particle is projected from the surface of the earth with a speed of $20\ ms^{-1}$ at an angle $30°$ with the horizontal. Determine the*

(a) *time of flight*

(b) *range*

(c) *maximum height, and*

(d) *equation of trajectory for this projectile motion [Take $g = 10\ ms^{-2}$]*

Solution Here, it is given that

$$u = 20\ \text{ms}^{-1} \quad \text{and} \quad \theta = 30°$$

So, $$T = \frac{2u\sin\theta}{g} = \frac{2 \times 20 \times \sin 30°}{10} = 2\ \text{s}$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \times \sin 60°}{10} = 20\sqrt{3}\ \text{m}$$

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{(20)^2 \times (\sin 30°)^2}{2 \times 10} = 5\ \text{m}$$

Equation of trajectory is,

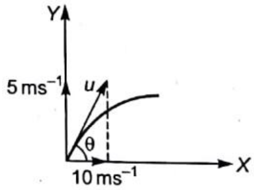
$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\Rightarrow \quad y = x\tan 30° - \frac{10 \times x^2}{2 \times (20)^2 \cos^2 30°}$$

$$\Rightarrow \quad y = \frac{x}{\sqrt{3}} - \frac{x^2}{60}$$

Illustration | 11 *A particle is projected from the surface of earth with a speed such that its horizontal and vertical components of the velocity are $10\ ms^{-1}$ and $5\ ms^{-1}$, respectively. Determine the angle which the particle makes with the vertical when it is projected.*

Solution The situation is shown clearly in the figure. Let θ be the angle of projection with horizontal, so the required angle is $\frac{\pi}{2} - \theta$.



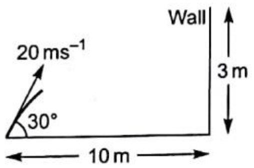
$$\tan\theta = \frac{u_y}{u_x} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Thus, the required angle *ie*, the angle which the projectile makes with the vertical is,

$$\alpha = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}(2)$$

Illustration | 12 *A particle is projected from the surface of earth with a velocity of* $20\ ms^{-1}$ *at an angle of* $30°$ *to the horizontal as shown in figure. A wall of height* 3 m *is there at a horizontal distance of* 10 m *from the point of projection. Find out whether the particle hits the wall or not.* [*Take* $g = 10\ ms^{-2}$]



Solution To proceed in this types of questions, find the value of vertical displacement y when horizontal displacement is $x = 10$ m (in present case). If value of y comes out to be less than height of wall then the projectile hits the wall otherwise not. This can be done by using trajectory equation or by using the equations of horizontal and vertical displacements.

(A) Trajectory equation method :

From equation of trajectory,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

For $x = 10$ m,

$$y = 10 \times \tan 30° - \frac{10 \times 10^2}{2 \times 20^2 \cos^2 30°}$$

$$\Rightarrow \quad y = \frac{10}{\sqrt{3}} - \frac{5}{3} = 4.107 \text{ m}$$

As value of y for $x = 10$ m is less than 3 m (height of wall), so the projectile won't hit the wall.

(B) Horizontal and vertical displacement method : Let particle's horizontal displacement is 10 m at any time t, then from equation $x = u\cos\theta \times t$

$$\Rightarrow \quad 10 = 20\cos 30° \times t$$

$$\Rightarrow \quad t = \frac{10}{20 \times \sqrt{3}/2} = \frac{1}{\sqrt{3}} \text{ s}$$

Let vertical displacement in this much time is y, then

$$y = u\sin\theta t - \frac{1}{2}gt^2$$

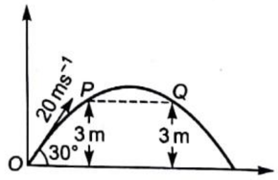
$$= 20 \times \frac{1}{2} \times \frac{1}{\sqrt{3}} - \frac{1}{2} \times 10 \times \frac{1}{3}$$

$$= \frac{10}{\sqrt{3}} - \frac{5}{3} = 4.107 \text{ m}$$

The same result as we get above.

Illustration | 13 *A particle is projected from ground with a velocity of* $20\ ms^{-1}$ *at an angle of* $30°$ *with the horizontal. Determine the time at which the vertical displacement of projectile is* 3 m. [*Take* $g = 10\ ms^{-2}$]

Solution Let the vertical displacement of projectile be 3 m at time t, then from



$$y = u\sin\theta\, t - \frac{1}{2}gt^2$$

$$\Rightarrow \quad 3 = 20 \times \sin 30° \times t - \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow \quad 3 = 20 \times \frac{1}{2} \times t - 5t^2$$

$$\Rightarrow \quad 5t^2 - 10t + 3 = 0$$

From theory of quadratic equations, the solutions of above equation are

$$t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 5 \times 3}}{2 \times 5}$$

$$= \frac{10 \pm \sqrt{40}}{10} = 0.37 \text{ s and } 1.63 \text{ s}$$

Here, two values of t are found, then you may have to decide which has to be our answer. The concept is the particle is having vertical displacement of 3 m at two different instants, one while going up and other while descending. Here $t = 0.37$ s corresponds to motion from O to P, while $t = 1.63$ s corresponds to motion from O to Q.

Untill now we have discussed the case when a projectile is projected from a point and at the end of motion it again comes to the same level, this type of projectile motion is termed as plane to plane projectile motion. Some other types of projectile motions are also there, which we mention as below :

- Projectile motion from a height.
- Projectile motion on an inclined plane.
- Projectile motion from moving vehicle.

Here, we shall discussion only the first one *ie*, projectile motion from a height and the remaining two types you will study later on.

Projectile Motion from a Height

In this case a particle is projected from some height and it moves under earth's gravity effect. The three different ways in which a particle is projected from a height have been shown below :

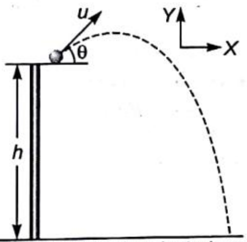


Fig. 4.17 (a) Upward projected projectile motion from a height

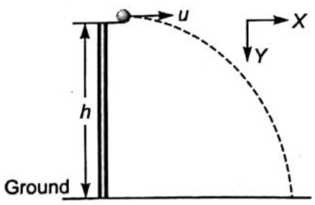


Fig. 4.17 (b) Horizontal projectile motion from a height

Along with the diagrams, we have also shown the +ve X and +ve Y directions which we will consider for writing the equation. In each of the situation we are considering the point of projection as the origin. Here we are providing you only the equation of motion for horizontal and vertical motions, and whatever is required to determine we can find by using these equations.

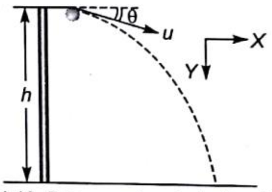


Fig. 4.18 Downward projected projectile motion from a height

Horizontal projectile motion from a height :

Horizontal Motion (X)	Vertical Motion (Y)
$u_x = u$	$u_y = 0$
$a_x = 0$	$a_y = g$
$v_x = u$	$v_y = gt$
$x = ut$	$y = \frac{gt^2}{2}$

Upward projected projectile motion from a height :

Horizontal Motion (X)	Vertical Motion (Y)
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$
$v_x = u\cos\theta$	$v_y = u\sin\theta - gt$
$x = u\cos\theta \times t$	$y = u\sin\theta \times t - \frac{gt^2}{2}$

Downward projected projectile motion from a height :

Horizontal Motion (X)	Vertical Motion (Y)
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = g$
$v_x = u\cos\theta$	$v_y = u\sin\theta + gt$
$x = u\cos\theta \times t$	$y = u\sin\theta \times t + \frac{gt^2}{2}$

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Illustration | 14 *A particle is projected horizontally from the top of a tower of height 19.6 m, with a speed of 10 ms^{-1} as shown in the figure. Determine*

(a) *the time in which projectile reaches the ground.*

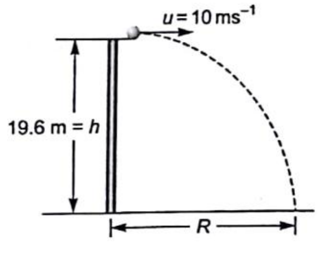
(b) *the horizontal displacement of projectile when it lands on the ground.*

(c) *the speed of the projectile when it lands on ground.*

(d) *the angle made by velocity vector of projectile with the horizontal when it reaches the ground.*

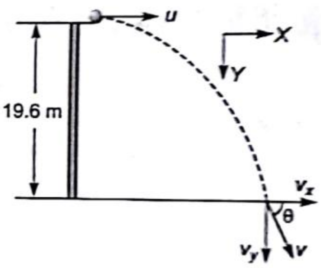
(e) *the instant at which both horizontal and vertical components of velocity have the same numerical value.*

[Take $g = 9.8$ ms^{-2}]



Solution (a) Let particle takes time t to reach the ground, then for vertical motion equation

$y = \frac{1}{2}gt^2$ [Choice of positive X and Y directions are shown in figure.]



When the particle reaches the ground, its vertical displacement is 19.6 m, so from above equation

$$19.6 = \frac{1}{2} \times 9.8 \times t^2$$

$$\Rightarrow \quad t = T = 2\text{ s}$$

(b) The horizontal displacement of projectile during time t can be computed by using $x = ut$.

$\Rightarrow$ Horizontal displacement

$$= R = u \times T$$
$$= 10 \times 2 = 20\text{ m}$$

(c) The horizontal component of velocity of projectile is constant, so v_x at $t = 2$ s is 10 ms^{-1}.

Vertical component of velocity at any time t is given by, $v_y = gt$. So, v_y at $t = T = 2$ s is, $v_y = 9.8 \times 2 = 19.6$ ms^{-1}.

So, speed of particle when it reaches ground is,

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\Rightarrow \quad v = \sqrt{19.6^2 + 10^2} \approx 22\text{ ms}^{-1}$$

(d) The angle which the velocity vector of projectile makes with the horizontal when it reaches the ground is given by,

$$\therefore \quad \tan\theta = \frac{v_y}{v_x}$$

$$\Rightarrow \quad \tan\theta = \frac{19.6}{10}$$

$$\Rightarrow \quad \theta = \tan^{-1}(1.96)$$

(e) Let at time t, $v_x = v_y$

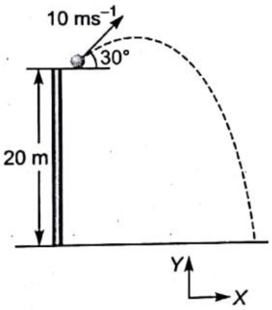
as v_x is always equal to 10 ms^{-1} and v_y at any time t is given by $v_y = gt$, so to compute time t when $v_x = v_y$

$$10 = gt$$

$$\Rightarrow \quad t = \frac{10}{9.8} = 1.02\,\text{s}$$

Illustration | 15 *A particle is projected from a height as shown in figure. Determine the time in which the particle reaches the ground.*

[Take $g = 10\ ms^{-2}$]



Solution Let us consider point of projection as origin, vertical upward direction as +ve Y-axis and horizontally right direction as +ve X-axis as shown in figure. Equation for vertical displacement at any time t is given by

$$y = u\sin\theta t - \frac{1}{2}gt^2$$

where symbols have their usual meanings.

Let T be the time taken by projectile to reach ground. When the particle reaches the ground, its vertical displacement is $y = -20$ m, take care of sign [as vertical downward direction is –ve and vertical displacement in time T is 20 m in downward direction, so $y = -20$ m]. From vertical displacement equation,

$$-20 = 10\sin 30° \times T - \frac{1}{2} \times 10 \times T^2$$

$$\Rightarrow \quad 5T^2 - 5T - 20 = 0$$

$$\Rightarrow \quad T^2 - T - 4 = 0$$

$$\text{So,} \quad T = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times (-4) \times 1}}{2 \times 1}$$

$$\Rightarrow \quad T = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

$$= -1.56\text{ s and } 2.56\text{ s}$$

As negative time is not an appropriate solution as particle is projected at $t = 0$ so, the required time is $T = 2.56$ s.

Illustration | 16 *In illustration 15, determine the vertical displacement of projectile after 2 s of its projection. Assume the +ve X and Y directions as mentioned in solution of illustration 15.*

Solution Let y be the vertical displacement of projectile at $t = 2$ s, then by using $y = u\sin\theta\, t - \frac{gt^2}{2}$, we can find out y.

$$y = 10 \times \frac{1}{2} \times 2 - \frac{1}{2} \times 10 \times 2^2 = -10\text{ m}.$$

Negative sign tells that the particle is below the plane of projection.

–Remember unknown quantities come out with appropriate signs.

Concept of Relative Velocity

Suppose you are travelling in a train and another train moving in the same direction as the train in which you are, is overtaking your train on a parallel track, and you are peeping out for the overtaking train from the window, then you may be able to see all the

compartments of the other train very clearly, you may be able to read name of the train, coach number etc. But if the same train is moving on a direction opposite to that of yours on same parallel track, then hardly you would be able to count the number of coaches. This is because of the fact that in first case when the trains are moving in same direction the other train seems to be moving slower to you even though it is moving faster than yours train *wrt* some observer on ground, while in second case *ie*, when trains are moving in opposite directions, then the other train seems to be moving faster to you even though it is moving slower than your's train *wrt* some stationary observer on ground. You may have also observed this, that when your train is standing on the platform and on a parallel track other train is moving, then you feel that your train is moving. To explain and make you understand the concept behind these observations, we will make use of the concept of relative velocity.

The detailed explanation and derivation of key equation of relative velocity concept required the usage of **calculus**, so it is not possible for us to provide the derivation of this key equation here, but to maintain the continuity and completeness of the concepts, here we are providing you only the key equation and the meaning of each term appearing in this equation.

Calculus is a vast field, which you will study later on.

The only and most important equation in the concept of relative velocity is, $\vec{\mathbf{v}}_{PS} = \vec{\mathbf{v}}_{PS'} + \vec{\mathbf{v}}_{S'S}$ which is read as, "velocity of particle P *wrt* frame of reference S is equal to vector sum of velocity of particle P *wrt* frame of reference S' and velocity of frame of reference S' *wrt* frame of reference S", in somewhat simpler words to remember—"Velocity of P *wrt* S is equal to velocity of P *wrt* S' plus velocity of S' *wrt* S".

The two points you have to keep in mind while using relative velocity equation—I relative velocity equation is a vector equation and not simply an algebraic equation, and II the relative velocity equation is valid only when objects are moving with speed much less than speed of light.

Let us take few cases (of one dimensional motion) which make the concept of relative velocity crystal clear. Consider a person A who is running on a flat rail road car with a velocity of 3 ms^{-1} *wrt* car, and the car in turn is moving with a velocity of 2 ms^{-1} *wrt* ground as shown in figure. If we are interested in finding the velocity of person *wrt* ground then we can make use of relative velocity equation.

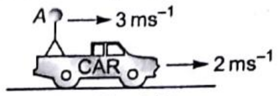


Fig. 4.19

To find $\vec{\mathbf{v}}_{\text{Preson, ground}}$ [Velocity of person *wrt* ground]

Given $\vec{\mathbf{v}}_{\text{Person, car}}$ [Velocity of person *wrt* car]

and $\vec{\mathbf{v}}_{\text{Car, ground}}$ [Velocity of car *wrt* ground]

If we compare the subscripts of above three velocities, with the subscripts of relative velocity equation $\vec{\mathbf{v}}_{PS} = \vec{\mathbf{v}}_{PS'} + \vec{\mathbf{v}}_{S'S}$, then we can consider

P as person
S as ground
S' as car

So, $\vec{\mathbf{v}}_{\text{Person, ground}} = \vec{\mathbf{v}}_{\text{Person, car}} + \vec{\mathbf{v}}_{\text{Car, ground}}$

As it is a one dimensional motion, we can avoid use of vector algebra, by simply assigning one direction as positive and other as –ve. Let in present case we are taking rightward as +ve, then $v_{\text{Person, ground}} = 2 + 3 = 5\ \text{ms}^{-1}$ towards right.

Let us consider another example, when two persons are approaching each other with velocities v_A and v_B (both the velocity have to be measured *wrt* same frame of reference, in present case these are *wrt* ground), then the relative velocity of A *wrt* B is given by,

Fig. 4.20

$$v_{AB} = v_A + v_B \qquad \text{(towards right)}$$

Velocity of B *wrt* A in above case is,

$$v_{BA} = v_B + v_A \qquad \text{[towards left]}$$
$$= -(v_B + v_A)$$

The above expressions have been found by using relative velocity equation only. The concept of relative velocity concept can also be interpreted in other ways also, like if two objects A and B are moving (both with ground frame of reference) in same direction with some separation between them with A ahead of B, as shown in figure. Now we want to find the time in which B catches up with A.

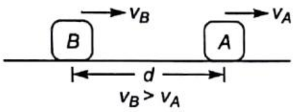


Fig. 4.21

You can easily solve this situation by using the concepts you know till now, but still for the comparison we are providing the solution here.

1st method : Let B catches A in time t at X as shown in the figure. Then in this time A will travel a distance of $QX = v_A t$ while B will travel a distance of $PX = v_B t$.

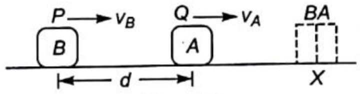


Fig. 4.22

From the situation shown in figure,

$$v_B t = d + v_A t \;\Rightarrow\; t = \frac{d}{v_B - v_A}$$

2st method : Now we are solving the above question with the help of relative velocity concept. Let us first find velocity of B relative to A, $v_{BA} = v_B - v_A$. Initially, the relative separation between the two objects is d and finally when B catches A, the relative separation is zero. If t is the required time, then it means the relative separation reduces from d to O in time t,

So, $\quad (v_B - v_A)\,t = d$

$$\Rightarrow \qquad t = \frac{d}{v_B - v_A}$$

The above situation can be interpreted as that A has been stopped and B is approaching it with velocity v_{BA}.

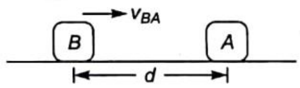
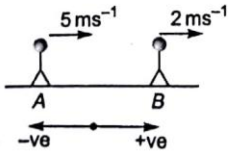


Fig. 4.23

C-BIs

Concept Building Illustrations

Illustration | 17 *Two persons A and B are moving with velocities of* 5 ms^{-1} *and* 2 ms^{-1} *respectively in the directions as shown in figure. [Both velocities are mentioned wrt ground frame of reference]. Positive and negative directions are also shown in the figure. Determine*



(a) *the velocity of A wrt B.*

(b) *the velocity of B wrt A.*

Solution $v_{AB} = v_A - v_B$

[From $\vec{v}_{AB} = \vec{v}_{AG} + \vec{v}_{GB}$

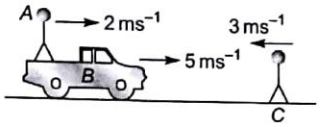
$\Rightarrow \qquad \vec{v}_{AB} = \vec{v}_{AG} - \vec{v}_{BG}$]

as $\qquad \vec{v}_{BG} = -\vec{v}_{GB}$

$\Rightarrow \qquad v_{AB} = 5 - 2 = 3\ \text{ms}^{-1}$

$$v_{BA} = v_B - v_A = 2 - 5$$
$$= -3\ \text{ms}^{-1}$$

Illustration | 18 *A person A is running on a car with a velocity of 2 ms^{-1} (wrt car) which in turn is running with a velocity of 5 ms^{-1} wrt ground. Another person C is approaching the car with a velocity of 3 ms^{-1} wrt ground. Determine the velocity of A wrt C.*



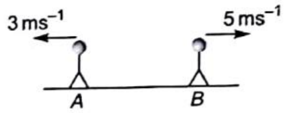
Solution $\vec{v}_{AC} = \vec{v}_{AG} - \vec{v}_{CG}$

$\vec{v}_{AG} = \vec{v}_{AB} + \vec{v}_{BG}$

Now, using right side as +ve and leftward as –ve,

$$v_{AG} = 2 + 5 = 7 \text{ ms}^{-1}$$
$$v_{AC} = 7 - (-3) = 10 \text{ ms}^{-1}$$

Illustration | 19 *Two persons A and B are receding away from each other, as shown in the figure. Determine the velocity of A wrt B. [Take leftward as –ve.]*



Both velocity are mentioned wrt ground.

Solution From $\vec{v}_{AB} = \vec{v}_{AG} - \vec{v}_{BG}$

$\Rightarrow$ $$v_{AB} = (-3) - (5) = -8 \text{ ms}^{-1}$$

Negative sign tells that relative velocity is towards left.

Here we have given you a brief (underived) view of relative velocity concept and illustrated the applications in one dimensional motion. But major application of relative velocity concept comes in the 2 dimensional motion, which we are not discussing here and leaving it for you to study in your later classes.

However, an inquisitive mind and labourious student can make proper use of underived most important equation $\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$ in two dimensional motion by himself/herself.

Motion of an Object when Dropped from a Moving Vehicle

When we drop some object from a height we say that the initial velocity of the object is zero, this is the case which we already discussed in the previous chapter. But what happens when object is dropped from a moving vehicle—say from a moving train or car, is still the initial velocity zero ! Is it zero *wrt* ground or *wrt* the moving vehicle ? What about the acceleration of the dropped body after releasing it ? Questions like this and many more may come to your mind in such a solution, here we are trying to give the answers of your curious questions.

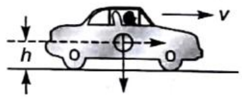


Fig. 4.25

Let us consider you are in a car which is moving on a straight horizontal highway, with a constant velocity of v ms^{-1}, and then you drop a ball out from the car as shown. Let us assume that height of the ball at the time of dropping *wrt* ground level is h. As the ball is dropped its velocity *wrt* you is zero, but due to car velocity and yours, the ball acquires an additional velocity v if we see the things from ground frame of reference. So the velocity of the car from ground frame of reference is v in horizontal direction. Now, if we consider the motion of ball from ground frame of reference, it is moving under earth's gravity effect as shown in figure. As soon as the ball leaves your hand, the only force

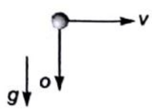


Fig. 4.26

acting on ball is the gravity force and hence its acceleration would be equal to acceleration due to gravity. So this situation is equivalent to a horizontal projectile motion from a height and hence we can use the similar equation. Let t be the time taken by ball to reach ground, then

$$h = \frac{1}{2} g t^2$$

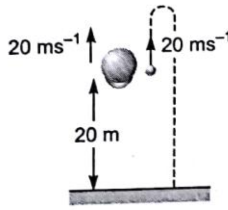
$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

Another interesting fact in this particular situation is that the ball always remains below your hand during its course of motion as the ball's horizontal velocity and your hand's horizontal velocity are the same, so in any time-interval, the ball and your hand travel the same horizontal distances.

C-BIs

Concept Building Illustrations

Illustration | 20 *An hot air balloon is moving up with a constant velocity of* $20\ ms^{-1}$*. When balloon is at a height of* $20\ m$ *from ground, a stone is dropped from it. Determine the time taken by stone to reach the ground.*
[Take $g = 10\ ms^{-2}$*]*



Solution As the stone is dropped from a moving balloon, it acquires the same velocity as the balloon has at the time of dropping the stone. Afterwards it moves under gravity effect. Let the required time be t, consider the upward direction as +ve and downward direction as –ve. At time t, the displacement of stone is –20 m and initial velocity is $+20\ ms^{-1}$. Using displacement equation,

$$-20 = 20t - \frac{1}{2} \times 10t^2$$

$$\Rightarrow \quad 5t^2 - 20t - 20 = 0$$

$$\Rightarrow \quad t^2 - 4t - 4 = 0$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-4)}}{2}$$

$$= \frac{4 \pm \sqrt{32}}{2} = \frac{4 \pm 4\sqrt{2}}{2}$$

$$= 4.83 \text{ s and } -0.83 \text{ s}$$

So, the answer is 4.83 s.

Towards Proficiency Problems
Exercise 1

A. Subjective Discussions

1. A projectile is thrown at an angle θ with horizontal. Is there any instant when velocity and acceleration of projectile are perpendicular to each other ? If it is then when and where ? Is there any point on the trajectory, where the acceleration and velocity are parallel or antiparallel ? If so, where ? Discuss the situation in detail.
2. From a height h above the ground, a ball is projected horizontally with speed v and at the same time another ball is dropped from same point. Which ball reaches the ground first ? Which ball is having higher speed while reaching the ground ?
3. Is it possible to have constant speed, yet a non-zero acceleration ? Is it possible to have constant acceleration and varying speed ?
4. There are two initial velocities for which a given projectile has same range ? How many initial velocities are there to have same maximum height ? The same time of flight ? [Velocity means include both magnitude and direction].
5. You are running in the rain. At what angle you should hold the umbrella to protect yourself from the rain ?
6. Suppose you run at constant speed and wish to throw a ball so that you can catch it as it comes back down. In which direction should you throw the ball ?
7. If you know the position vectors of a particle at two points along its path and also know the time it took to move from one point to another, can you determine the particle's instantaneous velocity. Its average velocity ? Explain.
8. Two projectiles are thrown with the same magnitude of initial velocity, one at an angle θ from ground and other at $90° - \theta$. Both projectiles will strike the ground at the same distance from the projection point. Will both projectiles be in the air for same time interval ?
9. An horizontally moving plane drops a bomb. If the plane is moving with constant velocity then the bomb would be always vertically below the plane. Comment on this statement.
10. A projectile is launched from ground with some initial velocity making an angle θ with the horizontal. Is the projectile a freely falling body for its entire duration ? What is the acceleration of projectile in vertical direction ? What is its acceleration in the horizontal direction ?
11. When a rifle is fired at a distant target, the barrel is not exactly lined up on the target. Why ? Does the angle of correction depends upon the distance of target ?

B. Numerical Answer Types

1. A particle starts from origin with an initial velocity of 5 ms^{-1} along positive Y-axis, under a constant acceleration whose X and Y components are 2 ms^{-2} and 1 ms^{-2}, respectively. Determine the velocity (magnitude and direction both) of the particle at $t = 4$ s.

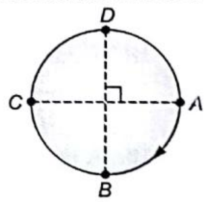
2. A particle is moving in X-Y plane, which is initially at $x = y = 2$ m. The particle's initial velocity is 2 ms^{-1} along +ve X-axis and acceleration is 1 ms^{-2} along +ve Y-axis. Determine the position of particle at $t = 5$ s.

3. A particle is moving along a curve in X-Y plane. At $t = 0$ its position is given by coordinate (3, 4) and at $t = 5$ s it is at (6, 3). Determine the displacement vector and the magnitude of displacement for this 5 s duration.

4. Take east-west line as X-axis and north-south line as Y-axis, (consider east and north as +ve X and +ve Y, respectively). If you first move 10 m towards north, then turn by 90° and move 10 m towards east, then 5 m along south, and then again 10 m along west, what would be the displacement (magnitude and direction both) for this motion of yours ?

5. Consider the XY as mentioned in previous question. A particle starts moving from origin in such a way that first it moves along east for 10 m, then takes 90° left turn and moves another 20 m, then takes 90° right turn and after moving for 10 m, it turns by 45° left to cover another 10 m. If the time taken by the particle for entire motion is 50 s, then determine the
(a) final position of particle.
(b) displacement of particle for this 50 s duration.
(c) average velocity of particle for this 50 s duration.
(d) average speed of particle for this 50 s duration.

6. A particle moves along a circle of radius 10 m with constant speed. The particle is revolving in clockwise direction. If the particle takes 5 s to cover one quarter of circle, then determine the magnitude of the
(a) displacement in first 5 s.
(b) displacement in first 10 s.
(c) displacement in first 15 s.
(d) average velocity in first 5 s.
(e) average velocity in first 10 s.
(f) average velocity in first 15 s.
(g) average speed in first 5 s.
(h) average speed in first 10 s.
(i) average speed in first 15 s.
(j) displacement in first 20 s.
(k) average velocity in first 20 s.
(l) average speed in first 20 s.



7. A particle's velocity changes from 10 ms^{-1} north to 20 ms^{-1} south in a time-interval of 5 s. Find the average acceleration (magnitude and direction both) of the particle for this 5 s duration.

8. A particle's velocity changes from 10 ms^{-1} north to 20 ms^{-1} east in a time interval of 5 s. Determine the average acceleration (magnitude and direction both) of the particle for this 5 s duration.

9. A car runs 85 m due south in 17 s, it starts from rest and stops momentarily (for a negligible time) at the end of the run. Then the car starts again and runs due east by 63 m in 21 s. During the second run the acceleration of the car is constant. For this entire 38 s duration, determine the car's (a) average velocity (b) average acceleration.

10. A particle is moving with a velocity of 3 ms^{-1} along +ve X-axis. As it crosses origin, a constant acceleration of 2 ms^{-2} starts acting on particle in +ve Y-axis. Consider this instant as $t = 0$. Determine
(a) the velocity of particle at $t = 3$ s.
(b) the position of particle at $t = 5$ s.
(c) the instant at which X and Y components of velocity have same numerical value ?
(d) the angle made by velocity vector with X-axis at $t = 3$ s.

11. A particle starts from rest from origin and moves along +ve X-axis under the action of an acceleration 2 ms^{-2}. At $t = 10$ s the acceleration remains same in magnitude but changes its direction to −ve X-axis. Determine
(a) the velocity of particle at $t = 15$ s.
(b) the position of particle at $t = 15$ s.
(c) the average velocity of particle for the interval 5 s to 15 s.

12. A particle is moving in X-Y plane, which crosses origin at $t = 0$ with a velocity of $10\ \text{ms}^{-1}$ along +ve X-axis. A constant acceleration, whose X and Y components are $-2\ \text{ms}^{-2}$ and $1\ \text{ms}^{-2}$ respectively is acting on particle. Determine
(a) the time when velocity vector becomes parallel to Y-axis.
(b) the position of the particle in above situation.
(c) the time when the particle crosses Y-axis.
(d) the speed of the particle when it crosses Y-axis.

13. A footballer kicks a football at an angle of 30° above the horizontal. Because of the kick, the football attains a speed of $20\ \text{ms}^{-1}$. Determine the maximum height achieved by the football. [Take $g = 10\ \text{ms}^{-2}$]

14. In above question, determine the time for which the football remains in air before it strikes the ground first.

15. A plane is moving at a speed of $250\ \text{ms}^{-1}$. If the vertical component of its velocity at any instant be $200\ \text{ms}^{-1}$, then determine the horizontal component of its velocity and the angle made by the velocity vector of plane with the horizontal.

16. The vertical component of a particle is $6.8\ \text{ms}^{-1}$, at the same time, the shadow of the particle is moving along the ground at a speed of $15.5\ \text{ms}^{-1}$ when the sun is directly overhead. Find the magnitude of the particle's velocity at this instant.

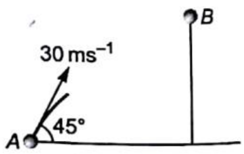
17. A particle is projected with a speed u at an angle α with the horizontal. Determine the speed of the projectile when its velocity vector makes an angle of β with the horizontal.

18. A particle is projected with a speed of $20\ \text{ms}^{-1}$ at an angle of 60° with the horizontal. Determine the instant(s) when the velocity vector of projectile is making an angle of 30° with the horizontal. [$g = 10\ \text{ms}^{-2}$]

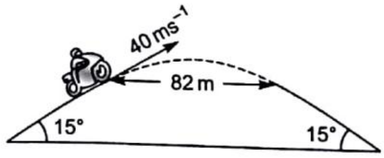
19. A projectile is launched from ground and returns to ground level. The horizontal range of the projectile is $R = 175$ m. If the horizontal component of projectile's velocity at any instant be $25\ \text{ms}^{-1}$, then determine the vertical component of the launch speed and the launching angle *wrt* the horizontal. [Take $g = 10\ \text{ms}^{-2}$]

20. In previous question, determine the time of flight of the projectile.

21. A ball A is projected from ground with a speed of $30\ \text{ms}^{-1}$ at an angle of 45° with the horizontal so that it can hit point B. When the ball hits point B, it ball is moving horizontally. Determine the time taken by the ball to reach B and the shortest distance between the point of projection and point B. [Take $g = 10\ \text{ms}^{-2}$]



22. A motorcyclist moving at a speed of $40\ \text{ms}^{-1}$ leaves the ramp as shown in figure. The horizontal separation between two ramps is 82 m. Will the motorcyclist safely reaches on the other ramp? Take $g = 10\ \text{ms}^{-2}$ and neglect the length of motorcycle.

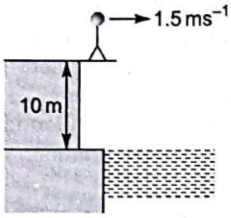


23. A projectile is projected from ground at a speed of $30\ \text{ms}^{-1}$ at an angle of 30° with the horizontal. At what instant the vertical displacement of projectile is 5 m ? Interpret the answer. [Take $g = 10\ \text{ms}^{-2}$]

24. A projectile projected with a speed of 25 ms^{-1} at an angle of 30° with the horizontal, is just able to clear two hurdles of heights 5 m each. For this situation determine the
(a) time taken by projectile to cross first hurdle as measured from instant of projection.
(b) time interval in which the projectile passes two hurdles is in between two hurdles.
(c) the horizontal separation between two hurdles. [Take $g = 10\ ms^{-2}$]

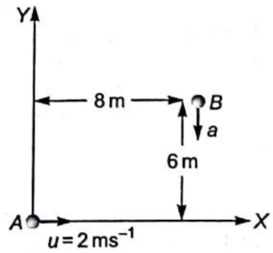
25. A volleyball is hit so that it has an initial velocity of 15 ms^{-1}, directed downward at an angle of 30° below the horizontal. Determine the horizontal component of the ball's velocity when it has been received by other player after 3 s of its hitting.

26. A swimmer dives off horizontally from a platform as shown in figure. Determine

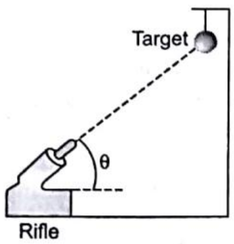


(a) the time taken by swimmer to reach water.
(b) the velocity of swimmer just before striking the water surface.
(c) the horizontal displacement of swimmer during its motion in air. [Take $g = 10\ ms^{-2}$]

27. The position of two particles A and B are as shown in figure at $t = 0$. The particle A moves along X-axis with constant velocity of 2 ms^{-1} while initial velocity of B is zero and it moves under constant acceleration a. Determine the value of a so that the particles collide and the time when they will collide.



28. A rifle is aimed towards a target as shown in figure. After aiming the target the rifle has been fired, but at the same instant when the gun is fired, the string supporting the target breaks and the target falls freely under gravity.



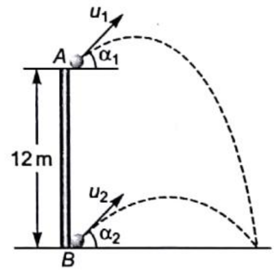
Show that whatever be the situation, (Initial velocity of bullet) the bullet will always hit the target. Assume that the bullet strikes the target before it hits the ground.

29. A projectile is projected at a speed of 75 ms^{-1} from ground level, at an angle of 60° above the horizontal. A wall of height 11 m is located at a distance of 27 m from the point of projection. Determine by how much amount the projectile will clear the top of the wall.
[Take $g = 10\ ms^{-2}$]

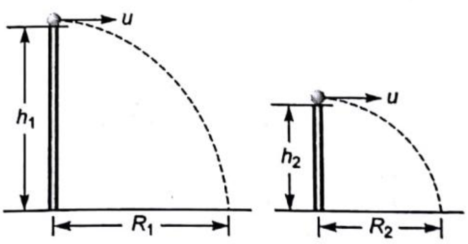
30. A horizontal rifle is fired at a target, the barrel of the rifle is pointed directly at the target, but the bullet strikes 0.05 m below the target. If the speed of bullet is 650 ms^{-1}, then determine the horizontal separation between the front of the rifle and the target. [Take $g = 10\ ms^{-2}$]

31. A projectile is fired from ground with a speed of 20 ms^{-1} at an angle of 30°, with the horizontal. At what time, the velocity vector of projectile becomes perpendicular to initial velocity ?

32. Two stones A and B are projected simultaneously as shown in figure. A is projected from the top of tower of height 12 m while B is projected from base of tower. It is known that $u_1 \neq u_2$ and $\alpha_1 \neq \alpha_2$. It has been observed that both the stones reach the ground at the same place after 3 s of their projection. Determine the
(a) difference of their horizontal components of launch velocities, and
(b) difference of their vertical components of launch velocities.



33. Two balls A and B are projected horizontally from the top of two different buildings with same initial speed as shown in figure. If $R_1 = 3\ R_2$, then determine $\frac{h_1}{h_2}$.

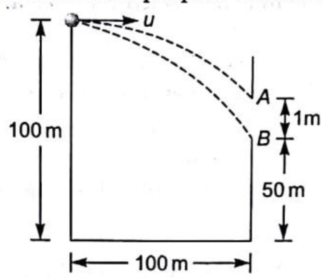


34. A particle starts from origin at $t = 0$ with a velocity of 5 ms^{-1} along +ve X-axis and moves in X-Y plane under action of a constant acceleration $\vec{a} = (3\ \hat{i} + 2\ \hat{j})\ ms^{-2}$. Determine the
(a) Y-coordinate of the particle at the instant its X-coordinate is 84 m.
(b) speed of the particle at this moment.

35. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of hall ?

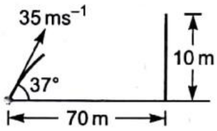
36. A fighter plane flying horizontally at an altitude of 1.5 km with a speed of 200 ms^{-1} passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell to hit the plane. The speed of the shell is 600 ms^{-1}. At what minimum altitude should the pilot fly the plane to avoid being hit ? [Take $g = 10\ ms^{-2}$]

37. From the top of a building of height 100 m, a ball has to be projected horizontally with a speed u such that it can enter into another building through a window AB of height 1 m as shown in figure. Determine the range of u so that purpose would be served.



38. A ball has been hit by a cricket bat at a height of 1 m above the ground, while leaving the bat the ball is moving with a speed of 25 ms^{-1} and at an angle of 45° with the horizontal. At the boundary which is at a horizontal distance of 56 m from the batsman position, a fielder is standing who can take the catch up to a height of 2.25 m. Would the fielder able to stop the boundary or six ? If fielder won't be there, then by what figure score will increase (four or six) ?

39. A ball is projected from ground with a speed of 35 ms^{-1} at an angle of 37° with the horizontal as shown in the figure.

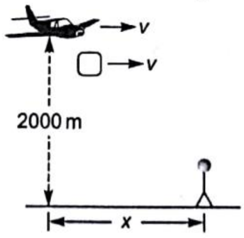


A wall of height 10 m is there at a distance of 70 m from the point of projection as shown. Determine whether the ball will hit the wall or not. If the ball hits the wall, then determine where it will hit and if not then by how much it will fail to do so ? [Take $g = 9.8\ ms^{-2}$]

40. Two men are running on straight track along north-south. The person A moves north with a speed of 5 ms^{-1} while B moves south with a speed of 2 ms^{-1}. Determine the velocity of
(a) A *wrt* B.
(b) ground with respect to A.
(c) B *wrt* A.

41. Two parallel rail tracks run north-south. Train A moves north with a speed of 70 kmh^{-1} and train B moves south with a speed of 90 kmh^{-1}. A monkey is running on the roof of a train A along south with a speed of 18 kmh^{-1} *wrt* train A. Determine the
(a) relative velocity of monkey *wrt* ground.
(b) relative velocity of monkey *wrt* train B.

42. A food packet is to be dropped by a plane on a flood relief mission. The plane is moving horizontally with a constant speed of $v = 50\ ms^{-1}$ as shown in figure. At the time of dropping the food packet, the plane is at a height of 2 km from ground level. For the person shown in the figure to receive the packet, determine the velocity of x.



43. A boat is travelling due south at a speed of 5 ms^{-1} relative to the water. Relative to the boat a passenger walks towards the back of the boat at a speed of 1.5 ms^{-1}.
(a) What is the magnitude and direction of the passenger's velocity relative to the water ?
(b) How long does it take for the passenger to walk a distance of 27 m on the boat ?
(c) How long does it take for the passenger to cover a distance of 27 m on the water ?

44. An eagle is flying horizontally with a speed of 6 ms^{-1}, with an object in its claws. The object accidentally dropped from the claws of eagle.
(a) How much time passes before the object's speed doubles ?
(b) How much additional time would be required for the object's speed to double again ?

C. Fill in the Blanks

1. For a projectile (plane to plane), the speed of particle is .... at its highest point.
2. Two stones are projected with the same velocity but make different angles with the horizontal. Their horizontal ranges are equal. If the angle of projection of one is $\pi/3$ and its maximum height is H, then the maximum height of the other will be ....
3. A rocket is accelerating upwards with an acceleration of 9.8 ms^{-2}. Near the earth's surface it releases a projectile. Immediately after releasing the projectile, its acceleration is having magnitude of .... with direction .... [Take $g = 9.8\ ms^{-2}$]
4. A particle is moving along a curved path in a plane, then the distance travelled by the particle in any time interval is always .... than the magnitude of displacement in corresponding time interval.
5. A particle is projected in vertical upward direction so that it can attain a maximum height of 20 m. If the same particle is projected with same speed at an angle of 45° with horizontal then it has a range of ....

D. True/False

1. The minimum speed of the projectile in plane to plane projectile motion is zero and this is the instant when projectile is at its highest point.
2. The acceleration and velocity of a projectile in plane to plane projectile motion are always perpendicular to each other.
3. The acceleration of the projectile is zero when it is at its topmost point.
4. Displacement of a particle in a given time interval is independent of choice of origin.
5. For a particle moving in a plane along some curved path, distance is always greater than magnitude of displacement.
6. A particle can't accelerate if its speed is constant.
7. A particle can have constant velocity and varying speed.
8. A particle is moving in XY plane along some curved path, then tangent drawn to trajectory drawn at any point gives the direction of instantaneous velocity at that point.
9. Relative velocity equation is a scalar equation.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. A golf ball is driven horizontally from a point which is 96 ft above a level plane. If it strikes the plane at a point distant 150 yards horizontally from the start, then the initial velocity of the ball is [1 yard = 3 ft and $g = 32\ \text{fts}^{-2}$]

(a) $120.4\ \text{fts}^{-1}$ (b) $183.7\ \text{fts}^{-1}$
(c) $48\ \text{fts}^{-1}$ (d) $246\ \text{fts}^{-1}$

2. Mark out the correct statement for a projectile motion.

(a) Time of flight is independent of vertical component of velocity.
(b) Time of flight is independent of horizontal component of velocity.
(c) Maximum height is independent of vertical component of velocity.
(d) Both (b) and (c)

3. A projectile is fired with a projection speed of $v_0\ \text{ms}^{-1}$ at an angle 65° with the horizontal with range R_0. If projection speed is kept the same, then range would be same R_0 for projection angle

(a) 30° (b) 25°
(c) 45° (d) No other angle

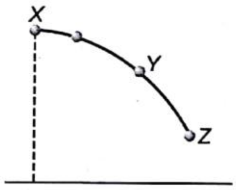
4. Which of the following is not an example of accelerated motion ?

(a) Vertical component of a projectile motion
(b) Circular motion at constant speed
(c) Earth's motion about sun
(d) Horizontal component of a projectile motion

5. The velocity of a projectile equals to its initial velocity added to

(a) a constant horizontal velocity
(b) a constant vertical velocity
(c) a continuously increasing horizontal velocity
(d) a continuously increasing downward velocity

6. A stone is thrown horizontally from a height and follows the path XYZ as shown. The direction of acceleration of the stone at point Y is



(a) $\downarrow$ (b) $\searrow$
(c) $\swarrow$ (d) None of these

7. Identical guns fire identical bullets horizontally at the same speed from the same height above level planes, one on the earth and one on the moon. Which of the following three statement(s) is/are true ?

I. The horizontal distance travelled by the bullet is greater for the moon.
II. The time of flight is less for the bullet on the earth.
III. The velocity of the bullets at impact are the same.

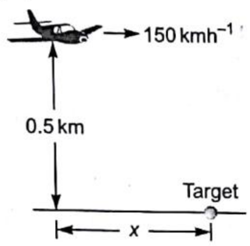
(a) III only (b) II only
(c) I and II only (d) I and III only

8. Two bodies are falling with negligible air resistance side by side, above a horizontal plane. If one of the bodies is given an additional horizontal acceleration during its descent, then it

(a) strikes the plane at the same time as the other body
(b) strikes the plane before other body strikes

(c) follows a straight line path along resultant acceleration vector
(d) None of the above

9. The aeroplane is shown here on level flight at an altitude of 0.5 km and at a speed of 150 kmh^{-1}.



At what distance x should it release a bomb to hit the target ? [Take $g = 10\ ms^{-2}$]
(a) 150 m (b) 420 m
(c) 2000 m (d) 800 m

10. A particle has an initial velocity of $(3\hat{i} + 4\hat{j})\ ms^{-1}$ and a constant acceleration of $(4\hat{i} - 3\hat{j})\ ms^{-2}$. Its speed after 1 s will be
(a) zero (b) 10 ms^{-1}
(c) $5\sqrt{2}\ ms^{-1}$ (d) 25 ms^{-1}

11. A river is flowing from west to east at a speed of 5 m min^{-1}. A man on the south bank of the river, capable of swimming at 10 m min^{-1} *wrt* river, wants to swim across the river in the shortest time. He should swim in the direction
(a) due north
(b) 30° east of north
(c) 30° west of north
(d) 60° east of north

12. A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 s, the velocity changes to 5 ms^{-1} northwards. The average acceleration of particle for this time duration is
(a) Zero
(b) $\frac{1}{\sqrt{2}}\ ms^{-2}$ towards north-east
(c) $\frac{1}{\sqrt{2}}\ ms^{-2}$ towards north-west
(d) $\frac{1}{2}\ ms^{-2}$ towards north

13. The equation of trajectory of a projectile in a vertical planes is given by $y = ax - bx^2$ where a and b are constants and, x and y are the horizontal and vertical displacements of the particle from the point of projection. The angle of projection of projectile from the horizontal is
(a) $\tan^{-1}\left[\frac{1}{b}\right]$ (b) $\tan^{-1}\left[\frac{a}{b}\right]$
(c) $\tan^{-1}[2a]$ (d) $\tan^{-1}[a]$

14. A particle has been projected from ground with an initial speed of 20 ms^{-1} at an angle of 30° to the horizontal. The time at which the projectile's velocity vector becomes perpendicular to its acceleration vector is [Take $g = 10\ ms^{-2}$]
(a) 1 s (b) 2 s
(c) 3 s (d) 4 s

15. A balloon is ascending up with a constant acceleration of 3 ms^{-2}. When it is at a height of 100 m from ground, its velocity is 5 ms^{-1} upward. At this instant a stone is dropped from balloon. The time in which stone reaches the ground is [Take $g = 10\ ms^{-2}$]
(a) 3 s (b) 1.3 s
(c) 5 s (d) 6 s

16. A stone is released from a high flying balloon that is descending at a constant speed of 10 ms^{-1}. After 20 s of dropping, the velocity of the stone would be [Take $g = 10\ ms^{-2}$]
(a) 190 ms^{-1} down
(b) 200 ms^{-1} down
(c) 210 ms^{-1} down
(d) 10 ms^{-1} down

17. If you are carrying a ball and running at constant speed and wish to throw the ball so that you can catch it as it comes back down, then you should throw it
(a) at an angle of 45° above the horizontal
(b) straight upward into the air and decrease your speed to catch it
(c) straight upward into the air and maintain your speed to be same
(d) None of the above

18. A projectile is fired from ground level in such a way that its initial velocity has horizontal and vertical components of 30 ms^{-1} and 20 ms^{-1}, respectively. The distance between the launching and landing points is [Take $g = 10\ ms^{-2}$]
(a) 40 m (b) 60 m
(c) 120 m (d) 180 m

B. More Than One Options Correct

1. Mark out the correct statements for plane to plane projectile motion.
   (a) Horizontal component of the velocity always remains constant.
   (b) Minimum speed of the projectile is at its highest point and is non-zero.
   (c) Speed of the projectile remains constant.
   (d) Speed of the projectile at the time of launching and landing are same.

2. A bomber flying in level flight with constant speed must release its bomb over the target. Neglecting air resistance, mark out the correct statement(s).
   (a) The bomber will be over the target when the bomb strikes.
   (b) The acceleration of the bomb is constant.
   (c) The bomb travels in a curved path *wrt* ground.
   (d) The time of flight of the bomb is independent of the velocity of the plane.

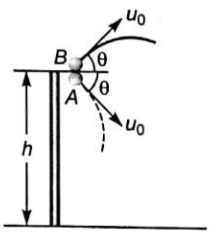
3. A projectile has an initial velocity v_0 at an angle θ above the horizontal. It reaches the highest point of its trajectory in time T after the launch. The highest point is at a vertical distance H and at the horizontal distance d from the point of projection. The speed of projectile at its highest point is v. For this situation, mark out the correct statement(s).
   (a) $d = v_0 \cos\theta \times T$
   (b) $v = v_0 \cos\theta$
   (c) $H = \dfrac{(v_0 \sin\theta)^2}{2g}$
   (d) $H = \dfrac{gT^2}{2}$

4. A particle is moving in xy plane, then which of the following is not necessarily same for its two one dimensional component motion ?
   (a) Velocity (b) Acceleration
   (c) Time (d) Displacement

5. The two projectiles would have same maximum height if their
   (a) time periods of flight are the same
   (b) ranges are the same
   (c) vertical components of initial velocity are the same
   (d) horizontal components of initial velocity are the same

6. Two projectiles are projected simultaneously from the top of a tower. A is dropped and B is thrown horizontally. For this situation mark out the correct statement(s).
   (a) Both reach the ground simultaneously.
   (b) Both reach the ground with same speed.
   (c) Both reach the ground with different speed.
   (d) Both have the same vertical displacement when they reach the ground.

7. Two particles A and B are projected from same point with same speed at same time as shown in the figure.



Particle A is projected down and B is projected up. For this situation mark out the correct statements.
   (a) Both reach the ground at same time.
   (b) Both reach the ground at different time.
   (c) Both reach the ground with same speed.
   (d) Both have same horizontal displacement when they reach the ground.

C. Assertion & Reason

Directions (Q. Nos. 1 to 5) *Some questions (Assertion-Reason type) are given below. Each question contains* ***Statement I (Assertion)*** *and* ***Statement II (Reason)****. Each question has 4 choices (a), (b), (c) and (d) out of which* ***ONLY ONE*** *is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**

(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**

(c) **Statement I** is True, **Statement II** is False

(d) **Statement I** is False, **Statement II** is True

1. **Statement I** Two balls are projected from the same point (from the top of tower) at the same instant. The ball A is thrown horizontally with some non-zero initial velocity while B is dropped from rest. In this case both the balls reach the ground simultaneously.
 Statement II The vertical motions of the two balls described in above statement are identical.
2. **Statement I** As object is shot from the back of a truck moving at a constant velocity of 40 $\mathrm{km\,h^{-1}}$ on a straight horizontal road. The launcher is aimed upwards, perpendicular to the bed of the truck, then the object falls on the truck.
 Statement II In above described situation, the acceleration of the object *wrt* ground is equal to acceleration due to gravity.
3. **Statement I** The two one dimensional motions along mutual perpendicular directions are independent of each other.
 Statement II The component of a vector in a direction perpendicular to it is zero.
4. **Statement I** The two projectiles having same time of flight would have same range.
 Statement II The two projectiles having same time of flight would have the same maximum height.
5. **Statement I** In a plane to plane projectile motion, the horizontal component of velocity remains constant.
 Statement II In a plane to plane projectile motion, the horizontal component of acceleration is zero.

D. Comprehend the Passage Questions

Passage I

A ball A is projected from O with an initial velocity $v_0 = 7\ \mathrm{ms^{-1}}$ in a direction 37° above the horizontal. Another ball B, 3 m from O on a line 37° above the horizontal is released from rest at the instant A starts, as shown in figure.

[Take $\sin 37° = \frac{3}{7}$; $\cos 37° = \frac{4}{5}$; $g = 9.8\ \mathrm{ms^{-2}}$]

Based on above information, answer the following questions :

1. After how much time from the instant of projection of A, the two balls collide ?
 (a) 1/7 s (b) 3/7 s
 (c) 2/5 s (d) 9/7 s
2. How far will B have fallen when it is hit by A?
 (a) $\frac{1}{10}$ m (b) $\frac{9}{10}$ m
 (c) $\frac{19.6}{25}$ m (d) $\frac{81}{10}$ m

3. What is the speed of A when it hits B?

(a) $\frac{28}{5}$ ms^{-1} (b) $\frac{56}{5\sqrt{3}}$ ms^{-1}

(c) $\frac{56}{5}$ ms^{-1} (d) None of these

4. In which direction A is moving when it hits B? [Angle made by horizontal]

(a) 0°

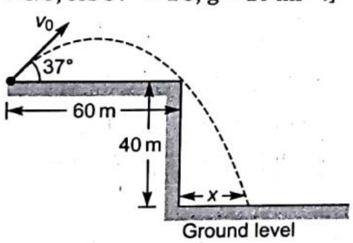
(b) 30° upward

(c) 30° downward

(d) Can't be predicted

Passage II

A ball is projected with a speed v_0 at an angle of 37° with the horizontal, from the top of a cliff as shown in the figure. The ball has been thrown with the aim so that it just hits directly the edge of the cliff, but the ball just misses the target and falls on ground as shown in figure. [Take $\sin 37° = 3/5$; $\cos 37° = 4/5$; $g = 10$ ms^{-2}.]



Based on above information, answer the following questions :

5. The value of v_0 is

(a) 25 ms^{-1} (b) 30 ms^{-1}

(c) 45 ms^{-1} (d) 18 ms^{-1}

6. The time taken by ball to reach ground is

(a) 4.7 s (b) 3.8 s

(c) 4.1 s (d) 5.0 s

7. The value of x is

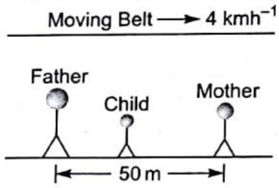
(a) 94 m (b) 64 m

(c) 34 m (d) zero

Passage III

On a long horizontally moving belt, a child runs to and fro with a speed of 9 km h^{-1} (*wrt* the belt) between his mother and father. The separation between the mother and father is 50 m. The belt moves with a constant speed of 4 km h^{-1} in the direction shown in figure.

[Take 1 km h^{-1} = $\frac{5}{18}$ ms^{-1}]



Based on above information, answer the following questions :

8. The velocity of child *wrt* ground when he is running towards his mother is

(a) 13 km h^{-1}

(b) 5 km h^{-1}

(c) 9 km h^{-1}

(d) 4 km h^{-1}

9. The time taken by the child to reach to his mother from his father is

(a) 10 s (b) 20 s

(c) $\frac{180}{13}$ s (d) 45 s

10. The velocity of child *wrt* ground when he is running towards his father is

(a) 13 km h^{-1}

(b) 5 km h^{-1}

(c) 9 km h^{-1}

(d) 4 km h^{-1}

11. The time taken by the child to reach to his father from his mother is

(a) 10 s (b) 20 s

(c) $\frac{180}{13}$ s (d) 45 s

E. Match the Columns

1. In Column I, some of the physical quantities associated with a plane to plane projectile motion are mentioned, while in Column II, the description with time is given. Match the entries of Column I with the entries of Column II. While matching consider the part of motion also, like for example, if acceleration increases first and then decreases then go for both the matchings.

Column I	Column II
(A) Horizontal component of velocity	(P) Increasing
(B) Speed of projectile	(Q) Constant
(C) Acceleration of projectile	(R) Decreasing
(D) Horizontal and vertical displacement of projectile as measured from point of projection	(S) Changing

2. Match the entries of Column I with the entries of Column II.

Column I	Column II
(A) For a plane to plane projectile motion	(P) Acceleration is constant
(B) For a particle projected horizontally from a height	(Q) Horizontal component of velocity remains constant
(C) For a particle projected upward (angle not equal to 90° from horizontal from a height)	(R) Velocity is changing in magnitude as well as direction
(D) For a particle projected in vertically upward direction from ground	(S) Acceleration is changing

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. $(8\hat{i} + 9\hat{j})\ ms^{-1}$ **2.** [12 M, 14.5 M] **3.** $(3\hat{i} - \hat{j})$ m, $\sqrt{10}$ m **4.** $(5\hat{j})$ m

5. (a) $\left(20 + \frac{10}{\sqrt{2}}, 20 + \frac{10}{\sqrt{2}}\right)$, (b) $(20\sqrt{2} + 10)$ m, (c) 0.7656 ms^{-1}, (d) 1 ms^{-1}

6. (a) $10\sqrt{2}$ m, (b) 20 m, (c) $10\sqrt{2}$ m, (d) $2\sqrt{2}$ m/s, (e) 2 m/s, (f) $\frac{2\sqrt{2}}{3}$ m/s, (g) π m/s, (h) π m/s, (i) π m/s (j) 0, (k) 0, (l) π m/s

7. 6 ms^{-2} towards south **8.** $2\sqrt{5}\ ms^{-1}$ at $\tan^{-1}\left(\frac{1}{2}\right)$ south of east

9. (a) 2.78 ms^{-1}, (b) 0.16 ms^{-2} **10.** (a) $3\hat{i} + 6\hat{j}$, (b) (15, 25), (c) $\frac{3}{2}$ s, (d) $\tan^{-1}(2)$

11. (a) 0, (b) (175 m, 0), (c) 15 ms^{-1} **12.** (a) 5 s, (b) (25 m, 12.5 m), (c) 10 s, (d) $10\sqrt{2}\ ms^{-1}$

13. 5 m **14.** 2 s **15.** 150 ms^{-1}, $\tan^{-1}\left(\frac{4}{3}\right)$ **16.** 16.3 ms^{-1}

17. $\frac{u \cos \alpha}{\cos \beta}$ **18.** $\frac{2}{\sqrt{3}}$ s, $\frac{4}{\sqrt{3}}$ s **19.** 35 ms^{-1}, $\tan^{-1}\left(\frac{7}{5}\right)$ **20.** 7 s

21. $\frac{3}{\sqrt{2}}$ s, 92.7 m **22.** No **23.** 0.38 s, 2.62 s **24.** (a) $\frac{1}{2}$ s, (b) $\frac{3}{2}$ s, 64.95 m

25. $\frac{15\sqrt{3}}{2}\ ms^{-1}$ **26.** (a) 1.414 s, (b) 14.23 ms^{-1}, (c) 2.121 m **27.** $\frac{3}{4}\ ms^{-2}$m, 4 s **29.** 33.172 m

30. 65 m **31.** Not possible **32.** (a) 0, (b) 4 ms^{-1} **33.** 9 : 1

34. (a) 36 m, (b) 35.12 ms^{-1} **35.** 148.33 m **36.** $\cos^{-1}\left(\frac{1}{3}\right)$

37. 31.30 to 31.63 ms^{-1} **38.** Yes, six **39.** No, 11.875 m

40. (a) 7 ms^{-1} N, (b) 5 ms^{-1} S, (c) 7 ms^{-1} S **41.** (a) 52 kph towards N, (b) 142 kph towards N

42. 1 km **43.** (a) 3.5 ms^{-1} towards S, (b) 18 s, (c) 7.72 s **44.** (a) 1.04 s, (b) 1.284 s

C. Fill in the Blanks

1. minimum and non-zero **2.** $\frac{H}{3}$ **3.** 9.8 ms^{-2}, downward **4.** greater **5.** 40 m

D. True/False

1. F **2.** F **3.** F **4.** T **5.** T **6.** F **7.** F **8.** T
9. F

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (b) 2. (b) 3. (b) 4. (d) 5. (d) 6. (a) 7. (c) 8. (a) 9. (b) 10. (c)
11. (a) 12. (c) 13. (d) 14. (a) 15. (c) 16. (c) 17. (c) 18. (c)

B. More Than One Options Correct

1. (a, b, d) 2. (a, b, c, d) 3. (a, b, c, d) 4. (a, b, d) 5. (a, c)
6. (a, c, d) 7. (b, c)

C. Assertion & Reason

1. (a) 2. (b) 3. (a) 4. (d) 5. (a)

D. Comprehend the Passage Questions

1. (b) 2. (b) 3. (a) 4. (a) 5. (a)
6. (a) 7. (c) 8. (a) 9. (b) 10. (b)
11. (b)

E. Match the Columns

1. A—Q; B—P, R, S; C—Q; D—P, S
2. A—P, Q, R; B—P, Q, R; C—P, Q, R; D—P, R

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. $\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$

$v_x = u_x + a_x t$ and $v_y = u_y + a_y t$

$\Rightarrow \quad v_x = 0 + 2\times 4 = 8\ \text{ms}^{-1}$

and $\quad v_y = 5 + 1\times 4 = 9\ \text{ms}^{-1}$

$\vec{\mathbf{v}} = 8\hat{\mathbf{i}} + 9\hat{\mathbf{j}}$

2. $x = x_0 + \left(u_x t + \frac{1}{2}a_x t^2\right)$

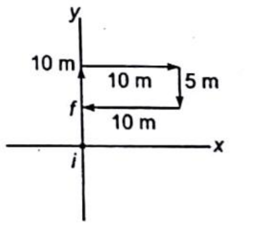
$= 2 + \left(2\times 5 + \frac{1}{2}\times 0\right) = 12\ \text{m}$

$y = y_0 + \left(u_y t + \frac{1}{2}a_y t^2\right)$

$= 2 + \left(0 + \frac{1}{2}\times 1\times 5^2\right) = 14.5\ \text{m}$

4. $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$

$= 5\hat{\mathbf{j}} - 0 = 5\hat{\mathbf{j}}$



7. $\vec{\mathbf{a}}_{av} = \dfrac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t} = 6\ \text{ms}^{-2}$ Southwards

12. $u_x = 10\text{ ms}^{-1}, u_y = 0$

$a_x = -2\text{ ms}^{-2}, a_y = 1\text{ ms}^{-2}$

$\vec{\mathbf{v}} = (10 - 2t)\hat{\mathbf{i}} + (t)\hat{\mathbf{j}}$

$\vec{\mathbf{r}} = (10t - t^2)\hat{\mathbf{i}} + \left(\frac{t^2}{2}\right)\hat{\mathbf{j}}$

(a) For $\vec{\mathbf{v}}$ to be parallel to y-axis,

$v_x = 0 \Rightarrow t = 5\text{s}$

(b) $\vec{\mathbf{r}}(t = 5\text{ s}) = \left[(10 \times 5 - 25)\hat{\mathbf{i}} + \frac{25}{2}\hat{\mathbf{j}}\right]$

$= (25\text{ m}, 12.5\text{ m})$

(c) $x = 0 \Rightarrow t = 10\text{s}$

(d) $\vec{\mathbf{v}}(10\text{s}) = -10\hat{\mathbf{i}} + 10\hat{\mathbf{j}}$

$v = 10\sqrt{2}\text{ ms}^{-1}$

13. $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{20^2 \sin^2 30°}{2g} = 5\text{ m}$

14. $T = \dfrac{2u\sin\theta}{g} = 2\text{ s}$

15. $\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$

$\Rightarrow \quad v^2 = v_x^2 + v_y^2$

$\Rightarrow \quad 250^2 = v_x^2 + 200^2$

$\Rightarrow \quad v_x = 150\text{ ms}^{-1}$

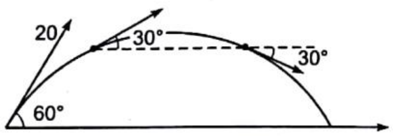
$\tan\theta = \dfrac{v_y}{v_x} = \dfrac{200}{150} = \dfrac{4}{3}$

17. Horizontal component of velocity of particle remains constant, so

$u\cos\alpha = v\cos\beta$

$\Rightarrow \quad v = \dfrac{u\cos\alpha}{\cos\beta}$

18. $\tan 30° = \dfrac{20\sin 60° - gt_1}{20\cos 60°}$



and $\quad \tan 30° = \dfrac{-(20\sin 60° - gt_2)}{20\cos 60°}$

21. The velocity of ball is along horizontal as it hits B, so it means that the point B is the highest point on the trajectory of motion of the particle.

Required time

$t = \dfrac{T}{2} = \dfrac{u\sin\theta}{g} = \dfrac{30}{10\sqrt{2}} = \dfrac{3}{\sqrt{2}}\text{ s}$

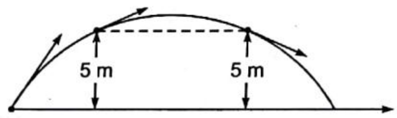
$AB = \sqrt{H^2 + \dfrac{R^2}{4}}$

where,

$H = \dfrac{u^2\sin^2\theta}{2g}$ and $R = \dfrac{u^2\sin 2\theta}{g}$

23. $5 = 30\sin 30° \times t - \frac{1}{2} \times 10t^2$

$\Rightarrow \quad t = 0.38\text{ s}$ and 2.62 s.



Two answers correspond to one during ascend and one during descend.

24. Let particle be at a height of 5 m after t sec of its projection, then

$5 = (25\sin 30)\,t - \frac{1}{2} \times 10t^2$

$\Rightarrow \quad t = \frac{1}{2}\text{s}$ and 2s

(a) $t_1 = \frac{1}{2}\text{ s}$

(b) $\Delta t = t_2 - t_1 = 2 - \frac{1}{2} = \frac{3}{2}\text{ s}$

(c) $\Delta x = 25\cos 30° \times \Delta t = 64.95\text{ m}$

26. For vertical motion, $+10 = \frac{1}{2} \times 10t^2$

$\Rightarrow \quad t = 1.414\text{ s}$

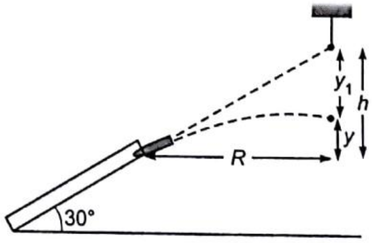
For horizontal motion,

$R = 1.5\,t = 1.5\sqrt{2} = 2.121\text{ m}$

$v = \sqrt{1.5^2 + (gt)^2} = 14.23\text{ ms}^{-1}$

28. Let bullet be shot at $t = 0$, and simultaneously the target is released.

$\tan\theta = \dfrac{h}{R}$



Let the particle (bullet) takes time t to cover horizontal distance of R, then

$$R = u\cos\theta \times t,$$

and $$y = u\sin\theta\, t - \frac{1}{2}gt^2$$

In this much time the target drops by,

$$y_1 = \frac{gt^2}{2}$$

$$y + y_1 = u\sin\theta \times t \quad \text{and} \quad R = u\cos\theta \times t$$

$$\Rightarrow \quad \frac{y + y_1}{R} = \tan\theta$$

$$\Rightarrow \quad y + y_1 = h$$

So, bullet always hits the target.

30. $0.05 = \frac{1}{2} \times 10 \times t^2 \quad \Rightarrow \quad t = 0.1\text{ s}$

$$R = 650 \times t = 65\text{ m}$$

32. As their horizontal ranges are same, so

$$u_1\cos\alpha_1 \times 3 = u_2\cos\alpha_2 \times 3$$

$$\Rightarrow \quad u_1\cos\alpha_1 - u_2\cos\alpha_2 = 0$$

In 3 s the vertical displacement of A is 12 m, and that of B is zero. So,

$$u_1\sin\alpha_1 \times 3 - u_2\sin\alpha_2 \times 3 = 12$$

$$\Rightarrow \quad u_1\sin\alpha_1 - u_2\sin\alpha_2 = 4\text{ ms}^{-1}$$

33. $h_1 = \frac{1}{2}gt_1^2$ and $h_2 = \frac{gt_2^2}{2}$

$R_1 = ut_1$ and $R_2 = ut_2$

$$R_1 = u\sqrt{\frac{2h_1}{g}}$$

and $$R_2 = u\sqrt{\frac{2h_2}{g}}$$

As $$R_1 = 3R_2$$

$$\Rightarrow \quad \sqrt{h_1} = 3\sqrt{h_2}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{9}{1}$$

35. Let particle is projected at an angle θ with the horizontal, then

$$25 = \frac{u^2\sin^2\theta}{2g} = \frac{40^2\sin^2\theta}{2g}$$

$$\Rightarrow \quad \sin^2\theta = \frac{5}{16}$$

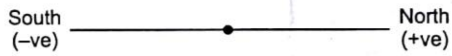
$$R_{\max} = \frac{u^2\sin 2\theta}{g} = 148.33\text{ cm}$$

39. Let y is the vertical displacement of ball when its horizontal displacement is 70 m, then from equation of trajectory.

$$y = 70\tan 37° - \frac{9.8 \times 70^2}{2u^2\cos^2 37°}$$

$$= 21.875\text{ m}$$

40. $v_A = +5\text{ ms}^{-1}, v_B = -2\text{ ms}^{-1}$

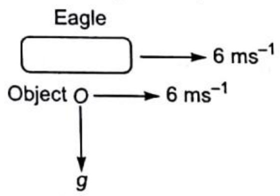


(a) $\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_A - \vec{\mathbf{v}}_B = 7\text{ ms}^{-1}$

(b) $\vec{\mathbf{v}}_{GA} = -\vec{\mathbf{v}}_{AG} = -v_A = -5\text{ ms}^{-1}$

(c) $\vec{\mathbf{v}}_A = -\vec{\mathbf{v}}_{AB} = -7\text{ ms}^{-1}$

44. As the object is dropped from eagle claws its velocity *wrt* ground is same as that of eagle and the object falls under gravity. Let speed of object is v at any time t, then



$$v = \sqrt{6^2 + (gt)^2}$$

For, $v = 12,\ t = 1.04\text{ s}$

$v = 24,\ t = 2.324\text{ s}$

Chapter 5

Newton's Laws of Motion

The First Steps' Learning

- Galileo's Law of Inertia
- Force
- Tension in a String
- Spring Force
- Pulley
- Newton's Laws of Motion
- Newton's First Law of Motion
- Newton's Second Law of Motion
- Newton's Third Law of Motion
- How to Solve Problems Based on Newton's Laws of Motion
- Mass and Weight

In previous two chapters we discussed about quantitative aspects of motion in terms of changes in displacement, velocity and acceleration. But till now we haven't bothered about what actually makes a body to be in motion. Is it in motion all by itself ? or Is there something which is causing its motion? If it is, then what it is and if not then why and how the motion takes place ? The answer of all these questions are contained in Newton's laws of motion, which are of central importance in classical physics, and are the subject-matter of this chapter. The branch of physics which deals with the causes of motion is called ***Dynamics****.*

To open your notebook while sitting in the chair, you need to make an effort to do so. If the ball in a soccer match going straight towards your opponent, then to make a pass to your team-mate, you must surely kick it in such a way so as to change its direction of motion. If you are pushing a heavy body on ground, then to move it faster surely you must exert more push on the block. All these physical situations involve the force about which we all are aware of and moreover which is responsible for the change in motion or we can say that force is the cause of motion.

He was the Galileo who first predicted that no ***force*** *is needed for a moving body to keep its motion as it is, and an external force is needed to change the state of rest or of uniform motion of a body. Later on Newton refined the works of Galileo and gave his theory in the form of three laws which are known as Newton's laws of motion or simply laws of motion. Without these laws it is next to impossible to understand the nature, although in 1905, Albert Einstein's theory of relativity revealed the limitations on scope of Newton's laws of motion. But as far as classical physics is concerned, these laws of motion are of utmost importance.*

Galileo's Law of Inertia

Before Galileo, it was the prevalent notion that "An external force is needed to keep a body in uniform motion". It seems to be a surprise how a wrong statement people could accept, but this is the fact that it took ages to find out whether a force is required or not to keep a body in uniform motion. Galileo was the first person to correctly explain the cause of motion through his experiments.

Only after the painstaking efforts of Galileo, the people worldwide started believing in experimental science.

Let us consider that a ball has been set rolling on a horizontal surface, then everyone will observe that the ball stops after sometime.

Galileo

A lot of common and similar observations you can make in your daily life, if you stop pedalling your bicycle it will stop after sometime, if you hit the ball from a bat (not very fast) it may stop by itself without any fielder, to keep on writing you have to continuously apply a force etc. From these observations what you can conclude may be that even if no external force is acting on the ball, it comes to rest by itself or you can say that for ball to keep on moving uniformly an external force is needed.

But now the question which may arise in your mind is that where you went wrong, the thing is that you have not considered the ever-present external force of *friction* which is exerted by ground on object and opposes the motion of object. What generally happens, is that we the common people try to formulate basic laws from our practical experience, but to get the true laws of nature for forces and motion, one has to imagine an ideal world in which uniform motion takes place with no opposing frictional forces, and this is what Galileo did. (*We shall discuss friction later on.*)

Galileo studied the motion of an object on an inclined plane and on horizontal surfaces of different levels of smoothness. We are illustrating in short, about one of the similar experiments conducted by Galileo.

Let us take an inclined plane, which ends with a horizontal surface as shown in the figure.

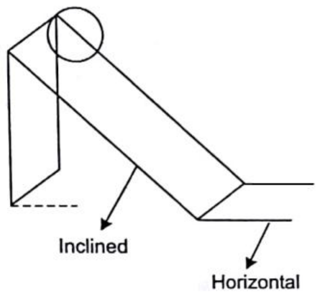


Fig. 5.1 Galileo released the ball from the top of the in li. e and measured the distance travelled by the ball on horizontal portion.

Now, release a ball from the top of the incline, as the ball moves down the incline the ball is accelerated and hence, it acquires some velocity when it reaches the horizontal portion. After moving some distance on horizontal portion, the ball comes to rest. Repeat the experiment by making the horizontal part more smoother, this time you will observer that the ball moves more distance on horizontal portion before it comes to rest. Continue to repeat the experiment again and again by increasing the smoothness of the horizontal section as compared to previous experiment and each time you will observe that ball is covering more distance before it stops, as compared to the previous one. Now, from this experimental observation what you conclude is that as the smoothness increases, the distance moved by the ball increases. Let us consider an idealised situation, if the horizontal section is perfectly smooth (practically this is not possible) then by how much distance the ball will move ? The answer is quiet obvious, *infinite,* as in this ideal situation ball will never stop and continues to move forever.

Here, only we have provided you with a similar type of experiment conducted by Galileo and from this only picture is somewhat more clear about the relation between force and motion. From a series of experiments conducted, Galileo arrived at a new conclusion about the cause of motion. He concluded that state of rest and the state of uniform motion are equivalent in dynamics *ie*, no net external force is needed to keep the body moving uniform motion.

The above conclusions in the form—"If the **net external force** acting on a body is zero, then a body at rest continues to be at rest and body in uniform motion continues to move with uniform velocity" is termed as Galileo's *law of inertia*. The word inertia means "resistance to change", thus we can say that inertia is the natural tendency of an object to resist a change in its state of rest or of uniform motion. But now the question arises, is the inertia for all objects the same ? The answer is no, let's consider an example. Two boxes, one heavy and the other light are kept on a smooth surface, then to move the boxes so that they move with same velocity, on which one you will have to exert more force? surely your answer would be the heavier box, which is absolutely correct. Now, the question arises, initially two objects of different masses were at rest and to change their state of rest, on heavier one we have to apply more force, it means heavy object is resisting more to change its state *ie*, heavy object is having more inertia. **Thus, we can say that mass of an object is a measure of its inertia.**

The work of Galileo had laid the foundation for the development of entire new mechanics, and this task has been accomplished almost single handed by Sir Isaac Newton. Taking Galileo's work as the groundwork, Newton formulated three laws of motion which constitute the central core of classical physics.

Before discussing Newton's laws of motion, let us discuss something about force and the components (like string, pulley, spring etc.) to be used in this chapter in detail.

Force

You want to throw a ball towards your friend, you want to write on your notebook using pen, you want to dig a pit to set up a plant, you want to hit a six, you want to perform the weightlifting, you want to stretch a spring, you want to hit the punching bag and a lot more are few of the daily chores, to do which you have to apply/exert some force on the objects. Force is defined as a **push or pull**, which **tries to change** or **changes the state** of rest or of uniform motion of a body. To explain the meaning of the words "tries to change or changes" in above definition of force, let us consider an object which is placed against a wall as shown in the figure. When you push the object, then also the block is not going to move. But, let us assume that if the wall were not there, then the block may have moved, so it means the force exerted by you on the block is trying to change the state of rest but due to the wall the block is not able to move. So in this situation force is trying to change the state of rest of block, similarly you are well aware of many situations in which the force changes the state of rest or of uniform motion.

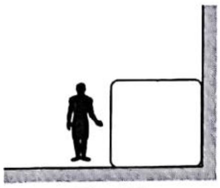


Fig. 5.2

Force is a vector quantity *ie*, it is associated with a magnitude and direction both. Its SI unit is **newton** (N). A force can't be seen, it can be only felt by the objects on which it is acting and the existence of the force can be only confirmed by observing the change in the state of the motion of the object. For example, if initially some object is at rest and suddenly it starts moving then it means, initially net external force acting on the object is zero and then it becomes non-zero. With regard to force always ask two questions from yourself-to understand the situation given in question in a more clear way.

1. Who is exerting the force?
2. On whom the force is being exerted?

For example, if you are pushing a block kept on the horizontal surface, then in this case you are exerting the force, and the block is experiencing the force [In the same way the block is also exerting a force on you - Details we shall discuss later on].

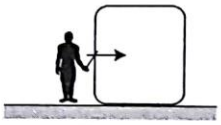


Fig. 5.3

Effect of a Force

However little we have discussed about the force, it is quiet obvious that a question will come to our mind—What force actually does? What is the effect of a force on an object? In this section we are to consider these questions. Consider a ball kept on the ground, when kicked, it starts moving and when this moving ball is approaching the goalpost, it has been caught by goalie and it comes to rest. In this illustration, when you kicked the ball, your foot exerts a force on the ball and hence it starts moving. Same way when the goalie snatches the ball, he exerts a force on the ball and makes it to stop. So, from this reasoning we can say that a *force can make a stationary body to move and can stop moving body as well.* From our day-to-day life experiences we can also conclude that—

- a force can change (increase or decrease) the speed of a body.
- a force can change the direction of motion of a moving body.
- a force can change the shape and/or size of a body.

Different Types of Forces

There are several different types of forces which exist in nature, like contact force, gravity force, friction force, tension in a string, spring force, viscosity force, electrostatic force, magnetic force, nuclear force, weak interaction force, buoyancy force etc. But broadly they have been classified into four categories only. These are as follows :

(a) Gravitational force
(b) Electromagnetic force
(c) Strong nuclear force
(d) Weak interaction force

To explain all the concepts related to these forces here is not a good idea, as this requires the knowledge of certain other concepts as well. So, we are skipping them till we are not done with other basic concepts, and here we are going to explain only those forces which are required in present chapters and the chapters that follow.

Contact Force

When you are writing, the force exerted by you on pen, the force exerted by pen-tip on a page of your notebook, the force exerted by notebook on table. When you play (for example cricket), the force exerted by you on bat, the force exerted by bat on ball. When you are standing still, then the force exerted by you on ground. All these are examples of contact forces, when two bodies come into physical contact, then they exert equal and opposite forces on each other, this force is termed as the contact force. All the contact forces are electromagnetic in nature (detailed discussion is beyond our scope here) and arise as a result of electromagnetic forces acting between the molecules of two objects present at the contact surface.

Force is a vector quantity, if many forces are acting on an object, then resultant force acting on the object is the vector sum of individual forces.

Let us consider a block moving on a horizontal surface, then both block and the surface exert a contact force on each other as shown in figure.

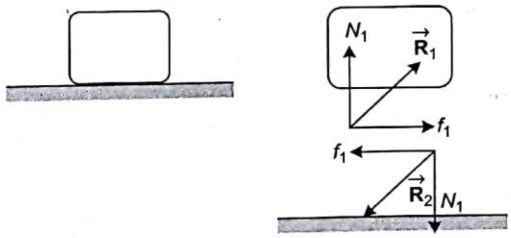


Fig. 5.4

$\vec{R}_1$ is the contact force exerted by surface on block and $\vec{R}_2$ is the contact force exerted by block on surface.

From Newton's third law, $\vec{R}_1 = -\vec{R}_2$. The magnitude and direction of contact force depends upon the nature of material of surface, motion of the surfaces in contact and also on other forces acting on them. If we resolve the contact force R into two components—one along the normal surface and other along the surface, then the component along the normal to surface is termed as *normal contact force* and is generally denoted by N. While the component along the surface is termed as *friction force* and is generally represented by f.

For frictionless surfaces (ideal smooth surfaces), the contact force would be along the normal only and friction force would be zero. In practice, it is not possible to find an ideal smooth surface but for smooth surfaces we will take $f = 0$ *ie*, only normal contact force would be there.

Until we haven't discussed friction in next chapter, we shall assume the surfaces to be smooth.

Forces at a Distance

In the previous section we have discussed about the contact forces between two objects which arise due to their physical contact (because of electromagnetic forces between the molecules at contact surface) and in general also our qualitative feeling for pushes and pulls arises due to a large extent from contact forces that we associate with a physical contact.

However, physical contacts are not necessary for forces to be operating. The force which arises between two bodies even when they are separated by a certain distance are known as **forces at a distance** or **action at a distance.** Examples of these types of forces are gravitational force between two objects, one bar magnet can attract another bar magnet without touching each other, two charged particles at a separation can attract or repel each other *via* electrostatic force, a ball thrown into air is acted upon by earth's gravity force throughout its motion.

A deeper analysis of forces shows that all forces act at a distance, now you may get confused that how this is possible? We have studied about contact forces and they are acting between two bodies in contact, how it could be that a force acts at a distance? The thing is that, the distances involved in the case of the contact forces we have discussed are of atomic sizes and hence this separation is not easily perceived by us. Or we can say, contact forces refer to forces acting over distances, too small to be visible to naked eye.

Earth's Gravity Force

If we release an object from rest in air above the earth's surface, then it is a very common observation that it falls back towards earth's surface. It means that some force is acting on it,this is the gravitational force experienced by the object due to the earth and this force is generally termed as earth's gravity force experienced by the object.

If the object is having mass m, then it will experience a gravitational force equal to mg where g is the acceleration due to gravity. The direction of this gravity force is always towards the centre of earth. If the distance of object from the earth's surface is negligible as compared to radius of the earth, then value of g can be taken as constant equal to $9 \cdot 8 \text{ m s}^{-2}$, and it is directed in vertical downward direction. This is an illustration of action at a distance and every object experiences it.

Tension in a String

Consider a string/rope which is connected to a block as sł own in the figure. If you pull the string from free end then the block will move (surface is assumed frictionless), this happens because of the force exerted by rope on block. This force transmission from one end of the rope to other leads us to discussion about tension in the string. Any substance in made up of molecules and at their equilibrium separation

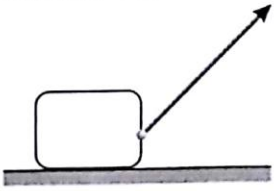


Fig. 5.5

the intermolecular forces are zero, if the separation between the molecules is changed from their equilibrium separation by some means then intermolecular forces (which we call internal restoring force) develop which act on different molecules in such a way so as to bring them back to equilibrium separation (also termed as natural position). For illustration consider two molecules of some substance, let us say that their equilibrium separation is r_0, if by some means this separation is increased to $r(>r_0)$, then an internal restoring force develops, which is attractive in nature as shown is figure and tries to bring back the molecules to their equilibrium separation. Although here we have considered only two molecules for the sake of convenience, but in actual this happens with every molecule.

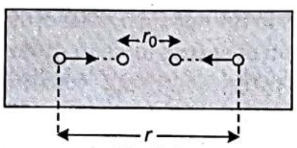


Fig. 5.6

The above described explanation is responsible for development of tension in string and the spring force.

Let us consider an inextensible string, which is pulled apart from its two ends with the help of two equal and opposite forces as shown in figure. Due to the application of forces, the intermolecular separation tends to increase and as a result internal restoring force develops which causes the

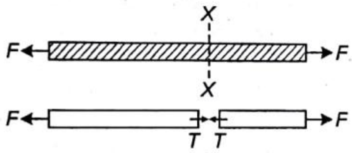


Fig. 5.7

tension to be developed in the string. This internal restoring force itself is the tension in string. Let us imagine that the string breaks at cross-section XX, then right part of the string is pulled towards left by left part and left part is pulled towards right by right part as shown in figure and this force with which the two adjacent sections are pulling each other is the tension force.

Some important points which we have to keep in mind, while dealing with string we mention here :

1. If no information about the string is mentioned in question, then assume the string to be inextensible. The different objects connected to the same inextensible string would have same magnitude of acceleration.
2. If no information about the mass of the string is provided in the question, then assume the string to be light. [Light string means its mass is negligible as compared to other objects connected to it and string mass can be taken as zero for calculation purposes, although massless string is not possible in practice]. The tension in light string is the same everywhere.

   The string having zero mass and perfectly inextensible is termed as the ideal string.
3. If string is having some mass, then tension at different cross-sections would be different.
4. Every string can bear a maximum force without breaking, this maximum force is termed as breaking strength of string.

Spring Force

You all may be familiar with the spring, which consists of a wire. When the wire is wound in the form of helix (as shown in figure) the spring is known as helical spring. It is a quiet common experience that to stretch (elongate) or compress a spring we have to apply some force on it. When we elongate or compress the spring from its natural length or relaxed state, then an internal restoring force develops which brings back the spring to its relaxed state when external force is removed. This internal restoring force developed is known as spring force. For small elongation or

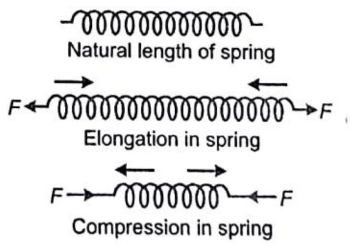


Fig. 5.8

compression from its natural length spring force is given by, $F = kx$ where k is known as force constant of spring or simply spring constant and x is the elongation or compression in spring, from its natural length. The direction of spring force is in such a way that it will try to bring back the spring in its natural position. The arrows in the spring in above diagram show the direction of spring force in different cases.

The value of spring constant depends upon the nature of material of spring, length of spring, length of wire used in constructing the spring etc. The SI unit of spring constant is Nm^{-1}.

C-BIs

Concept Building Illustrations

Illustration | 1 *A spring of spring constant $200\ Nm^{-1}$ is elongated by $0.05\ m$ from its natural length, then determine the spring force.*

Solution Spring force, $F = kx$

where, x is the elongation or compression in spring from its natural length. Here, elongation $x = 0.05$ m and $k = 200\ Nm^{-1}$.

So, $F = 200 \times 0.05 = 10$ N

Illustration | 2 *A spring of spring constant $200\ Nm^{-1}$ is compressed by 0.05 from its natural length, then determine the spring force.*

Solution Spring force, $F = kx$

where, x is the elongation or compression in spring from its natural length. Here, compression $x = 0.05$ m.

So,
$$F = 200 \times 0.05$$
$$= 10 \text{ N}$$

In above two illustrations, magnitude of spring force is the same as elongation and compression in same spring in two cases are the same ?
But, remember direction of F would be different in above cases!

Illustration | 3 *A spring of spring constant $200\ Nm^{-1}$ and relaxed length of 20 cm is elongated to 20.5 m with the application of a certain force. Determine the spring force.*

Solution Spring force, $F = kx$

Here,
$$x = 20.5 \text{ cm} - 20 \text{ cm}$$
$$= 0.5 \text{ cm}$$
$$\Rightarrow \quad F = 200 \times \frac{0.5}{100} = 1 \text{ N}$$

Pulley

A pulley will be one of the components of the various systems which we are going to analyse frequently in the present and subsequent chapters. A pulley is a wooden or iron disk, with a groove cut on its periphery so that a string can be wound on the disk which is used to pull heavy objects. You may have seen the pulley mounted on a well to pull out a bucket of water from it.

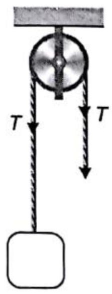


Fig. 5.9

If rotational motion of pulley is neglected and the pulley is light and smooth, then the tension in string on two sides of pulley would be the same.

Newton's Laws of Motion

The science of mechanics is based on three natural laws, which were first recognized by Sir Isaac Newton. These three laws are known as Newton's laws of motion. Starting with the work of Galileo, Sir Isaac Newton formulated three laws which provide us the basic for understanding the effects of forces on an object. Newton's laws of motion establish a relation between the forces applied on a body and the state of motion acquired by it. Without Newton's laws of motion it would have been impossible to understand the classical mechanics. Here, we shall discuss these laws one-by-one.

It should not be inferred that science of mechanics began only with Newton. It is a very old one, but correct interpretation was first understood by Galileo which laid the foundations of the work by Newton.

Newton's First Law of Motion

Newton's first law describes what happens to atoms, apples, balls, asteroids, galaxies or any other bodies, *moving or at rest,* when they are left alone *ie,* when no force is acting upon them from outside. Newton developed the insights into these situations from Galileo's work. Newton's first law states—

"An object continues to be in a state of rest or in a state of uniform motion, unless compelled to change that state by a **net non-zero external** force."

The bold phrase in above statement of Newton's first law has a wider significance. The word external means, external to the body, for example, you are inside the car sitting on back seat and we are interested in analysing the motion of car (including you). If you start pushing the front seat, then that force would be an internal and has not to be considered according to first law, and if some of your friend is pushing the car from back of the car (obviously outside the car), then that would be the external to the body considered, whose motion has to be analysed.

Generally, many forces act simultaneously on a body, like in above illustration the force exerted by your friend on car, the gravity force, the force exerted by ground on car. The word net means, the net force, the vector sum of all the forces acting on the body. The individual forces don't cause the motion of a body, the individual forces matter only to the extent that they contribute in the total force, it is the total (net) external force acting on the body which causes the change in the state of rest or of uniform motion of the body.

A system means the body or a set of bodies which are under observation. In above illustration, car and you constitute the system.

In other words, Newton's first law can be stated as :

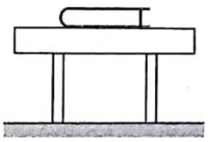
"Until and unless a non-zero net external force is applied on a system, it can't change its state of rest or its state of uniform motion".

"If a body/system is left alone, it maintains its state of rest or of uniform motion".

"When a body is at rest or moving with constant velocity, then the resultant of all of the forces exerted on the body is zero".

Newton's first law indicates that a state of rest and a state of constant velocity are completely equivalent in the sense that neither one requires an application of a net force to sustain it and also that net external force acting in both the situations is zero. The purpose of a non-zero net external force acting on a body is to change the velocity of body and not to sustain it.

Newton's first law can be used very effectively to find the magnitude and direction of some unknown forces. Consider for illustration, the book kept on table, as the book is at rest so from Newton's first law we can predict that net external force acting on the book is zero. We know that, a gravitational force mg (where m is mass of book) is acting upon the book in vertical downward direction and another force acting on the book is by the table, let it be N. As net external force acting on book is zero, so resultant of N and mg must be zero so we can say N is equal to mg and is in vertical upward direction. Let us consider another case in which you put your hand on the book as a result of which you exert a force F on the book in vertical downward direction and the book remains at rest,then again from first law you can find the force exerted by table on book. In this case it comes out to be $N_1 = mg + F$ in vertical upward direction.



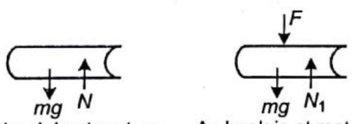


Fig. 5.10

This is only one of the simplest illustration we have given here, a lot of other structures like beams, bridges, buildings, tables, legs of table etc, are at rest generally. In all these cases we can find certain forces which are not known to us directly.

Inertia and Newton's First Law of Motion

Galileo's law of inertia and Newton's first law are equivalent, in fact we can say that Newton's first law is nothing but an extension of Galileo's law of inertia. Here, we give some illustrations which can be explained by law of inertia or by first law of motion.

(i) When a vehicle starts suddenly, the passenger falls backward, this is due to the fact, as lower portion of the passenger's body is in contact with seat of vehicle it comes into the motion with vehicle, but upper portion tries to maintain its state of rest due to inertia and hence the passenger falls backward.

(ii) When a vehicle stops suddenly, the passenger falls forward.

(iii) When we shake a tree vigourously, its fruits and leaves fall down, this happens because the fruits and leaves were at rest initially and as tree is shaken vigourously, the tree moves to and fro but the force is not acting on leaves and fruits and they try to maintain their states of rest due to inertia and hence fall.
If we shake the tree slowly, then fruits don't fall, instead they will make to and fro motion. Think why?

(iv) If we suddenly and rapidly pull the table cloth on which dishes are placed, then dishes remain on the table and the cloth comes out from the table. This is because of the fact that dishes were initially at rest and due to their inertia they try to maintain their state of rest as force exerted by us on table cloth is not transmitted to the dish bowls.

Inertial Frame of Reference

Newton's first law and also the second law, can appear to be invalid to certain observers. Let's look into these situations : Consider a trolley car in which a box is kept. The trolley car is accelerating on horizontal road with acceleration a as shown in the figure. If an observer A (inside the car) applies the Newton's first law on this block, then he will conclude that the net external force acting on the block is zero, as block is at rest with respect to him (observer A).

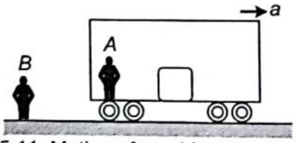


Fig. 5.11 Motion of an object as observed by two observers

So according to observer A, F_{ext} acting on the block is zero *ie*, $F_{ext} = 0$.

If observer B (who is standing on ground) observes the block, then he will conclude that net external force acting on block is non-zero as it is accelerating with respect to ground. So according to observer B, $F_{ext} \neq 0$. Now the question arises, how it is possible that same quantity (net external force) is having two values?

It is obvious that one of them is wrong, then you may ask,which one is wrong? Here,the solution to this contradiction is—"Newton's laws of motion are valid only in certain frames of reference". Now you may ask, how will we come to know that in which frame of reference, Newton's laws are valid? Then, the answer to your question is, Newton's laws are valid only in inertial frame of reference. Hope, you are ready with the next question. What is meant by inertial frame of reference? Now from the definition of inertial frame of reference, "those frames of reference in which Newton's laws are valid are inertial frame of reference". Now the situation seems to be more confusing, "Newton's laws are valid only in inertial frames of reference and inertial reference frames are those frames in which Newton's laws are valid", something like two person are running along a circle and you are asked to tell which one is ahead. This problem has been solved by assuming earth itself as an inertial frame, which is a good approximation to an inertial frame of reference. Strictly speaking, earth is also not an inertial frame of reference, but we will consider it as inertial frame of reference as it is near to perfect inertial frame of reference. Now the question arises,is there only one inertial frame of reference? The answer is no. All frames of reference which are not accelerating with respect to earth would be considered as inertial frames of reference, *ie*, all frames of reference which are at rest or moving with constant velocity with respect to earth would be considered as inertial frames of reference.

It means in above illustration observation of B is correct as the frame of reference attached to B is an inertial one and Newton's laws are valid in this frame of reference.

Equilibrium of a Body

A body is said to be in equilibrium when net external force acting on the body is zero. For example, a book kept on a table, a car moving with constant velocity etc, are in equilibrium as net external force acting on the respective objects is zero.

Equilibrium is of two types :

(a) Static equilibrium, and

(b) Dynamic equilibrium

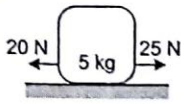
When the object is at rest, then it is said to be in static equilibrium, for example, a book kept on the table. And when the object is moving with constant velocity, then equilibrium is said to be a dynamic one.

Remember that it is not necessary that a body will be at rest when net external force acting on the body is zero *ie*, when it is in equilibrium.

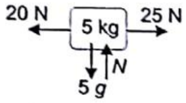
C-BIs

Concept Building Illustrations

Illustration | 4 *Two horizontal forces of 25 N and 20 N respectively are acting on a block of mass 5 kg as shown in the figure. Determine the net external force acting on the block in horizontal direction? and in vertical direction ? Also determine the contact force between the surface and block (Assume surface to be smooth, and take $g = 10\ ms^{-2}$).*



Solution There are four forces acting on the block:



1. Horizontal force of 25 N towards right.
2. Horizontal force of 20 N towards left.
3. Gravity force of $5\,g$ in vertical downward direction.
4. Normal contact force, say N by surface in vertical upward direction.

Net horizontal force,

$F_H = (25 - 20)\ \text{N} = 5\ \text{N}$ towards right

Net vertical force, $F_y = 0$ as block is at rest in vertical direction.

From $F_y = 0$, $5\,g - N = 0$

$\Rightarrow \quad N = 5\,g = 50\ \text{N}$

Illustration | 5 *Which one of the following objects can be taken as an inertial frame of reference?*

(a) A car moving with constant velocity.
(b) A car with brakes applied.
(c) A lift falling freely.
(d) A space-ship moving with constant velocity in space.
(e) A train moving on an incline with constant velocity.

Solution The objects which are moving with a constant velocity or are at rest with respect to earth, could be considered as inertial frames of reference, so a, d, e are correct answers.

Illustration | 6 *Three children, each pull the same platform (board), all the forces are in the horizontal plane. The three forces on the board have vectorial decomposition as : $\vec{F}_1 = 5\,\hat{i}$ units, $\vec{F}_2 = 5\,\hat{j}$ units, and $\vec{F}_3 = (-5\,\hat{i} - 5\,\hat{j})$ units.*

What is the net force on the board? What can you conclude about its subsequent motion? Neglect gravity.

Solution As gravity force is not acting on the board, so the net force acting on the rod is given by vector sum of forces acting on board by children.

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$
$$= 5\,\hat{i} + 5\,\hat{j} + (-5\,\hat{i} - 5\,\hat{j})$$
$$= 0$$

As the net external force acting on the board is zero, it continues its initial state of motion. If it is at rest initially, then it will remain at rest and if it is moving with some velocity, then it continues to move with the same velocity.

Newton's Second Law of Motion

Newton's first law states that if no net external force acts on a system, then it continues to be in its state of rest or of uniform motion, *ie*, Newton's first law gives us a qualitative idea that some non-zero net external force is needed to change the state of rest or of uniform motion of the body, but it doesn't give any quantitative idea that how

much force is needed and what happens when a net force acts on the system? This, Newton explains in his second law which is more popularly known as Newton's second law of motion.

In the words of Newton, second law is stated as "Rate of change of *quantity of motion* is equal to net external force acting on the system". To understand the statement of Newton's second law itself we require some preperatory training about the phrase— **quantity of motion**. Let us first consider few physical situations from our everyday life.

If you are standing at the middle of a road and a loaded truck is standing still in front of you then you won't fear but if the truck is moving even very slowly say at 10 kmh^{-1}, then you will try to escape from its path *ie*, you will move so that it doesn't hit you.

If a piece of thermocol is thrown towards you at very high speed, then you won't try to save yourself from the thermocol because you know that even if it hits you it won't hurt you but if the thermocol is replaced by a stone, then the situation would be reverse.

A bullet fired from a gun damages the human body severely, but if the same bullet is thrown by hand it won't be that much damaging. A table-tennis ball may not hurt the player much on hitting, while a cricket ball on hitting the player causes more pain. To push a light box a smaller force is needed than to push a heavy box so that both acquire the same speed in same time.

From the above physical situations and many more, it can be interpreted that the force required to change the state of rest or of uniform motion of a body depends upon both the mass and velocity of body. And from this interpretation, Newton introduced "the quantity of motion in a body" which depends on its mass and velocity. This quantity of motion is termed as **momentum**. Momentum of a body/particle is defined as the product of its mass m and velocity $\vec{\mathbf{v}}$. It is denoted by $\vec{\mathbf{p}}$.

$$\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}$$

Momentum is a vector quantity, whose direction is same as that of velocity. Its SI unit N-s and MKS unit is kg-ms^{-1}. Since, the net external force is responsible for the change in the velocity of a system, and hence for the change in momentum. Thus, we can say a net external force is needed to change the momentum of the system. From the mentioned physical situations it is clear that a greater force is needed to cause greater change in momentum. Now, let's have a look on another aspect of the picture.

If a person jumps barefooted from a height on a cemented floor, then it will hurt him more as compared to when he jumps from the same height on sand. While taking a catch, the fielder draws his hands backwards, as a result, the ball does not hurt his hands. From these observations, it is clear that force not only depends on change of momentum but also on how fast the momentum has been changed. If the same change in momentum is to be carried out in smaller time, then the force needed is more.

These qualitative illustrations lead to the Newton's second law of motion which we stated earlier. In simpler words, the second law can be stated as—*"The rate of change of momentum of an object is equal to net external force acting upon it".*

In mathematical form, Newton's second law is expressed as $\vec{\mathbf{F}} = \dfrac{\Delta \vec{\mathbf{p}}}{\Delta t} = \dfrac{\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i}{t_f - t_i}$ where,

$\Delta \vec{\mathbf{p}} \rightarrow$ change in momentum of system in time Δt.

This expression is not absolutely correct completely, as understanding it requires the use of calculus.

For a system of constant mass, Newton's second law can be written as,

$$\vec{\mathbf{F}} = \frac{m\,\vec{\mathbf{v}}_f - m\,\vec{\mathbf{v}}_i}{t_f - t_i} = m\left(\frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i}\right)$$

$$\vec{\mathbf{F}} = m \times \vec{\mathbf{a}}$$

where, $\vec{\mathbf{a}}$ is the acceleration of an object of mass m when net external force $\vec{\mathbf{F}}$ is acting upon it.

Thus, we can say, net external force acting upon a system is equal to product of its mass with acceleration of system *ie*, $F = ma$.

Response of a body to a net force is an acceleration.

Some points required to be kept in mind related to second law are :

1. Newton's first law is consistent with second law.
2. $\vec{F} = m\vec{a}$ is a vector equation *ie*, a force $\vec{F}$ produces acceleration in the direction of $\vec{F}$ only. This could be resolved into its three components as :
$F_x = ma_x$; $F_y = ma_y$ and $F_z = ma_z$ where F_x, F_y and F_z are X, Y and Z components of force respectively while a_x, a_y and a_z are corresponding components of acceleration.
3. In $\vec{F} = m\vec{a}$, the force $\vec{F}$ is the net external force acting upon the system, any internal force is not to be included in $\vec{F}$.

C-BIs

Concept Building Illustrations

Illustration | 7 *A particle of mass 5 kg is moving with a constant velocity of $10\ ms^{-1}$ along positive X direction. What is the momentum of particle? What is the net external force acting upon the particle?*

Solution Momentum of the particle is, $\vec{p} = m\vec{v}$

As, $m = 5$ kg,

and $\vec{v} = 10\ \text{ms}^{-1}$ along positive X-axis

$= (10\hat{i})\ \text{ms}^{-1}$

So, $\vec{p} = 5 \times 10\,\hat{i}\,(\text{N-s}) = 50\,\hat{i}\,(\text{N-s})$

ie, momentum of the particle is 50 N-s along positive X-axis.

As velocity of particle is constant, so no net external force acts on the particle.

Illustration | 8 *A constant force acts on an object of mass 2 kg for a duration of 3 s. This force increases the velocity of object from $3\ ms^{-1}$ to $10\ ms^{-1}$ without change in direction. Determine the magnitude and direction of force.*

Solution From Newton's second law,

$$\vec{F} = \frac{\vec{p}_f - \vec{p}_i}{t}$$

Now as the direction of the object's velocity remains the same *ie*, it is not changing direction and speed is increasing it means force is acting in same direction as that of velocity :

$$F = \frac{m \times v_f - m \times v_i}{t}$$

$$F = \frac{2 \times (10 - 3)}{3} = \frac{14}{3}\ \text{N}$$

Illustration | 9 *A 10 N force causes an acceleration of $2\ ms^{-2}$ to an m kg body, then what acceleration does same force cause to a body of 2 m kg?*

Solution From, $F = ma$

$\Rightarrow \quad 10 = m \times 2 \Rightarrow m = 5$ kg

For body of mass $2m$ kg *ie*, of 10 kg,

$10 = 10 \times a \Rightarrow a = 1\ \text{ms}^{-2}$

Illustration | 10 *Which will require the least force to stop the objects, think and answer qualitatively and from our daily observation.*

(a) *A bullet fired from a gun.*
(b) *A loaded truck standing still in parking area.*
(c) *A toy car driven by child's hand.*
(d) *A loaded truck running on highway.*

Solution All objects except in option (b) have a non-zero momentum, and hence require

certain force to make them stop, but in option (b) the loaded truck is stationary and hence, no force is needed to stop it, so (b) is the correct option.

In addition, in ascending order (the least force required first) are as $b < c < d < a$.

Illustration | 11 *An object of mass 2 kg is moving with a velocity of $5\ mg^{-1}$ along +ve X-axis, under the action of a constraint,* its velocity changes to $5\ ms^{-1}$ along $-ve$ X-axis in duration $4\ s$. Determine the force acting on object.

Solution Here, $\vec{p}_i = 2 \times 5\ \hat{i} = 10\ \hat{i}$ N-s

$$\vec{p}_f = -2 \times 5\ \hat{i} = -10\ \hat{i}\ \text{N-s}$$

From,
$$\vec{F} = \frac{\vec{p}_f - \vec{p}_i}{t}$$

$$\vec{F} = \frac{-10\hat{i} - 10\hat{i}}{4} = -5\ \hat{i}\ \text{N}$$

Newton's Third Law of Motion

Newton's second law relates the net external force acting on system with its acceleration, but till now we have not bothered about the origin/source of force. In Newtonian mechanics, any external force on a body is always due to some other body. Newton's third law states "Forces always occurs in pairs. Force on a body A by B is equal and opposite to the force on the body B by A". Consider a pair of bodies A and B. B gives rise to an external force on A, and equal and opposite force is exerted by B on A as shown in figure,

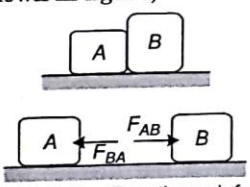


Fig. 5.12 Action-Reaction pair for two bodies kept contact

ie, $\vec{F}_{AB} = -\vec{F}_{BA}$ where $\vec{F}_{AB}$ and $\vec{F}_{BA}$ are forces exerted by B on A and by$A$ on B respectively. Let us consider few physical situations which explain Newton's third law :

1. If you kick a ball barefoot, then the ball flies away and you also feel something on your foot. Here in this case your foot exerts a force on ball and the ball exerts equal and opposite force on your foot.
2. While playing if you collide with another player, both of you get hurt, the thing is you are exerting a force on other player and the other player is exerting an equal and opposite force on you.
3. A stone is experiencing a downward force due to earth's gravity and the earth is also experiencing equal and opposite force due to the gravitational pull of stone. Although this situation seems not to be obvious but it is according to Newton's third law.

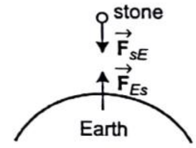


Fig. 5.13 Action-Reaction pair for earth and a stone

4. If you start walking, then you push the ground in backward direction and as a result ground push you back with equal and opposite force and because of this force only you are able to walk.

Fig. 5.14

5. If you press a spring, then spring is compressed by the force of your hand, the

compressed spring in turn exerts an equal and opposite force on your hand.

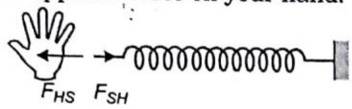


Fig. 5.15

6. If you are standing on ground, you are exerting a force on ground, then the ground will also exert an equal and opposite force on you.

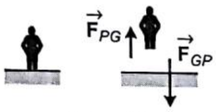


Fig. 5.16

7. When a book is kept on table, then the book exerts a downward force on table and the table exerts an equal and opposite force on the book.

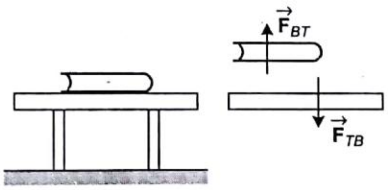


Fig. 5.17

Thus, according to Newtonian Mechanics, force never occurs single in nature. It always occurs in pair, it is the mutual interaction between two bodies. The exact wording's of Newton's third law is as—"To every action, there is an equal and opposite reaction". Here action and reaction words have been used for forces, it may be sometimes confusing to you that different words have been used for the same physical quantity, but it is what Newton used ! Now, we mention some of the important points related to Newton's third law :

1. The terms 'action' and 'reaction' are used for the same physical quantity, force, and any force can be considered as action or reaction *ie*, if one force you considered as action, then other would be considered as reaction. For example, in book-table system if you consider $\vec{\mathbf{F}}_{BT}$ as action, then $\vec{\mathbf{F}}_{TB}$ would be reaction and *vice-versa.*
2. Action and reaction always act at the same instant *ie*, there is no time-delay between action and reaction.
3. Action and reaction always act on different bodies, for example in book-table system if we want to analyse the motion of book then among action-reaction pairs only $\vec{\mathbf{F}}_{BT}$ has to be considered as $\vec{\mathbf{F}}_{TB}$ is acting on table, and not on the book. If we consider book-table as system, then $\vec{\mathbf{F}}_{TB}$ and $\vec{\mathbf{F}}_{BT}$ would be the internal forces.
4. If we consider two bodies A and B and only mutual interaction force is acting on them *ie*, $\vec{\mathbf{F}}_{AB}$ on A by B and $\vec{\mathbf{F}}_{BA}$ on B by A, then $\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}$ according to Newton's third law, but it is not necessary that accelerations of A and B are the same *ie*, their accelerations are also equal and opposite. Their accelerations may be different as they depend upon their masses too.

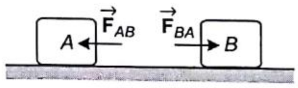


Fig. 5.18

5. Any pair of equal and opposite forces need not to be necessarily an action-reaction pair. For example consider a book placed on a table, now the forces acting on the block are — the contact force between book and table which we denote by N and the gravitational force of the book *ie*, mg. As the book is at a rest *ie*, in equilibrium, the net force acting on book is zero. From equilibrium condition we get $N = mg$ *ie*, contact force acting on book by table and

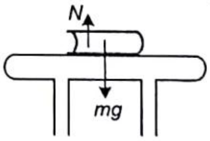


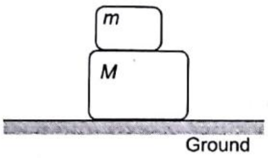
Fig. 5.19

gravitational force experienced by book due to the earth are equal and opposite, but these two forces N and mg don't constitute an action-reaction pair. The reason is — these two forces are acting on the same body while an action-reaction pair acts on different bodies and these two forces are not the forces applied by two bodies on each other. If we consider N as the action, then its reaction will be the force experienced by the table due to book and the reaction of mg would be the force experienced by the earth due to book.

C-BIs

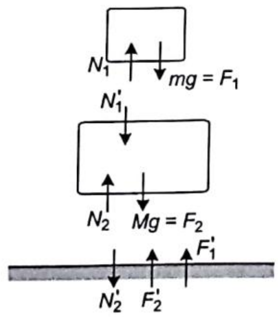
Concept Building Illustrations

Illustration | 12 *A block of mass m is kept at rest over another block of mass M which in turn is kept on ground as shown in figure. Identify different action-reaction pairs on this system.*



Solution N_1, N_1' $\rightarrow$ Action-reaction pair between m and M.

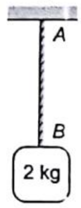
N_2, N_2' $\rightarrow$ Action-reaction pair between M and ground.



F_1, F_1' $\rightarrow$ Action-reaction pair between m and earth.

F_2, F_2' $\rightarrow$ Action-reaction pair between M and earth.

***Illustration* | 13** *A block of mass 2 kg is suspended from a fixed support with the help of a string as shown in the figure. Determine the force exerted by string on block and the force exerted by block on string. (Take $g = 10\ ms^{-2}$)*



Solution The forces acting on the block are :

(a) The gravity force $F = mg = 2 \times 10$ N acting downward.

(b) The force exerted by string on block in upward direction say T.

As the block is in equilibrium, the resultant of F and T would be zero,

ie, $F = T$

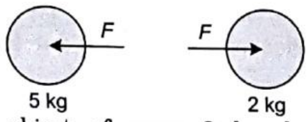
$\Rightarrow$ $T = 20$ N

So, the force exerted by the string on block is 20 N in vertical upward direction.

Using Newton's third law, the force exerted by the block on the string is equal and opposite to that of the force exerted by string on block. Thus, the block exerts a force of 20 N on the string in vertical downward direction.

Illustration | 14 *Two objects of masses 2 kg and 5 kg are moving under their mutual interaction force only ie, no other force is acting on objects except mutual action-reaction pair. If at any instant acceleration of 2 kg object is $10\ ms^{-2}$ towards right, then at this instant what is the acceleration of 5 kg object?*

Solution Let us say at the instant considered the two objects are exerting a force F and each other as shown in figure.



For object of mass 2 kg, by applying Newton's second law

$$F = 2a$$

$$\Rightarrow F = 2 \times 10 = 20 \text{ N}$$

For 5 kg object

$$F = 5a'$$

$$20 = 5a'$$

$$a' = 4 \text{ ms}^{-2} \text{ towards left.}$$

How to Solve Problems Based on Newton's Laws of Motion

Here in this section we are providing you step by step procedure to solve questions based on Newton's laws of motion.

Step I : Identify the system : First of all it is necessary that you identify your system for which you are going to apply the Newton's laws to analyse the motion of system. You can chose an individual block, a combination of blocks, combination of blocks and strings etc, as a system under some restriction. The restriction imposed in chosing a system is not necessary at this level, about this you will study in your higher classes. In the present context we shall consider individual objects or springs or strings as the system.

Step II : Identify the external forces acting upon the system : Once you have identified your system, the next task is to identify various external forces acting upon the system. It is better to make a list of various forces acting on the system. Remember while identifying the forces acting on the system, you have not to consider the internal forces and the forces exerted by the system on other surrounding bodies.

Step III : Draw a free body diagram (FBD) : In this step, you represent your system by a point and exert all the forces identified in the above steps, on the system by arrows, in their respective directions.

Step IV : Writing Newton's second law equation : Now the most important step comes *ie*, to use Newton's second law equation. In this step first of all assume the probable direction of acceleration of system and mark it as positive X-axis and perpendicular to it as positive Y-axis. Now resolve all the forces acting on the system along X and Y-axes and compute the net force along positive X-axis and positive Y-axis, let these be denoted by ΣF_x and ΣF_y, respectively where Σ is the greek symbol, and read as "sum of ". Then use $\Sigma F_x = ma_x$ where m is the mass of system and a_x is the component of acceleration of system along X-axis and then use $\Sigma F_y = ma_y$ where a_y is the component of acceleration of system along Y-axis.

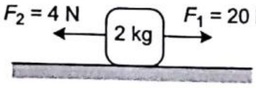
If $a_y = 0$, then last equation reduces to $\Sigma F_y = 0$.

Here it is important to note that forces have to be resolved in that direction along which motion of the system takes place, however, if the system is in equilibrium then we can resolve that force along any two mutual perpendicular directions. Another point to keep in mind is that $m \times a$ is simply the product of mass and acceleration of system and is equal to the net force acting upon the system in the direction of assumed acceleration, this quantity ma doesn't act upon the system.

C-BIs

Concept Building Illustrations

Illustration | 15 *A block of mass 2 kg is moving on a smooth horizontal surface under the action of two forces $F_1 = 20\ N$ and $F_2 = 4\ N$ as shown in the figure.*



Determine

(a) *the acceleration of block.*

(b) *the contact force between block and horizontal surface. (Take $g = 10\ ms^{-2}$)*

Solution Let us follow these step :

Step I : We will consider the block as a system.

Step II : The system *ie*, block is experiencing following forces :

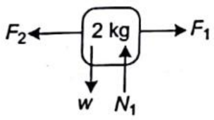
(a) The applied force $F_1 = 20$ N towards right.

(b) The applied force $F_2 = 4$ N towards left.

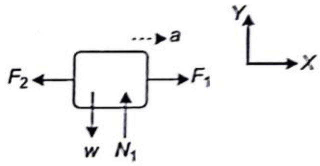
(c) The gravitational force experienced by the block due to earth in vertical downward direction, $w = mg = 2 \times 10$ N.

(d) The contact force exerted by the surface on block, as surface is smooth so it will exert the contact force along the normal only. Let us consider it to be N_1 in vertical upward direction.

Step III : Draw a free body diagram.



Step IV : From logical reasoning and practice you can easily find out that in this case the probable direction of acceleration is horizontal towards right. So, we consider X and Y-axes as shown in figure. Let a be the acceleration of block towards right, then find the net force along X and Y-axes. Here, in present case, there is no need to resolve the forces as already all the forces are either along X or Y-axes.



The net force along positive X-axis is

$$F_x = F_1 - F_2 = 20 - 4 = 16\ \text{N}$$

The net force along positive Y-axis is

$$F_y = N_1 - w$$

Using Newton's second law equation

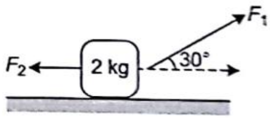
$$F_x = m \times a \quad \text{and} \quad F_y = 0$$

$$\Rightarrow \qquad 16 = 2 \times a$$

$$\Rightarrow \qquad a = 8\ \text{ms}^{-2}$$

From $F_y = 0$, $N_1 = w = mg = 20$ N.

Illustration | 16 *Consider the same situation as in Illustration 15, with the only difference that F_1 is now making an angle of 30° with horizontal.*

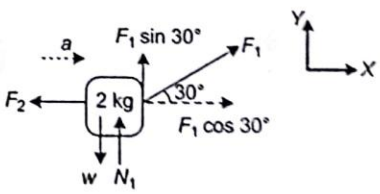


Determine the

(a) *acceleration of block.*

(b) *force exerted by the surface on block.*

Solution The free body diagram of block is as shown in the figure. Let the block moves towards right with acceleration a, then after resolving the forces along X and Y axes as shown in figure.



$F_x = F_1 \cos 30° - F_2$

$= 20 \times \frac{\sqrt{3}}{2} - 4$

$= 10\sqrt{3} - 4 = 13.32$ N

$F_y = F_1 \sin 30° + N_1 - w$

$= 20 \times \frac{1}{2} + N_1 - 20 = (N_1 - 10)$ N

By using Newton's second law equation $F_x = ma$ and $F_y = 0$.

$\Rightarrow\ 13.32 = 2a$

$\Rightarrow\ a = 6.66\ \text{ms}^{-2}$ towards right.

From $F_y = 0$

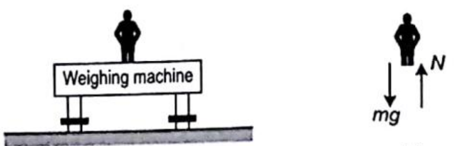
$\Rightarrow\ N_1 = 10$ N

Mass and Weight

Earlier we have seen that mass of a body is the measure of its inertia, and it is a property of substances. Its value is irrespective of the location and time. It is measured with the help of a beam balance, although the beam balance works on the principle of gravity but mass is independent of gravity.

Weight is the force experienced by body due to some heavenly body like earth, moon etc. We consider the weight of the body as the force experienced by it due to gravitational influence of earth. Its value is equal to mg where m is the mass of body and g is the acceleration due to gravity at the location of body. The value of weight depends on the location as for different locations, the value of g is different. The weight of an object is measured with the help of spring balance or weighing balance.

Consider a person of mass m, standing on a weighing machine which in turn is kept on the ground. The free body diagram of the person is as shown in figure. From the equilibrium of person, $N = mg$. The spring balance measures nothing but the force exerted on it *ie*, it reads the value of N and as N = weight of person so we are able to measure the weight of person who is standing on weighing machine. In some situations, N may not be equal to weight of person and in those situations the reading of weighing balance is not giving the true weight of person.



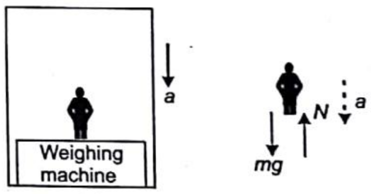
(a) (b)

Fig. 5.20

Consider one such situation, a person of mass m is standing on a weighing balance which in turn is kept in a box which is moving down with acceleration a as shown in figure. If we draw the free body diagram of person, then it would be as shown in figure.

Writing Newton's second law equation for person,

$$mg - N = ma \Rightarrow N = m(g - a)$$



(a) (b)

Fig. 5.21

The reading of spring balance is $N = m(g - a) <$ true weight of the person. In such a situation the reading of spring balance is termed as apparent weight. So in above situation apparent weight < true weight. What will happen if $a = g$ *ie*, the box is falling freely in above situation? From $N = m(g - a)$ the value of N comes out to be zero, this situation is termed as apparent weightlessness as here apparent weight is equal to zero. Any object in freely falling vehicle/enclosure/cabin is in a state of apparent weightlessness.

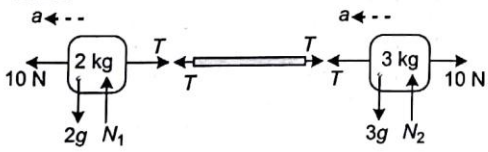
Proficiency in Concepts (PIC)

Problems

Problem | 1 *Two blocks of masses 2 kg and 3 kg are connected with the help of a light, inextensible string as shown in figure. Two forces each of 10 N are acting on the blocks as shown in figure. The horizontal surface is smooth.*

(a) *Draw the free body diagram of both the blocks, and the string.*
(b) *Determine the acceleration of 2 kg and 3 kg blocks.*
(c) *Determine the tension in string.*
(d) *Determine the force exerted by 3 kg block on string.*

Solution Let us first of all draw the free body diagram of both the blocks, and string. Let T be the tension in string and as string is light, tension at different points of the string are same.



The free body diagram of all three objects are shown in figure. As the two blocks are connected to same string they would have same acceleration. Let acceleration of 2 kg and 3 kg blocks is a towards left, then by writing Newton's second law equation for both the blocks

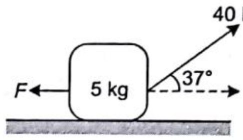
$$10 - T = 2a$$
$$T - 10 = 3a$$

The above two equations would possess only one solution *ie*, $a = 0$ and if $a = 0$ then $T = 10$N.

So acceleration of both the blocks are zero and tension in string is 10 N.

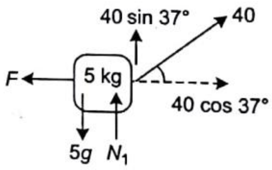
The force exerted by 3 kg block on string is same as of tension in the string at the end connected to block of mass 3 kg.

Problem | 2 *A block of mass 5 kg is at rest on a smooth horizontal surface. Two forces are acting on block as shown in figure. Determine the value of horizontal force F and the normal contact force between the block and table.*



(*Take* $g = 10\ ms^{-2}$, $\sin 37° = \frac{3}{5}$, $\cos 37° = \frac{4}{5}$)

Solution Draw the free body diagram of block and resolve the forces along any two mutual perpendicular direction (as block is in equilibrium). Here we are resolving the forces along horizontal and vertical directions.



Along horizontal direction,

$$40 \cos 37° - F = 0$$

$$\Rightarrow \quad F = 40 \times \frac{4}{5} = 32\text{ N}$$

Along vertical direction,

$$40 \sin 37° + N_1 - 5g = 0$$

$$\Rightarrow \quad 40 \times \frac{3}{5} + N_1 - 50 = 0$$

$$\Rightarrow \quad N_1 = 26\text{ N}$$

Problem | 3 *A ball has been thrown with a speed of 20 ms^{-1} at an angle of 30° with the horizontal. How its momentum will change with time?*

Solution As the ball is moving up its speed decreases and becomes minimum at the topmost point of flight and then again starts increasing.

So, the momentum of ball first decreases, reaches minimum, and then increases.

Problem | 4 *A body of mass 4 kg is subjected to a constant force F as a result of which its speed changes from 3 ms^{-1} to 7 ms^{-1} in a duration of 2s. The direction of motion of the body is not changing during its course of motion. Determine the change in magnitude of its momentum.*

Solution Final momentum,

$$p_f = mv_f = 4 \times 7 = 28 \text{ kg-ms}^{-1}$$

Initial momentum,

$$p_i = mv_i = 4 \times 3 = 12 \text{ kg-ms}^{-1}$$

Change in magnitude of its momentum is

$$= |p_f| - |p_i|$$
$$= 28 - 12 = 16 \text{ kg-ms}^{-1}$$

Problem | 5 *In above question, if direction of motion reverses once during the motion of body, then determine the—*

(a) magnitude of change in momentum of the body.
(b) change in the magnitude of momentum of the body.
(c) value of F.

Assume that, initially the body is moving along positive X-axis.

Solution Initial momentum,

$$\vec{\mathbf{p}}_i = 4 \times 3\,\hat{\mathbf{i}} = 12\,\hat{\mathbf{i}} \text{ kg-ms}^{-1}$$

Final momentum,

$$\vec{\mathbf{p}}_f = 4 \times (-7\,\hat{\mathbf{i}}) = -28\,\hat{\mathbf{i}} \text{ kg-ms}^{-1}$$

(a) Change in momentum,

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = -28\,\hat{\mathbf{i}} - 12\,\hat{\mathbf{i}}$$
$$= -40\,\hat{\mathbf{i}} \text{ kg-ms}^{-1}$$

Magnitude of change in momentum of the body is, $|\Delta\vec{\mathbf{p}}|$

$$|\Delta\vec{\mathbf{p}}| = 40 \text{ kg-ms}^{-1}$$

(b) Change in magnitude momentum

$$= |\vec{\mathbf{p}}_f| - |\vec{\mathbf{p}}_i|$$
$$= 28 - 12 = 16 \text{ kg-ms}^{-1}$$

(c) $|\vec{\mathbf{F}}| = \dfrac{\Delta\vec{\mathbf{p}}}{\Delta t} = \dfrac{40}{2} = 20$ N.

Problem | 6 *Determine the force required to produce an acceleration of 2 ms^{-2} to a body of mass 5 kg.*

Solution From Newton's second law, $F = ma$

Force required, $F = 5 \times 2 = 10$ N.

Problem | 7 *An unloaded truck having a mass of 1200 kg can have a maximum acceleration of 1.25 ms^{-2}. If the truck is loaded so that its total mass (including load) is 2500 kg, then determine the maximum acceleration of loaded truck.*

Solution Here, the word maximum acceleration is making a lot of meaning to solve the question. As the engine of truck remains the same whether the truck is loaded or unloaded, so it will exert same value of maximum force on ground which will push back the truck. So it means in both the cases, the force experienced by the truck is same.

For unloaded truck,

$$F = 1200 \times 1.25 = 1500 \text{ N}$$

For loaded truck,

$$F = 1500 = 2500 \times a$$

$$\Rightarrow \quad a = \frac{3}{5} = 0.6 \text{ ms}^{-2}.$$

Problem | 8 *The same force produces an acceleration of 3 ms^{-2} to a body of mass m_A and an acceleration of 5 ms^{-2} to a body of mass m_B. Determine the*

(a) ratio $\dfrac{m_A}{m_B}$.

(b) acceleration of the body whose mass is $m_A + m_B$, when same force is applied to it.

Solution Let F be the value of force, then from Newton's second law

$$F = m_A \times 3 = m_B \times 5$$

$$\Rightarrow \quad \frac{m_A}{m_B} = \frac{5}{3}$$

For a body of mass $m_A + m_B$, let acceleration be a.

$$F = (m_A + m_B)\,a = \left(m_A + \frac{3}{5}m_A\right)a$$

$$= \frac{8}{5}m_A \times a = m_A \times 3$$

$$\Rightarrow \quad a = \frac{15}{8}\ \text{ms}^{-2}.$$

Problem | 9 *A bullet of mass 0.04 kg moving with a speed of 90 ms^{-1} enters a heavy wooden block and is stopped after travelling a distance of 60 cm. Determine the average resistive force exerted by the block on the bullet.*

Solution The velocity of bullet get zero from 90 ms^{-1} while travelling a distance of 60 cm. So from equations of motion [acceleration assumed constant as average value of resistive force has to be determined],

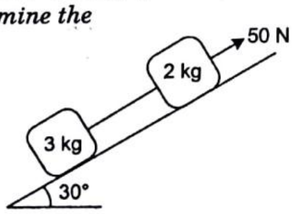
$$0 = 90^2 - 2a \times 0.6$$

$$\Rightarrow \quad a = 6750\ \text{ms}^{-2}$$

Average retarding force,

$$F = ma = 0.04 \times 675 = 270\ \text{N}.$$

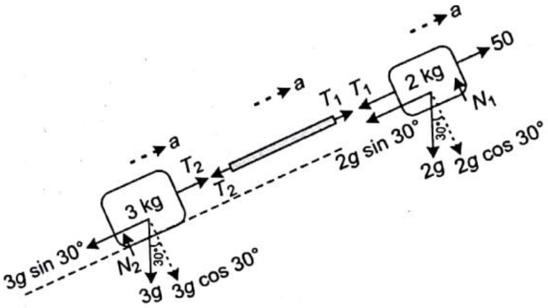
Problem | 10 *Two blocks of masses 3 kg and 2 kg are connected by a light string as shown in figure. The entire system is pulled up the fixed incline with the help of a force $F = 50\,N$ acting parallel to the incline. Determine the*



(a) acceleration of 2 kg block.
(b) acceleration of 3 kg block.
(c) tension in string.
(d) normal contact force between 3 kg block and incline. [Take $g = 10\ ms^{-2}$]

Solution As the two blocks are connected by the same string, so both the blocks will move with same acceleration.

Let us first of all draw the free body diagram of both the blocks, and the string.



We have considered that acceleration of blocks is a up the incline and the force exerted by string on block of mass 2 kg is T_1 and on block of mass 3 kg is T_2. Here we have taken tension in different parts of the string as different, so that we can prove that tension in a light string is same everywhere.

For the 2 kg block,

$$50 - T_1 - 2g\sin 30° = 2a \qquad \text{...(i)}$$

$$N_1 - 2g\cos 30° = 0$$

For the 3 kg block,

$$T_2 - 3g\sin 30° = 3a \qquad \text{...(ii)}$$

$$N_2 - 3g\cos 30° = 0$$

For string,

$$T_1 - T_2 = 0 \times a \qquad \text{...(iii)}$$

Adding Eqs. (i), (ii) and (iii), we get

$$50 - 2g\sin 30° - 3g\sin 30° = 5a$$

$$\Rightarrow \quad a = \frac{50 - 5 \times 10 \times 1/2}{5} = 5\ \text{ms}^{-2}$$

From Eqs. (iii), $T_1 = T_2$ *ie*, tension in light string is the same everywhere.

From Eq. (i),

$$T_1 = 50 - 2 \times 10 \times \frac{1}{2} - 2 \times 5$$

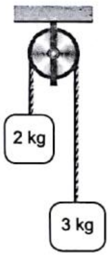
$$= 30 \text{ N.}$$

From $N_2 - 3\,g\cos 30° = 0$

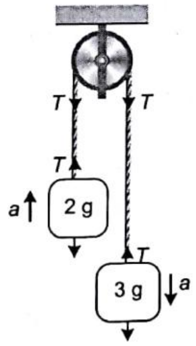
$$\Rightarrow \quad N_2 = 3 \times 10 \times \frac{\sqrt{3}}{2}$$

$$= 15\sqrt{3} \text{ N}$$

Problem | 11 *Two blocks of masses 2 kg and 3 kg are tied to a string and hanged over a light and frictionless pulley as shown in figure. If the system is released from rest, then find the acceleration of blocks and tension in the string? (Take $g = 10\ ms^{-2}$)*



Solution As both the blocks are connected by same string, they would have same acceleration let us say a and as pulley is light and frictionless, tension in string on either side of it is the same, say T.



The free body diagram of two blocks is as shown in figure.

For 3 kg block,

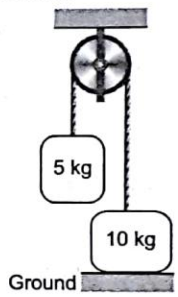
$$3\,g - T = 3a$$

For 2 kg block,

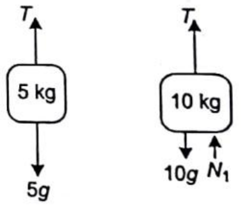
$$T - 2\,g = 2a$$

Adding these two equations, we get $a = g/5 = 2\ \text{ms}^{-2}$. Substituting this value of a in any of the equation, we get $T = 24$ N.

Problem | 12 *Two blocks are tied to a string and the string is wrapped over the pulley as shown in figure. The block of mass 10 kg is placed on the ground. Determine the tension in the string and contact force between 10 kg block and ground.*



Solution It is obvious that system won't move as 5 kg block is not able to pull up 10 kg block. Free body diagram of both the blocks are as shown.



For equilibrium of 5 kg block,

$$T - 5\,g = 0$$

$$\Rightarrow \quad T = 5\,g$$

For equilibrium of 10 kg block,

$$T + N_1 - 10g = 0$$

$$\Rightarrow \quad N_1 = 10g - T = 5g$$

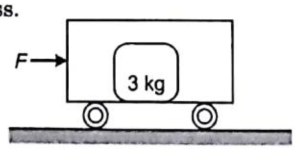
Towards Proficiency Problems
Exercise 1

A. Subjective Discussions

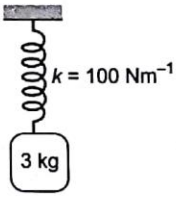
1. The net external force acting on an object is zero. Is it possible for the object to be travelling with a velocity that is not zero? If your answer is yes, state whether any conditions must be placed on the magnitude and direction of the velocity. If your answer is no, provide a reason for your answer.
2. Draw the free body diagram of a body falling freely.
3. Three non - zero external forces are acting on an object, then it must be accelerating. Comment on this statement.
4. When a body is moved from one place to another, then what may change the body's mass or its weight?
5. Give one physical situation other than mentioned in text, with the help of which you can verify Galileo's law of inertia.
6. Two bodies one heavy and other light are kept at rest on a smooth horizontal surface. Now different forces are applied on both the blocks so that both the blocks acquire same speed in same interval of time. Which force would be greater?
7. For a light string, tension is same at all points of string. Prove this statement qualitatively and quantitatively.
8. Explain the significance of "net external force" in Newton's first and second laws of motion.
9. If action and reaction are always equal and opposite, why don't they always cancel one another and leave no net force for accelerating a body?
10. Object A weighs twice as much as object B at the same spot on the earth. Would the same be true at a given spot on Mars? Explain your answer.
11. Consider a book lying on a table. The weight of the book and the normal force by the table on the book are equal and opposite. Is this an illustration of Newton's third law?
12. Two blocks of unequal masses are tied by a spring. The blocks are pulled stretching the spring slightly and the system is released on a frictionless horizontal platform. Are the forces due to the spring on the two blocks equal and opposite? If yes, is it an action-reaction pair?

B. Numerical Answer Types

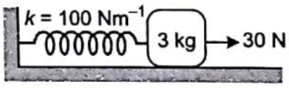
1. A car of mass 200 kg is accelerating under the action of a force $F = 3000$ N as shown in figure. A block of mass 3 kg is at rest on the surface of car. [3 kg is included in 200 kg]. Determine the net force acting on the 3 kg mass.



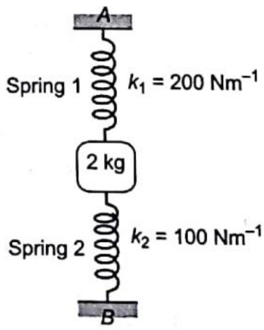
2. A bicycle has a mass of 13.1 kg and the rider's mass is 84.7 kg. The rider is bicycling hard so that a net horizontal force of 9.78 N acts on the bicycle. What is the acceleration of bicycle?
3. Two forces $\vec{F}_A$ and $\vec{F}_B$ are applied to an object of mass 8 kg. $\vec{F}_A$ is larger in magnitude among two forces. When both the forces are pointing due east, the object's acceleration is 0.5 ms^{-2} due east. However, when $\vec{F}_A$ points due east and $\vec{F}_B$ points due west, the acceleration is 0.4 ms^{-2} due east. Determine the magnitudes of both $\vec{F}_A$ and $\vec{F}_B$.
4. A block of mass 3 kg is suspended from a light spring of spring constant 100 Nm^{-1} as shown in figure. Determine the elongation in spring when the block is in equilibrium. (Take $g = 10$ ms^{-2})



5. A block of mass 3 kg is connected to an ideal spring as shown in the figure. Determine the elongation of spring when the block is in equilibrium.

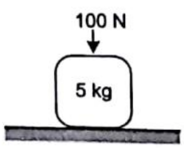


6. A block of mass 2 kg is connected to two vertical springs as shown in figure. Initially, the springs are in relaxed state and then the block is slowly brought to the equilibrium position. Determine the



(a) elongation in spring 1.
(b) compression in spring 2. (Take $g = 10$ ms^{-2})

7. You apply a force of 100 N on a block of mass 5 kg kept on ground as shown in figure. Determine the normal contact force between the block and ground. (Take $g = 10$ ms^{-2})



8. A particle of mass 5 kg is moving with a velocity of 3 ms^{-1}. Then a force $\vec{F}$ acts on it for 1 s as a result of which its direction of motion deviates by 60° from its initial path without any change in its speed. Determine the

(a) change in magnitude of momentum of particle.

(b) magnitude of change in momentum of particle.

(c) the magnitude of $\vec{F}$.

9. A particle of mass 2 kg is moving with a speed of 2 ms^{-1} along +ve X-axis. Determine its momentum.

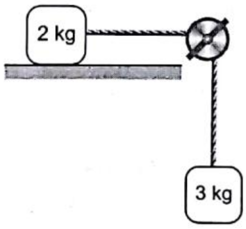
10. Under the action of a constant force $\vec{F}$ the momentum of a particle changes from $-10\,\hat{i}$ N-s to $+5\,\hat{i}$ N-s in a 2 s duration. Determine the magnitude of $\vec{F}$.

11. A lift weighing 8000 kg is given an upward acceleration of 1.5 ms^{-2}. Determine the tension in the supporting cable. [Take $g = 10\ m/s^2$]

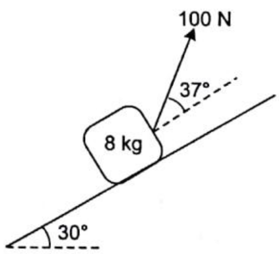
12. With what force will the elevator (lift) push upward a 60 kg passenger if the elevator is accelerating up with an acceleration of 2 ms^{-2}?

13. Two 3 kg and 2 kg blocks are connected with the help of a light inextensible string as shown in figure. A horizontal force of 30 N is applied to the 2 kg block. Determine the acceleration of 3 kg block and tension in string. [*Neglect friction everywhere*]

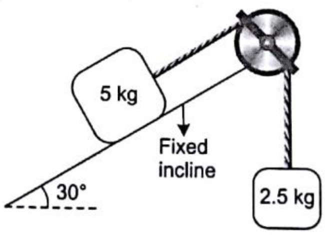
14 For the system shown in figure, determine the acceleration of 3 kg block and tension in string. Assume the string and pulley to be ideal. (Take $g = 10\ ms^{-2}$, and neglect friction.)



15. A block of mass 8 kg is pulled up on a smooth, fixed incline with the help of a 100 N force as shown in figure. Determine the acceleration of block and normal contact force between block and incline. (Take $g = 10\ ms^{-2}$, and neglect friction.)



16. For the system shown in figure, determine the acceleration of 5 kg block and tension in string. (Take $g = 10\ ms^{-2}$, and neglect friction.)



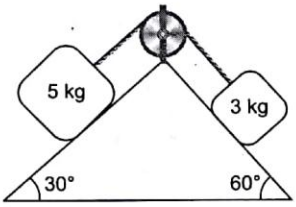
17. Repeat the above question if hanging block is having a mass of
(a) 1.5 kg (b) 4.0 kg

18. A 5 kg block is supported by a cord and pulled upward with an acceleration of $2\ ms^{-2}$. Determine the tension in cord. $(g = 9.8\ ms^{-2})$

19. In above question if the tension in cord is 49 N, then what sort of motion will the block perform?

20. A balloon is descending with a constant acceleration a, less than the acceleration of gravity g. The weight of the balloon, with its basket and contents is W. What weigh w, should be dropped from balloon so that it will ascend up with acceleration a? Neglect air resistance and assume that a constant force F is acting on balloon in upward direction.

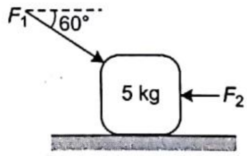
21. The system shown in figure is released from rest. All surfaces are frictionless, string is light and pulley is ideal.



Determine the tension in string and acceleration of 3 kg block.

22. A 10 kg suitcase is placed on a scale that is in an elevator. Is the elevator accelerating up or down when the scale reads (a) 75 N, (b) 120 N? Justify your answer. Also determine the acceleration of elevator in two cases.

23. Two forces $F_1 = 45$ N and $F_2 = 25$ N are acting on a block of mass 5 kg as shown in figure.



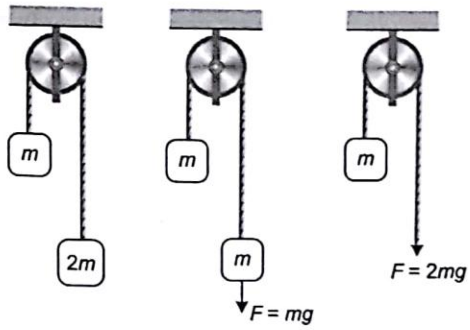
Determine the acceleration of block and the normal contact force between block and surface. Neglect friction. (Take $g = 10\ ms^{-2}$).

24. A ship of mass 1.7×10^8 kg is moving with a constant velocity in a straight line path. The engine of the ship generates a forward thrust of 7.4×10^6 N on the ship. Determine the resistive force being exerted on the ship by water.

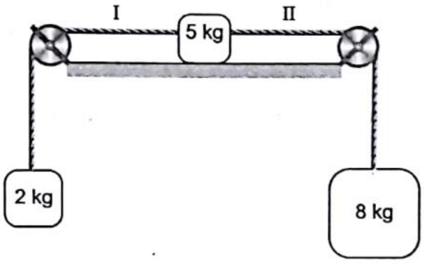
25. Three forces act on the object moving with constant velocity. One force has a magnitude of 80 N and acting along +ve Y-axis. Another has a magnitude of 60 N and acting along –ve X-axis, determine the magnitude and direction of third force.

26. A train consists of 50 cars (including engine), each of which has a mass of 6.8×10^3 kg. The train has an acceleration of $0.05\ ms^{-2}$. Neglecting friction, determine the tension in the coupling
(a) between the 30^{th} and 31^{st} cars.
(b) between the 49^{th} and 50^{th} cars.

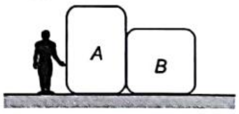
27. Three situations have been shown here. Determine the acceleration of block on left side and tension in string in all the three cases.



28. For the system shown in figure, determine the acceleration of 2 kg block and tension in strings I and II. (Take $g = 10\ ms^{-2}$, and neglect friction.)

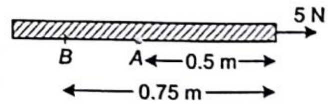


29. Draw free body diagram of the boy, and two blocks A and B for the situation shown in figure.

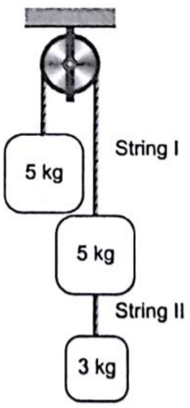


30. A body builder exerts a force of 150 N against a spring and compresses it by 20 cm. Determine the force constant of the spring.

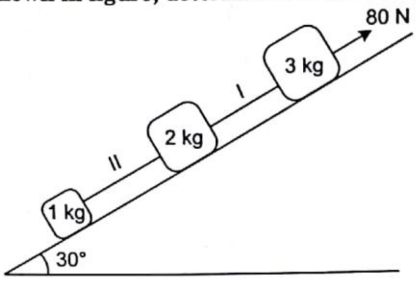
31. An uniform string of mass 2 kg and length 1 m is placed on a smooth horizontal surface. An horizontal force of 5 N is applied at one of its ends as shown in figure. Determine the tension in string at A and B.



32. For the arrangement shown in figure, determine the tension in strings I and II.



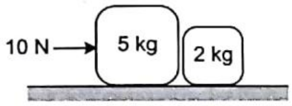
33. For the arrangement shown in figure, determine the tension in strings I and II.



34. A block of mass 2 kg placed on a long frictionless horizontal table is pulled horizontally by a constant force F. It is found to move by 10 m in first 2 s. Determine the value of F.

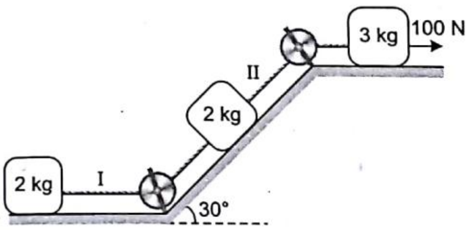
35. Two blocks A and B of masses m and $2m$ respectively are kept in contact on a smooth table. A person pushes the block A from behind so that both the blocks accelerate. If the block A exerts a force F on the block B, then determine the force exerted by the person on A.

36. For the arrangement shown in figure, determine the contact force between two blocks. Neglect friction everywhere.

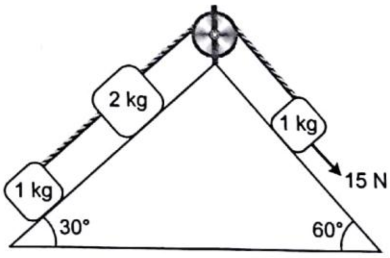


37. Repeat above question if force is applied on 2 kg block towards left.

38. For the arrangement shown in figure, determine the tension in strings I and II and acceleration of all the three blocks. Neglect friction. (Take $g = 10\ ms^{-2}$).



39. For the arrangement shown in figure all the surfaces are assumed frictionless. Determine the acceleration of 2 kg block.



40. Ten one-rupee coins are put on top of each other on a table. Determine the magnitude and direction of
(a) the force on the 6^{th} coin (counted from bottom) due to all the coins on its top.
(b) the force on the 6^{th} coin due to 7^{th} coin. [Take m as the mass of each coin]

41. A block of mass 5 kg is connected to a vertical spring, as a result, the spring elongates by 4 cm. If another block of mass 2 kg is attached to the block of mass 5 kg, then determine the further elongation in spring.

C. Fill in the Blanks

1. A body is falling down with constant velocity of 9.8 ms^{-1}, the net force acting on the body is ....
2. .... net external force is needed to keep a body in its state of uniform motion.
3. In Newton's second law of motion, the quantity of motion means ....
4. An inertial reference frame is one in which ....
5. Acceleration is always in the direction of ....
6. Two objects, one having three times the mass of the other, are dropped from the same height in a vacuum. At the end of their fall, their velocities are equal because their .... are equal.
7. In a tug of war, two men each pull on the rope with 400 N forces in opposite directions. The tension in the rope is ....

D. True/False

1. In $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ *ie*, Newton's second law equation, $\vec{\mathbf{F}}$ is the resultant of all internal and external forces acting on the system.
2. If a body is in equilibrium, then the component of the resultant of all the forces acting on it in any direction is zero.
3. Two equal and opposite forces always constitute an action-reaction pair.
4. An external force is needed to keep a body in uniform motion.
5. A non-zero net force acting on an object will always change its state of rest or of uniform motion.
6. A car travels towards east with constant velocity. The net force on the car is towards east.
7. A book rests on a table, exerting a downward force on the table. The reaction to this force is the force of earth on the book.
8. Beam balance is used to measure mass and can work in the gravity-free area.
9. Beam balance gives the same reading at the earth and moon.
10. A body is in equilibrium if it is in uniform linear motion.
11. A person wants to slide down a rope whose breaking strength (the maximum tension string can bear without breaking) is $\frac{3}{4}$ of his weight. He can come down safely.
12. A particle moving with constant acceleration may traverse a parabolic path.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. A 60 kg man pushes a 40 kg man by a force of 60 N. The 40 kg man has pushed the other man with a force of

(a) 40 N (b) zero
(c) 60 N (d) 20 N

2. A block of mass m is placed on a smooth inclined plane of inclination θ with the horizontal. The force exerted by the plane on the block has a magnitude

(a) mg (b) $\frac{mg}{\cos\theta}$
(c) $mg\cos\theta$ (d) $mg\tan\theta$

3. A body of mass 5 kg is acted upon by two perpendicular forces of 8 N and 6 N. The acceleration of the body is

(a) 1.6 ms^{-2} along 8 N force
(b) 1.2 ms^{-2} along 6 N force
(c) 2 ms^{-2} at an angle of $\tan^{-1}\left(\frac{3}{4}\right)$ with 8 N force
(d) 2.8 ms^{-2} at an angle of 45° with 8 N force

4. A constant force acting on a body of mass 3 kg changes its speed from 2 ms^{-1} to 3.5 ms^{-1} in 25 s. The direction of the motion of the body remains unchanged. The magnitude of force is

(a) 0.18 N (b) 0.6 N
(c) 0.66 N (d) None of these

5. A monkey of mass 40 kg climbs up and down on a rope, which can withstand a maximum tension of 600 N. In which of the following cases will the rope break up ? The monkey

(a) climbs up with an acceleration of 6 ms^{-2}
(b) climbs down with an acceleration of 4 ms^{-2}
(c) climbs up with a uniform speed of 5 ms^{-1}
(d) climbs down with an acceleration of g

6. Three spring scales (need not to be identical) are connected along a straight line as shown in figure :

The scale on the left is attached to the wall and the scale on the right is pulled with a force of 15 N. The reading of the leftmost and middle scale are respectively,

(a) 15 N each
(b) 5 N and 10 N
(c) 10 N and 15 N
(d) 10 N and 5 N

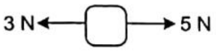
7. A 1000 kg aeroplane moves in straight flight with a constant velocity. The force of air friction is 1800 N. The net force on the plane is

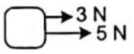
(a) zero
(b) 1800 N
(c) 9000 N
(d) 3600 N

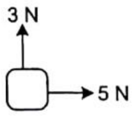
8. The mass and weight of a body

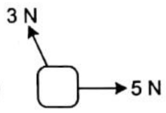
(a) differ by a factor of 9.8
(b) are identical
(c) are both a direct measure of the inertia of body
(d) have the same ratio as that of another body placed at that location

9. Two forces one with a magnitude of 3 N and the other with a magnitude of 5N, are applied to an object. For which orientation of the forces shown in the diagrams is the magnitude of the acceleration of the object would be the least ?

(a) 

(c) 

(b) 

(d) 

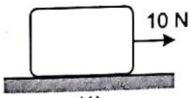
10. An object rests on a horizontal frictionless surface. A horizontal force of magnitude F is applied. This force produces an acceleration

(a) only if F is greater than weight of the object
(b) only if F is increasing
(c) always
(d) None of the above

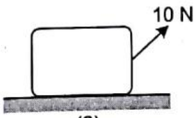
11. The "reaction" force does not cancel the "action" force because

(a) the action is greater than the reaction force
(b) they act upon different bodies
(c) they act at different instants
(d) the reaction comes into existence when action disappears

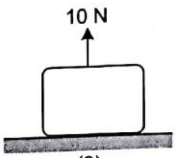
12. A crate (box) rests on a smooth horizontal surface and a person pulls it with a 10 N force. Rank the situations shown below according to the magnitude of the normal force exerted by the surface on the crate, from the least to the greatest.



(1)



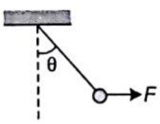
(2)



(3)

(a) 1, 2, 3 (b) 3, 2, 1
(c) 3, 1, 2 (d) 1, 3, 2

13. A 1 N pendulum bob is held in equilibrium at an angle θ with the vertical when a horizontal force $F = 2$ N is acting on it as shown in figure. The tension in the string is



(a) $\cos\theta$ N (b) $\dfrac{2}{\cos\theta}$ N
(c) $\sqrt{5}$ N (d) 1 N

14. When a 40 N force, parallel to the incline and directed up the incline is applied to a block on a frictionless incline that is 30° above the horizontal, the acceleration of the block is 2 ms^{-2} down the incline. The mass of the block is (Take $g = 10$ ms^{-2})

(a) 10 kg (b) $\dfrac{40}{3}$ kg
(c) $\dfrac{20}{3}$ kg (d) 8 kg

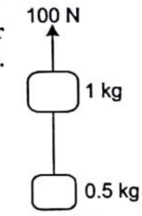
15. Mark out the correct statement.

(a) Newton's laws of motion are universal.
(b) Newton's laws of motion give same result at all times at all places.
(c) Newton's laws of motion define inertial frames of reference.
(d) Non-accelerating frames are non-inertial in nature.

16. A block is moving with constant velocity on a horizontal surface. Mark out the INCORRECT option.

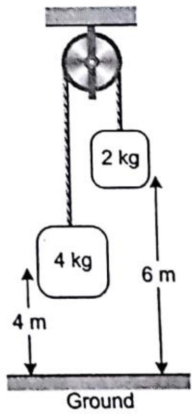
(a) Some forces must act on the block.
(b) Some forces may act on the block.
(c) Net force on the block must be zero.
(d) The block is in equilibrium.

17. Find the tension in the cable for the situation shown in figure. (Take $g = 10$ ms^{-2})



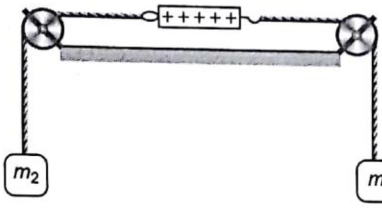
(a) 5 N
(b) 28.34 N
(c) 33.33 N
(d) 85 N

18. The system is released from rest in the position shown in figure. Determine the time taken by 4 kg block to reach ground. (Take $g = 10$ ms^{-2}).



(a) $\sqrt{\dfrac{18}{5}}$ s
(b) $\sqrt{\dfrac{12}{5}}$ s
(c) 3 s
(d) 2.8 s

19. Consider a light spring balance connected to two blocks as shown in figure. For this situation mark out the correct statement(s).



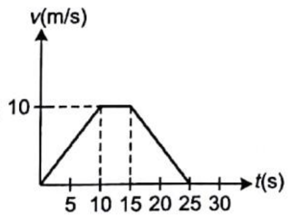
(a) The spring balance reading is $2mg$ if $m_1 = m_2 = m$.

(b) The spring balance reading is $m_1 g$ if $m_1 \neq m_2$.

(c) The spring balance reading is $m_2 g$ if $m_1 \neq m_2$.

(d) The spring balance reading can be greater than or less than $m_1 g$ if $m_1 \neq m_2$.

20. A body starting from rest is pulled along a horizontal frictionless surface by a constant force for 10 s. At the end of this interval the object in moving with a velocity of 10 ms^{-1}. For next 5 s interval the force is zero. At the end of this interval a force of one-half of the original force is acting on object in opposite direction of original force, until the body comes to rest. For this situation velocity-time graph for the motion of the object is

(a)


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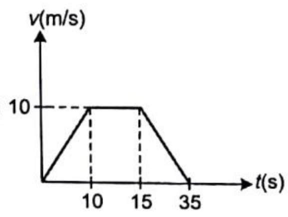

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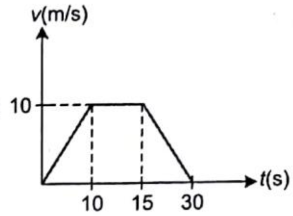
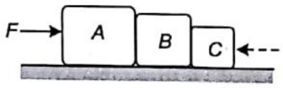


(d) Can't be drawn from the given information.

21. A block of mass 80 kg is to be lowered with the help of a string whose breaking strength is 500 N. The minimum acceleration with which the task would be performed so that string won't break is ($g = 10$ ms^{-2})

(a) 3.75 ms^{-2} (b) 16.25 ms^{-2}
(c) 5 ms^{-2} (d) 10 ms^{-2}

22. Three blocks A, B and C of masses 5 kg, 3 kg and 2 kg respectively are placed on a horizontal surface. If a force $F = 10$ N is applied on A as shown and then in second case on C (shown dotted), then the ratio of normal contact force between B and C in first to second case is [$g = 10$ ms^{-2}, and neglect friction everywhere.]



(a) 13/12 (b) 12/13
(c) 1/4 (d) 4/1

23. An object is moving towards east. From this information one can conclude

(a) that there is a single force acting on a surface towards east

(b) that net force acting on object is towards east

(c) nothing can be predicted about force

(d) that no force is acting on object

24. Determine the magnitude and direction of the net force acting on a stone of mass 0.2 kg, just after it is dropped from the window of a train which is accelerating at 5 ms^{-2}? ($g = 10$ ms^{-2})

(a) 2 N downward

(b) $\sqrt{5}$ N, making an angle of $\tan^{-1}(2)$ with the train's motion

(c) 2 N upward

(d) $\sqrt{5}$ N, making an angle of $\tan^{-1}(2)$ with vertices

25. A lead block is suspended from the ceiling with the help of a string. The reaction to the force of gravity on the block is the force exerted by

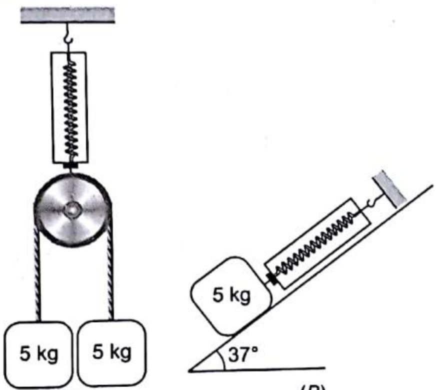
(a) the string on the block

(b) the ceiling on the block

(c) the block on earth

(d) the block on string

26. Which of the following statement regarding the relationship between the readings of the spring balance in arrangements (*A*) and (*B*) is correct?



(*A*) (*B*)

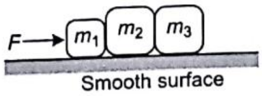
(a) *A* is reading more.

(b) *B* is reading more.

(c) Both scales read the same.

(d) Impossible to predict which is more.

27. Three blocks placed one after the other are pushed by a force *F* as shown in the figure.



Determine the normal reaction exerted by m_2 on m_3.

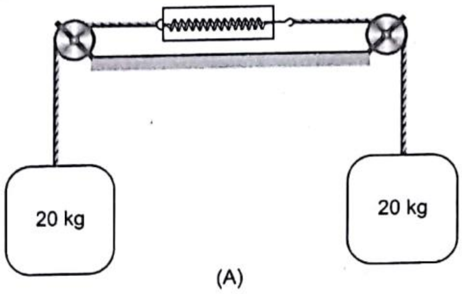
(a) $\dfrac{m_2F}{m_1+m_2+m_3}$

(b) F

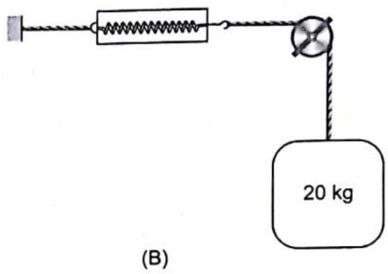
(c) $\dfrac{m_3F}{m_1+m_2+m_3}$

(d) $\dfrac{m_3F}{m_1+m_2}$

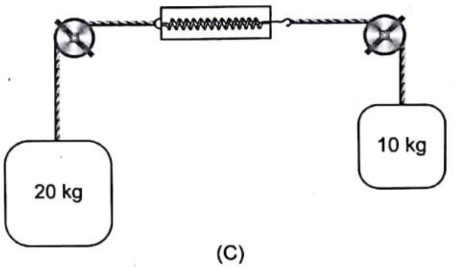
28. Three arrangement of a light spring balance are shown in the figure below.



(A)



(B)



(C)

Reading of the spring scales in three arrangements are respectively,

(a) $20\,g, 20\,g, 10\,g$ (b) $20\,g, 20\,g, \dfrac{40\,g}{3}$

(c) Zero, $20\,g, 10\,g$ (d) Zero, $20\,g, \dfrac{40\,g}{3}$

29. Mark out the most appropriate statement.

(a) The normal force is the same thing as the weight.

(b) The normal force is different from weight, but always has the same magnitude.

(c) The normal force is different from weight, but the two form an action-reaction pair according to Newton's third law.

(d) The normal force is different from weight, but the two may have same magnitude in certain cases.

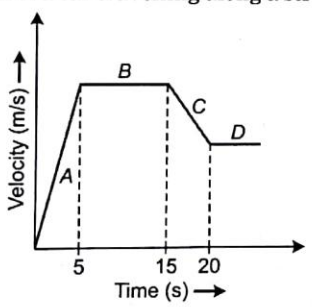
B. More Than One Options Correct

1. A force acting on an object
   (a) can change direction of its velocity
   (b) can change magnitude of its velocity
   (c) must change magnitude of its velocity
   (d) must change direction of its velocity

2. In equilibrium
   (a) the net force acting on object must be zero
   (b) no force must act on object
   (c) force may act on object
   (d) the object must be at rest

3. On which of the following no net force is acting
   (a) a drop of rain falling down with constant velocity
   (b) a cork of mass 10 g floating in water
   (c) a car moving with a constant velocity of 30 kmh^{-1} on a rough road
   (d) a kite skillfully held stationary in the sky

4. The figure below shows the velocity-time graph of a car travelling along a straight line.



For this situation below mark out the correct statement(s).
   (a) No net force acts on the car during interval B and D.
   (b) Non-zero net force is acting on the car during interval A and C.
   (c) Net force acting on the car during interval A is greater than that during C.
   (d) Some opposing force may act on the car during all intervals.

5. Which of the following statement(s) can be explained by Newton's second law of motion?
   (a) To stop a heavy body (say truck), greater force is needed than to stop a light body (say motorcycle) in the same time if they are moving with same speed.
   (b) For a body of given mass, the greater the speed, the greater is the opposing force needed to stop the body in a particular time duration.
   (c) To change the momentum of a body by given value, the force required is independent of time.
   (d) The same force acting on two different bodies for same time causes the same change in momentum for different bodies.

6. Some statements are given below, which in one way or the other can be explained by Newton's laws of motion. Mark out the correct statement(s).
   (a) In a tug of war, the team that pushes the ground harder, wins.
   (b) In a tug of war, the team that pushes the ground harder, (horizontally) wins.
   (c) If observers in two different inertial frames measure the same acceleration of a moving object, then the velocity of object with respect to two observers would be also same.
   (d) A horizontal force acts on a body that is free to move. It can produce an acceleration if this force is equal to half of the weight of the body.

7. Suppose a body that is acted on by exactly two forces is accelerated. For this situation mark out the incorrect statement.
   (a) The body can't move with constant speed.
   (b) The velocity can never be zero.
   (c) The resultant of two forces can't be zero.
   (d) The two forces must act in the same line.

8. Acceleration of a moving object as measured by two observers are found to be non-zero and different, it means
   (a) at least one of the observers is non-inertial in nature
   (b) at least one of the observers is inertial in nature
   (c) both the observers may be non-inertial in nature
   (d) if the object is moving with constant velocity with respect to ground, then both the observers must be non-inertial in nature

C. Assertion & Reason

Directions (Q. Nos. 1 to 4) *Some questions (Assertion-Reason type) are given below. Each question contains **Statement I (Assertion)** and **Statement II (Reason)**. Each question has 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**

(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**

(c) **Statement I** is True, **Statement II** is False

(d) **Statement I** is False, **Statement II** is True

1. **Statement I** In equilibrium of a body some forces might be acting on the body.
   Statement II The resultant of the forces acting on a body may be zero.
2. **Statement I** The acceleration of a particle as seen from an inertial frame can be zero.
   Statement II The net force acting on a particle could be zero.
3. **Statement I** A reference frame attached to the earth is an inertial frame of reference.
   Statement II Newton's laws of motion are valid only in inertial frames of reference.
4. **Statement I** A book is kept at rest on a table, its weight being balanced by the normal contact force between the book and table *ie*, weight and normal contact force between book and table are equal and opposite.
   Statement II In above statement, weight and normal contact force form an action-reaction pair.

D. Comprehend the Passage Questions

Passage I

A body hangs from a spring balance supported from the roof of an elevator. If the elevator moves up with an acceleration of 2 ms^{-2}, the spring balance reading is 120 N. (Take $g = 10$ ms^{-2}).

Based on above information, answer the following questions :

1. The true weight of the body is
   (a) 120 N (b) 100 N
   (c) 80 N (d) 140 N
2. The acceleration of lift so that balance reads 100 N is
   (a) 2 ms^{-2} up (b) Zero
   (c) 2 ms^{-2} down (d) 1.5 ms^{-2} down
3. The reading of the spring balance when the elevator cable breaks is
   (a) Zero (b) 180 N
   (c) 120 N (d) 240 N

Passage II

A stone of mass 0.05 kg is thrown in vertical upward direction. (Take $g = 10$ ms^{-2}). Neglect air friction.

Based on above information, answer the following questions :

4. The net force acting on stone during its upward motion is
   (a) 0.5 N, upward
   (b) 0.5 N, downward
   (c) 5 N, upward
   (d) Zero
5. The net force acting on stone during its downward motion is
   (a) 0.5 N, upward
   (b) 0.5 N, downward
   (c) 5 N, upward
   (d) Zero

6. The net force acting on stone at the highest point where it is momentarily at rest is
(a) 0.5 N, upward
(b) 0.5 N, downward
(c) 5 N, upward
(d) Zero

Passage III

A book is resting on the surface of a table. Consider the following four forces that arise in this situation.

(1) The force of the earth pulling on the book.
(2) The force of the table pushing on the book.
(3) The force of the book pushing on the table.
(4) The force of the book pulling on the earth.

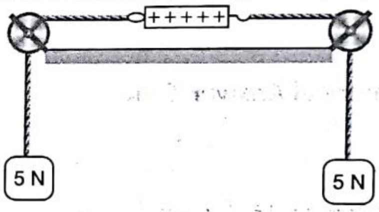
Based on above information, answer the following questions :

7. Which two forces form on action - reaction pair?
(a) 1 and 2 (b) 1 and 3
(c) 1 and 4 (d) 2 and 4

8. Which pair of forces, excluding "action - reaction" pairs, must be equal in magnitude and opposite in direction?
(a) 1 and 2
(b) 1 and 3
(c) 2 and 3
(d) 1 and 4

Passage IV

Two 5 N blocks are attached to opposite ends of a spring scale as shown in figure. The spring scale is light, and pulleys are light and frictionless. Strings are massless. (Take $g = 10\ \text{ms}^{-2}$)



Based on above information answer the following questions :

9. The reading of the scale is
(a) Zero (b) 5 N
(c) 2.5 N (d) 10 N

10. If the above arrangement is present in an elevator which is moving down with an acceleration of $2\ \text{ms}^{-2}$, then spring balance reading will be
(a) Zero (b) 5 N
(c) 4 N (d) 8 N

11. If the right block is replaced by another block of weight 10 N, then spring balance reading will be
(a) Zero (b) 10 N
(c) $\frac{10}{3}$ N (d) $\frac{20}{3}$ N

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. 45 N **2.** 0.1 ms^{-2} **3.** $F_A = 3.6$ N, $F_B = 0.4$ N **4.** 0.3 m
5. 0.3 m **6.** (a) 20/3 cm, (b) 20/3 cm **7.** 150 N
8. (a) Zero, (b) $15\sqrt{3}$ kg-ms^{-1}, (c) $15\sqrt{3}$ N **9.** $4\hat{i}$ Kg-ms^{-1} **10.** $7.5\hat{i}$ N **11.** 92000 N
12. 720 N **13.** 6 ms^{-2}, 18 N **14.** 6 ms^{-2}, 12 N **15.** 5 ms^{-2}, 9.3 N
16. 0 ms^{-2}, 25 N **17.** (a) 1.54 ms^{-2}, 17.31 N, (b) 1.67 ms^{-2}, 33.34 N **18.** 59 N
19. At restore in uniform motion **20.** $\frac{2Wa}{g+a}$ **21.** 0.123 ms^{-2}, 25.61 N
22. (a) down, 2.5 ms^{-2}, (b) up, 2 ms^{-2} **23.** 0.5 ms^{-2} towards left, 80 N **24.** 7.4×10^6 N
25. $60\hat{i} - 80\hat{j}$ **26.** (a) 6800 N, (b) 340 N
27. (a) $\frac{g}{3}, \frac{4}{3}$mg, (b) $\frac{g}{2}, \frac{3}{2}$mg, (c) g, 2 mg **28.** 4 ms^{-2} upward, 28 N, 48 N **30.** 750 Nm^{-1}
31. $T_A = 2.5$ N, $T_B = 1.25$ N **32.** $T_1 = \frac{80g}{13}$ N, $T_2 = \frac{30g}{13}$ N
33. $T_1 = 40$ N, $T_2 = \frac{40}{3}$ N **34.** 10 N **35.** $\frac{3}{2}$F **36.** $\frac{20}{7}$ N
37. $\frac{50}{7}$ N **38.** $T_1 = \frac{180}{7}$ N, $T_2 = \frac{430}{7}$ N, $a = \frac{90}{7}$ ms^{-2} **39.** 2.17 ms^{-2}
40. (a) 4 mg, down, (b) 4 mg, down **41.** 1.6 cm

C. Fill in the Blanks

1. Zero **2.** No/Zero **3.** momentum **4.** Newton's laws are valid
5. net force **6.** acceleration **7.** 400 N

D. True/False

1. F **2.** T **3.** F **4.** F **5.** T **6.** F **7.** F **8.** F
9. T **10.** T **11.** T **12.** T

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (c)	2. (c)	3. (c)	4. (a)	5. (a)	6. (a)	7. (a)	8. (d)	9. (a)	10. (c)
11. (b)	12. (b)	13. (c)	14. (b)	15. (c)	16. (a)	17. (c)	18. (b)	19. (d)	20. (b)
21. (a)	22. (c)	23. (c)	24. (a)	25. (c)	26. (a)	27. (c)	28. (b)	29. (d)	

B. More Than One Options Correct

1. (a, b)	2. (a, c)	3. (a, b, c, d)	4. (a, b, c, d)	5. (a, b, d)
6. (b, d)	7. (a, b, d)	8. (a, c, d)		

C. Assertion & Reason

1. (a)	2. (a)	3. (d)	4. (c)

D. Comprehend the Passage Questions

1. (b)	2. (b)	3. (a)	4. (b)	5. (b)
6. (b)	7. (c)	8. (a)	9. (b)	10. (c)
11. (d)				

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. The acceleration of block w.r.t. car is zero, so *wrt* ground both are moving with same acceleration. Let a be the required acceleration, then

$$F = (M + m)\,a$$

$$\Rightarrow \quad a = \frac{3000}{200} = 15\ \text{ms}^{-2}$$

Net force acting on block is,

$$F_1 = ma = 3 \times 15 = 45\ \text{N}$$

2. $a = \dfrac{F}{M}$

$$= \frac{9.78}{13.1 + 84.7} = \frac{9.78}{97.8} = 0.1\ \text{ms}^{-2}$$

3. Let F_A and F_B represent the magnitudes of two forces $\vec{F}_A$ and $\vec{F}_B$.

Now,

$$F_A + F_B = 8 \times \frac{1}{2} \qquad \text{...1st Case}$$

$$F_A - F_B = 8 \times 0.4 \qquad \text{...2nd Case}$$

After solving above equations, we get

$$F_A = 3.6\ \text{N}, \quad F_B = 0.4\ \text{N}$$

4. In equilibrium, $kx_0 = mg$

$$\Rightarrow \quad 100\,x_0 = 30$$

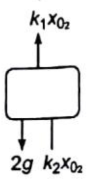
$$\Rightarrow \quad x_0 = 0.3\ \text{m}$$

5. In equilibrium $F = kx_0$

$$30 = 100\,x_0$$

$$\Rightarrow \quad x_0 = 0.3\ \text{m}$$

6. Let in equilibrium spring 1 is elongated by x_{0_1} and spring 2 is compressed by x_{0_2}, then the FBD of block would be as shown in figure.



$$x_{0_1} = x_{0_2}$$

[Both x_{0_1} and x_{0_2} are equal to the displacement of block]

$$(k_1 + k_2)x_{0_1} = 2g$$

$$\Rightarrow \quad x_{0_1} = x_{0_2} = \frac{20}{3} \text{ cm}$$

7. $100 + 5g = N$

$\Rightarrow \quad N = 150 \text{ N}$

8. $|\vec{\mathbf{P}}_i| = 15$ kg-ms^{-1} and $|\vec{\mathbf{P}}_f| = 15$ kg-ms^{-1} with an angle of 60° between $\vec{\mathbf{P}}_f$ and $\vec{\mathbf{P}}_i$

(a) $\Delta|\vec{\mathbf{P}}| = |\vec{\mathbf{P}}_f| - |\vec{\mathbf{P}}_i| = 0$

(b) $|\Delta\vec{\mathbf{P}}| = |\vec{\mathbf{P}}_f - \vec{\mathbf{P}}_i|$

$$= \sqrt{P_f^2 + P_i^2 + 2P_fP_i\cos\theta}$$

$$= \sqrt{15^2 + 15^2 + 2 \times 15^2 \times \frac{1}{-2}}$$

$$= \sqrt{3} \times 15 \text{ kg-ms}^{-1}$$

(c) $|\vec{\mathbf{F}}| = \left|\frac{\Delta\vec{\mathbf{P}}}{\Delta \mathbf{t}}\right| = \sqrt{3} \times 15 \text{ N}$

9. $\vec{\mathbf{P}} = m\vec{\mathbf{v}} = 4\hat{\mathbf{i}}$ kg - ms^{-1}

10. $\vec{\mathbf{F}} = \frac{\vec{\mathbf{P}}_f - \vec{\mathbf{P}}_i}{\Delta t} = 7.5\,\hat{\mathbf{i}}$ N

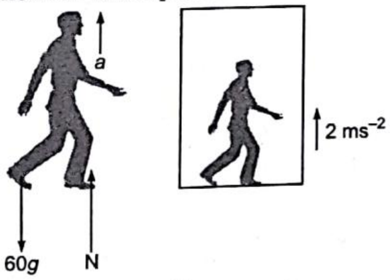
$$|\vec{\mathbf{F}}| = 7.5 \text{ N}$$

11. Let T be the required tension, then

$$T - 8000\text{ g} = 8000 \times 1.5$$

$\Rightarrow \quad T = 92 \text{ kN}$

12. The FBD of the person is as shown in figure.

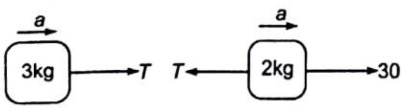


$$N - 60g = 60a$$

$\Rightarrow \quad N = 720 \text{ N}$

13. For 3 kg block,

$$T = 3a \quad \text{...(i)}$$



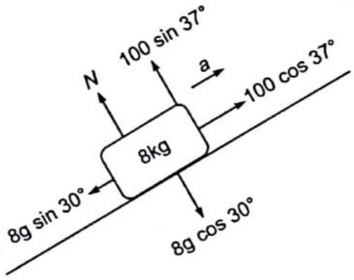
For 2 kg block,

$$30 - T = 2a \quad \text{...(ii)}$$

After solving Eqs. (i) and (ii), we get

$$a = 6 \text{ ms}^{-2} \quad \text{and} \quad T = 18 \text{ N}.$$

15. The FBD of the block is shown in figure.



Along the incline,

$$100\cos 37° - 8g\sin 30° = 8a$$

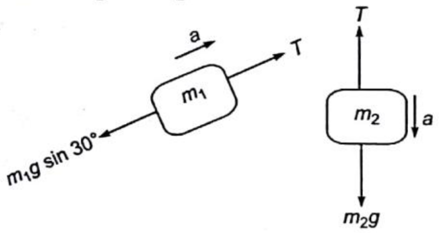
Perpendicular to incline,

$$-100\sin 37° + 8g\cos 30° = N$$

Solving above equations, we get

$$a = 5 \text{ ms}^{-2} \quad \text{and} \quad N = 9.3 \text{ N}$$

16 & 17. We are writing the equation for general masses m_1 and m_2.



$$T - m_1g\sin 30° = m_1a$$

$$m_2g - T = m_2a$$

18. $T - 5g = 5 \times 2$

$\Rightarrow \quad T = 59 \text{ N}$

19. If $T = 49$ N, then $T = mg$ and net force on the block is zero, so block will be at rest or perform uniform motion.

20. $W - F = \frac{W}{g}a$

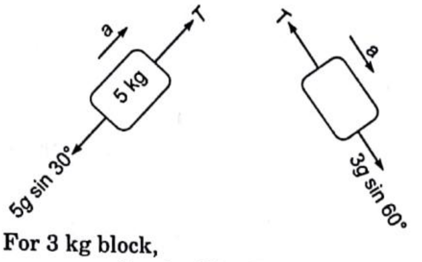
$$F - (W - w) = \frac{W - w}{g} \times a$$

Solving above equations, we get

$$w = \frac{2Wa}{g + a}$$

21. For 5 kg block,

$$T - 5g\sin 30° = 5a$$



For 3 kg block,

$$3g \sin 60° - T = 3a$$

24. As the ship moves with constant velocity it will experience a zero net force.
Resistive force exerted on ship by water = forward thrust force exerted by engine on the ship

$$= 7.4 \times 10^6 \text{ N}$$

25. $\vec{F}_1 = 80\hat{j}, \vec{F}_2 = -60\hat{i}$

Let $\vec{F}_3$ is the 3rd force, then $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$ as the object is moving with constant velocity.

$$\vec{F}_3 = 60\hat{i} - 80\hat{j}$$

26. (a) $T_1 = (20 \times 6.8 \times 10^3) \times 0.05 = 6800$ N

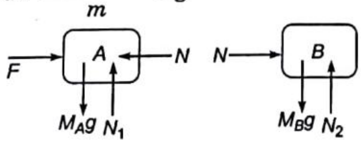
(b) $T_2 = 6.8 \times 10^3 \times 0.05 = 340$ N

27. (a) $a = \dfrac{2mg - mg}{3m} = \dfrac{g}{3}$

(b) $a = \dfrac{2mg - mg}{2m} = \dfrac{g}{2}$

(c) $a = \dfrac{2mg - mg}{m} = g$

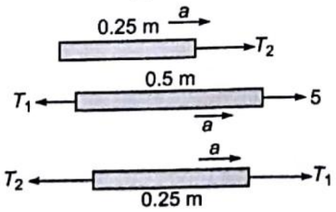
29.



31. Mass per unit length of string is $\dfrac{2 \text{ kg}}{1 \text{ m}} = 2 \text{ kg m}^{-1}$.

Let the tension at A and B be T_1 and T_2

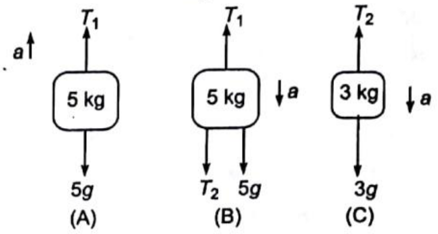
$$a = \frac{5}{2} = 2.5 \text{ ms}^{-2}$$



$$5 - T_1 = (0.5 \times 2) \times a$$
$$T_1 - T_2 = (0.25 \times 2) \times a$$
$$T_2 = (0.25 \times 2) \times a$$

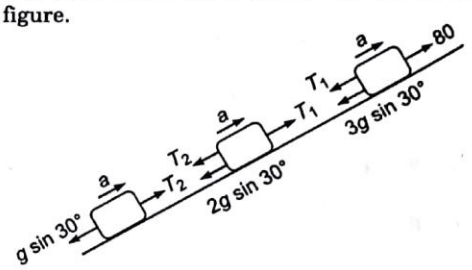
32. For block A,

$$T_1 - 5g = 5a$$



For block B, $T_2 + 5g - T_1 = 5a$
For block C, $3g - T_2 = 3a$

33. The FBD of the three blocks are shown in figure.

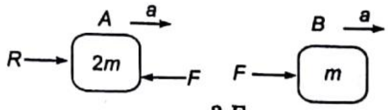


For 1 kg block, $T_2 - g \sin 30 = 1 \times a$
For 2 kg block, $T_1 - T_2 - 2g \sin 30 = 2 \times a$
For 3 kg block, $80 - T_1 - 3g \sin 30 = 3a$

34. For block, use $s = ut + \dfrac{1}{2}at^2$.

$$\Rightarrow \quad 10 = 0 + \frac{1}{2} \times a \times 2^2$$
$$\Rightarrow \quad a = 5 \text{ ms}^{-2}$$
$$F = ma = 2 \times 5 = 10 \text{ N}$$

35. For $A \rightarrow R - F = 2ma$
For $B \rightarrow F = ma$



$$\Rightarrow \quad R = \frac{3F}{2}$$

41. Let k be spring constant, then $kx = mg$.

$$\Rightarrow \quad k \times (0.04) = 5g$$

Let further elongation be x_0, then

$$k(0.04 + x_0) = 7g$$
$$\Rightarrow \quad x_0 = 1.6 \text{ cm}$$

Chapter 6

Friction

The First Steps' Learning

- The Nature of Friction
- Static and Kinetic Friction

In previous chapter we limited our discussion to the surfaces which were smooth ie, frictionless. In this chapter we are going to discuss about the friction which is a property of two surfaces in contact. Between the frictionless surfaces the contact force is along the normal to surface, but this is not the case with rough surfaces ie, surfaces having friction.

The Nature of Friction

If a toy car has been pushed by a child on a horizontal surface, then it stops by itself after travelling some distance. This is due to the ever- present opposing frictional force by the surface on toy car. If a pendulum bob is displaced from its equilibrium position and released then it will finally stop after making some oscillations, this is because of the air frictional force acting on bob. If you ride a bicycle on a levelled road and stop pedalling, then it will move for some time and it finally stops, this is again due to opposing frictional force by road on tyres. From these we may conclude that *"friction is a property between two surfaces which opposes motion"*. But before making final conclusion let us see some other physical situations. Have you ever tried to walk on ice ? If you have experienced this, then you know it is very difficult to walk on ice as it is very slippery and smooth. It is also a very common experience, that it is easier to run on a rough surface as compared to on a smooth surface without skidding. Now, from this you may conclude that *friction causes motion,* but how it is possible that friction can cause the motion as well as it can oppose the motion ? Here is the answer, in the correct interpretation of friction—**"Friction is the property of two surfaces by virtue of which it opposes the relative motion between two surfaces"**. The word 'relative' is the most important term to grasp the meaning of friction well.

Now, we are ready to explain how friction causes the motion in certain situations. Until now we have considered only smooth surfaces where the contact force between two surfaces is along the normal but this is not the case in general. When the surface is rough, the contact force between the surfaces is not along the normal, but it is at some angle to normal as shown in figure. The component of this contact force along the normal is termed as normal contact force and the component along the surface is termed as frictional force (f). The magnitude and direction of contact force (R) is dependent upon the nature of surfaces, the other forces applied and the state of motion on the surface.

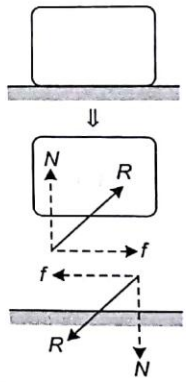


Fig. 6.1 Component of contact force along the surface is termed as friction.

Here we are skipping the cause of friction, we are leaving this for you to study in your later years of life. Here we are only going to give you some empirical relations that have been developed and which make possible to account for the effects of friction.

Static and Kinetic Friction

Consider a heavy box placed on a horizontal surface as shown in Fig. 6.2. If you exert a small horizontal force on it, then it won't move as a result of this force. From Newton's laws you can conclude that net force acting on it

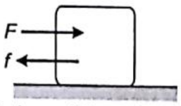


Fig. 6.2 A small force is not able to cause the motion of heavy block.

is zero in horizontal direction. But one force F, however small it may be, is acting on block towards right, if no other force is acting on the block then it must move, but as it is not moving it means some other external force is acting on block which is equal and opposite to F. This force is nothing but *frictional force* acting on the block *ie*, horizontal component of contact force between the block and horizontal surface. This small and simple observation discloses the existence of frictional force.

Now replace the applied small force by a linearly increasing force *ie*, the force F is not constant but an increasing one say a time varying force given by $F = 2t$ where t is in second.

So, at $t = 0.25\text{ s}$, $F = 0.5\text{ N}$,

at $t = 0.5\text{ s}$, $F = 1.0\text{ N}$ and so on.

It is quiet obvious that block doesn't move for some time say up to t_0 sec, and it starts to move only after $t = t_0$ sec. Now just try to analyse the situation, let us take $t_0 = 10\text{ s}$. Now at $t = 1$ s, the force applied is 2 N and as the block is not moving so frictional force is equal to 2 N and acting leftward. At $t = 2$ s, the force applied is 4 N and still the block is at rest and hence frictional force is equal to 4 N, and is opposite to the applied force. This reasoning continues till $t = t_0$, as the block is at rest up to this instant. The frictional force when there is no relative motion between the two surfaces is termed as *static friction*. Here, in this case, it is the situation up to $t = t_0$ sec. From above discussion we can easily conclude that static frictional force value is equal to applied force and directed opposite to the applied force. Eventually, we can say that **static friction is self-adjusting in nature *ie*, it adjusts its magnitude and direction in such a way that altogether with other forces the net force acting on the object is zero.**

Static friction opposes the **impending** motion, the term impending motion means the motion that would take place (but in actual, not taking place) under the action of applied force if friction were absent. In above illustration if friction won't be present, then the block tends to move towards right under the action of applied force, and hence frictional force is towards left to oppose this impending motion. In simpler words, we can say that to find the direction of static frictional force we will assume that in which way the body can move if no friction is present and hence opposite to direction of impending motion (imaginary motion) would be the direction of static frictional force.

So in short we can say that when there is no relative motion between two surfaces, the friction is static in nature and its value is equal to the applied force and direction is such as to oppose the impending motion. Still the exposition of static friction is not completed, let's have a look on other aspect.

From above discussion,we can say that static frictional force is equal and opposite to applied force, but what happens if applied force is continuously increasing? Will the block always remain at rest ? Obviously your answer would be no it means for some value of applied forces the relative motion between two surfaces will start, and there is a maximum value of applied force up to which there is no relative motion between the surfaces. This corresponds to the maximum value of static frictional force and is termed as **limiting friction (f_L)**. Thus, we can say that static frictional force can have any value between zero and f_L.

From experiments, $f_L \propto N$

where N is normal contact force between surfaces.

Removing the proportionality sign, we have

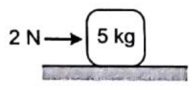
$$f_L = \mu_s N$$

where μ_s is the coefficient of static friction and its value depends upon the nature of surfaces. In general, its value can be anything between 0 and 1.

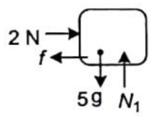
C-BIs

Concept Building Illustrations

Illustration | 1 *A block of mass 5 kg is at rest on a rough horizontal surface. An external horizontal force of 2 N is applied to the block as shown in figure. Determine the frictional force acting on block.*



Solution As the block is at rest, the net external force acting on block would be zero. The free body diagram of block is as shown in figure. As the block is at rest, the friction between block and surface is static in nature.



For horizontal equilibrium of block,

$$2\text{ N} - f = 0$$

$$\Rightarrow \quad f = 2\text{ N}$$

For vertical equilibrium of block,

$$N_1 - 5g = 0$$

$$\Rightarrow \quad N_1 = 5g$$

Illustration | 2 *In above illustration, if $\mu_s = 0.6$ and $g = 10\text{ ms}^{-2}$, then determine the value of frictional force.*

Solution Here first of all find the limiting frictional force,

$$f_L = \mu_s N_1 = 0.6 \times 5g = 30\text{ N}$$

As applied force is not able to cause the motion ($\because F < f_L$) so the friction is static in nature. So f = applied force $\Rightarrow f = 2$ N.

Illustration | 3 *In illustration 1 determine the minimum value of coefficient of static friction so that the block remains at rest.*

Solution Let μ_s is the value of coefficient of static friction, then limiting value of static friction is

$$f_L = \mu_s N_1 = \mu_s \times 5g$$

As the block has to remain at rest so, $f \le f_L$ where f is the frictional force. From equilibrium of block $f = 2$ N.

So, $$2 \le \mu_s \times 5g$$

$$\Rightarrow \quad \mu_s \ge \frac{2}{50} = 0.04$$

So minimum value of μ_s for block not to move is 0.04.

Now, just try to answer, what will happen if applied force becomes greater than limiting frictional force? Does the frictional force remain constant and equal to limiting frictional force or is it increasing or decreasing ? The friction which comes into existence when there is relative motion between two surfaces is termed as **kinetic friction**. From common experience and experimental observation, it is clear that external force required to maintain the motion of body is somewhat less than the external force required to start the motion of body, *ie*, in above described case up to $t = t_0$ sec, the motion of the body is not taking place but as $t > t_0$ sec the body starts moving and the frictional force between body and surface is termed as kinetic frictional force and this kinetic frictional force is constant and less than the limiting frictional force.

From experiments, the kinetic frictional force is given by

$$f_k = \mu_k N$$

where N is the normal contact force between two surfaces and μ_k is the coefficient of kinetic friction. The value of μ_k is generally less than μ_s and its value also depends on the same parameters as on which μ_s depends. Its value is also lying between 0 and 1 just like μ_s.

Thus, in short we can conclude, when there is relative motion between two surfaces the friction is kinetic in nature and the kinetic

frictional force is constant and its direction is such as to oppose the relative motion between two surfaces.

To have a clear idea about direction of kinetic frictional force let's consider two blocks A and B placed one over the other as

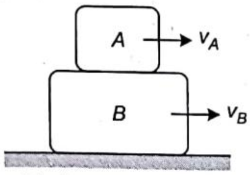


Fig. 6.3 Two blocks are moving one over another

shown in figure. Under the application of some external forces let us say that the blocks are moving in such a way that both the blocks are moving towards right with velocities v_A and v_B

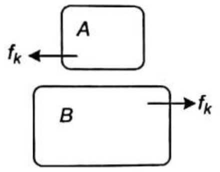


Fig. 6.4 $v_A > v_B$ so A is moving towards right *wrt* B.

such that $v_A > v_B$, then it means *wrt* B, A is moving towards right so the kinetic frictional force on A is towards left and on B is towards right. Above description can be summarized as, "The direction of kinetic frictional force on A due to B is opposite to the direction of velocity of A wrt B".

If we plot a graph between frictional force and continuously increasing applied force, then it would be like as shown in figure. For small values of F, the object is not moving and the friction is static in nature, as a result $f = F$, so f *versus* F curve is a straight line having slope 45°, this is shown by OA in figure. As F value goes beyond f_L, the block starts moving and the friction becomes kinetic in nature as $f_k < f_L$, so there is a small kink at $F = f_L$ shown by AB in figure and thereafter f *versus* F curve is a straight line parallel to force (F) axis represented by BC in figure.

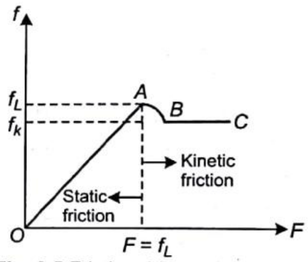


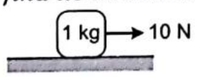
Fig. 6.5 Frictional force depends on applied force in certain cases

C-BIs

Concept Building Illustrations

Illustration | 4 *An horizontal force of* 10 *N is applied to a block of mass placed on a rough horizontal surface. The coefficient of static and kinetic friction are* 0.6 *and* 0.45, *respectively. Determine*

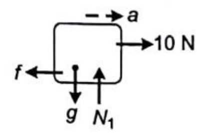
(*a*) *the value of frictional force acting on block and also find its direction.*



(*b*) *the acceleration of block.*

Is the friction static or kinetic in nature ? [*Take* $g = 10\,ms^{-2}$]

Solution First of all draw the free body diagram of block.



In vertical direction there is no motion of block and hence, $N_1 = g = 10$ N.

$$f_L = \mu_s N_1 = 0.6 \times 10 = 6\text{ N}$$

as f_L is less than applied force it means relative motion is there between block and surface and hence friction is kinetic in nature.

So, frictional force,

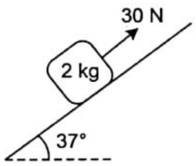
$$f = f_k = \mu_k N_1 = 4.5 \text{ N}$$

Let acceleration of block is a, then

$$10 - f = 1 \times a$$

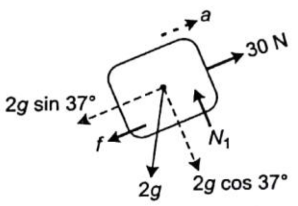
$$\Rightarrow \quad a = 10 - 4.5 = 5.5 \text{ ms}^{-2}$$

Illustration | 5 *A block of mass 2 kg is on a rough inclined plane making an angle of 37° with the horizontal as shown in figure. A force of 30 N parallel to the incline acts on the block. The coefficient of friction between the block and incline is 0.5. Determine the acceleration of block.*
(Take $g = 10 ms^{-2}$, $\sin 37° = 3/5$, $\cos 37° = 4/5$)



Solution Free body diagram of the block is as shown in figure. Let acceleration of block be a up the incline.

$$N_1 = 2g\cos 37° = 2 \times 10 \times \frac{4}{5} = 16 \text{ N}$$



$$f = \mu N_1 = 0.5 \times 16 = 8 \text{ N}$$

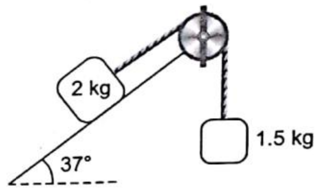
Writing the second law equation for block,

$$30 - 2g\sin 37° - f = 2 \times a$$

$$\Rightarrow \quad 30 - 2 \times 10 \times \frac{3}{5} - 8 = 2a$$

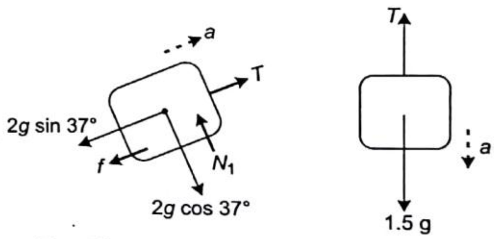
$$\Rightarrow \quad a = 5 \text{ ms}^{-2}$$

Illustration | 6 *Two blocks of masses 2 kg and 1.5 kg are connected with the help of a string as shown in figure. The coefficient of friction between 2 kg block and incline is 0.5. If the system is released from rest, then determine the acceleration of two blocks and the tension in string.* (*Take $g = 10 \ ms^{-2}$, $\sin 37° = \frac{3}{5}$, $\cos 37° = \frac{4}{5}$*)



Solution As the two blocks are connected to the same string, both will move with same acceleration. Let acceleration of 1.5 kg block is a in vertical downward direction and of 2 kg block is a up the incline and tension in string is T.

The free body diagram of blocks are as shown in figures.



For 2 kg block,

$$N_1 = 2g\cos 37° = 16 \text{ N} \quad \text{...(i)}$$

$$f = \mu N_1 = 0.5 \times 16 = 8 \text{ N} \quad \text{...(ii)}$$

$$T - 2g\sin 37° - f = 2a \quad \text{...(iii)}$$

$$\Rightarrow \quad T - 2 \times 10 \times \frac{3}{5} - 8 = 2a$$

$$\Rightarrow \quad T - 20 = 2a \quad \text{...(iv)}$$

For 1.5 kg block,

$$1.5g - T = 1.5a \quad \text{...(v)}$$

Adding Equations (iv) and (v), we have

$$1.5g - 20 = 3.5a$$

$$\Rightarrow \quad a = -\frac{5}{3.5} = -1.43 \text{ ms}^{-2}$$

The acceleration in above illustration comes out to be negative; it means that the assumed direction of acceleration for the solution was wrong. Before proceeding further let's be clear of one more concept.

If value of acceleration comes out negative, then it is obvious that our assumed direction of acceleration is wrong and the system may or may not move in the direction opposite to that of assumed direction. But you may ask what about the value of acceleration ? To answer this question categorise the situation in two—

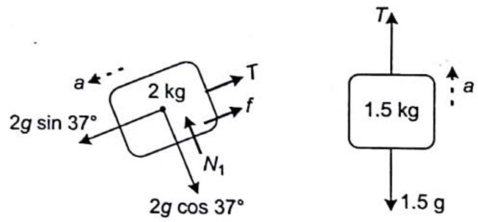
(a) When there is no friction.

(b) When there is friction.

(a) When there is no friction In this situation, if you get acceleration as –ve say –2 ms^{-2}, it means your assumed direction of motion was wrong and the system is moving in a direction opposite to that of assumed direction with acceleration equal to 2 ms^{-2} *ie*, magnitude of acceleration remains same even if direction is assumed wrong.

(b) When there is friction In this case, if you get $a = -1.43\ ms^{-2}$ like in Illustration 6, then it means your assumed direction of acceleration was wrong and what about the magnitude of acceleration ? Will it be 1.43 ms^{-2} ? No, in this case the magnitude of acceleration is not equal to 1.43 ms^{-2}. The reason is, if direction of acceleration and hence direction of motion reverse, the direction of frictional force also reverses and the equations corresponding to Newton's second law change and give different numerical values. So, in this case if acceleration comes out to be –ve, then start again from the very beginning, assume direction of acceleration opposite to that of already assumed direction and then write Newton's second law equation. Now you may ask, what will happen if acceleration comes out to be –ve from both ways, then it means the system is not moving and the friction is static in nature.

Again, we come back to Illustration 6. We assume that 2 kg block is moving down the incline with acceleration a. Free body diagrams of the blocks are as shown in the figures.



For 2 kg block,

$$N_1 = 2g\cos 37° = 16\text{ N}$$
$$f = \mu N_1 = 8\text{ N}$$
$$2g\sin 37° - T - f = 2a$$

For 1.5 kg block,

$$T - 1.5\ g = 1.5\ a$$

Solving above equations, we again get a value with (–ve) sign equal to $-3.14\ ms^{-2}$.

So, it means the blocks are not moving *ie*, their acceleration is zero.

For equilibrium of 1.5 kg block,

$$T = 1.5\ g = 15\text{ N}$$

Here, in this question, you can also find frictional force which is static in nature. You can find the magnitude and direction of frictional force by using the self-adjusting property of static friction.

Proficiency in Concepts (PIC)

Problems

Problem | 1 *An ice scooter is resting on horizontal patch of snow and the coefficient of static friction is 0.350. The ice scooter and its rider has a total mass of 100 kg. What is the magnitude of the maximum horizontal force that can be applied to ice scooter before it just begins to move ? (Take $g = 10\, ms^{-2}$)*

Solution For no motion of ice scooter the horizontal force acquires maximum value when its magnitude becomes equal to that of the limiting frictional force.

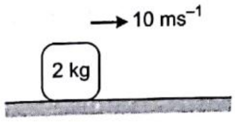
$$f_L = \mu_s N_1$$

where $N_1 = mg = 100 \times 10 = 1000$ N

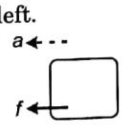
$$f_L = 0.35 \times 1000 = 350 \text{ N}$$

So, the magnitude of maximum horizontal force is 350 N.

Problem | 2 *A block of mass 2 kg is moving towards right with a velocity of $10\, ms^{-1}$ on a rough horizontal surface as shown in figure. The coefficient of friction is $\mu = 0.4$. Determine the distance travelled by block before it stops.*
(Take $g = 10\, ms^{-2}$)



Solution As the block is moving towards right relative to surface, the frictional force on it is acting towards left.



$$f = \mu \times mg = 0.4 \times 2 \times 10 = 8 \text{ N}$$

Acceleration of block is,

$$a = \frac{f}{m} = \frac{8}{2} = 4 \text{ ms}^{-2}$$

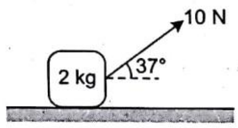
Let the block stops after travelling a distance's then by using the equations of motion,

$$v^2 = u^2 + 2as$$

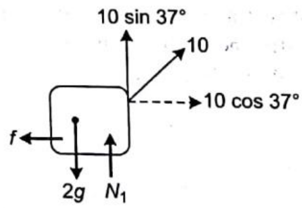
$$\Rightarrow \quad 0 = (10)^2 - 2 \times 4 \times s$$

$$\Rightarrow \quad s = \frac{100}{8} = 12.5 \text{ m}$$

Problem | 3 *For the block of mass 2 kg shown in figure, determine the frictional force acting on the block. The coefficient of friction is 0.5.*
(Take $g = 10\, ms^{-2}$, $\sin 37° = \frac{3}{5}$, $\cos 37° = \frac{4}{5}$)



Solution The free body diagram of the block is as shown in figure.



$$N_1 + 10 \sin 37° = 2g$$

$$\Rightarrow \quad N_1 = 20 - 10 \times \frac{3}{5} = 14 \text{ N}$$

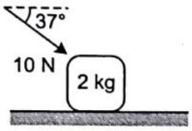
Limiting frictional force,

$$f_L = \mu N_1 = 0.5 \times 14 = 7 \text{ N}$$

As horizontal component of applied force *ie*, $10 \cos 37° = 8$ N is greater than f_L so friction is kinetic in nature and equals to 7 N.

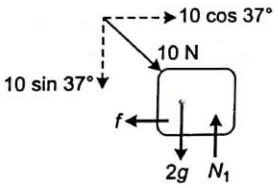
In this situation as only coefficient of friction is mentioned, so both μ_s and μ_k would be taken as same equal to μ.

Problem | 4 *A block of mass 2 kg is pushed from the back with a force of 10 N as shown in figure. The coefficient of friction between block and surface is $\mu = 0.5$. Determine the frictional force.*



(*Take* $g = 10\,ms^{-2}$, $\sin 37° = \frac{3}{5}$, $\cos 37° = \frac{4}{5}$.)

Solution Free body diagram of the block is as shown in figure.



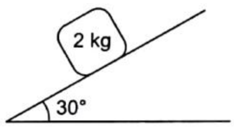
$$N_1 = 2\,g + 10 \sin 37° = 26\text{ N}$$

Limiting frictional force is,

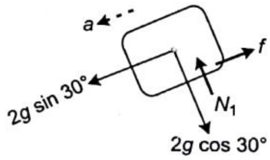
$$f_L = \mu N_1 = 0.5 \times 26 = 13\text{ N}$$

As $10 \cos 37° (= 8\text{ N})$ is less than f_L, so the block is not moving and friction is static in nature having value equal to 8 N *ie*, the horizontal component of applied force.

Problem | 5 *A block of mass 2 kg is released from rest on an inclined plane as shown in figure. The coefficient of friction between block and plane is $\frac{1}{2\sqrt{3}}$. Determine the acceleration of block.* (*Take* $g = 10\,ms^{-2}$)



Solution Let the block is moving down the incline with acceleration a, then the frictional force is acting up. Free body diagram of the block is as shown in figure.



$$N_1 = 2\,g \cos 30° = 2\,g \times \frac{\sqrt{3}}{2}$$

$$f = \mu N_1 = \frac{1}{2\sqrt{3}} \times 2\,g \times \frac{\sqrt{3}}{2} = 5\text{ N}$$

From Newton's second law equation,

$$2\,g \sin 30° - f = 2\,a$$

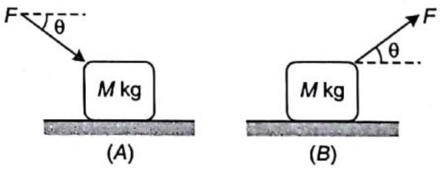
$$\Rightarrow \quad 2 \times 10 \times \frac{1}{2} - 5 = 2\,a$$

$$\Rightarrow \quad a = 2.5\text{ ms}^{-2}$$

Towards Proficiency Problems
Exercise 1

A. Subjective Discussions

1. Suppose that the coefficients of static and kinetic friction have values such that $\mu_s = 2\mu_k$ for a block in contact with rough horizontal surface. Does this mean that magnitude of static frictional force acting on block at rest would always be twice the magnitude of the kinetic frictional force acting on the moving block ? Explain your answer.
2. Which one is better—*A* or *B* to move the block ?



3. Why it is difficult to walk on solid ice ?
4. Can we accelerate up a car on a frictionless horizontal surface ?
5. Analyse the tug of war game in different circumstances.
6. A car is driven up a steep hill at constant speed. Discuss all the forces acting on the car. What pushes it up the hill ?
7. To push a box up the incline, is the force required is smaller if you push it horizontally or if you push it parallel to the incline ? Explain your answer.
8. A block rests on an inclined plane with enough friction to prevent it from sliding down. To start the block moving, is it easier to push it up the plane, down the plane, or sideways. Why ?
9. Can the force of kinetic friction acting on an object ever cause that object to speed up ? If it cannot, explain why not? If it can, give at least one example. Repeat the question for static frictional force.

B. Numerical Answer Types

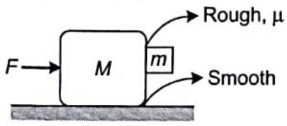
1. If the coefficient of friction between tyres of a vehicle and road is 0.5, then what is the shortest distance in which the vehicle can be stopped when travelling at 90 km h^{-1}? [Take $g = 10 \text{ ms}^{-2}$]
2. A block moving with 30 ms^{-1} on a rough horizontal floor travels 105 m before coming to stop. Determine the coefficient of friction between the block and surface. [Take $g = 10 \text{ ms}^{-2}$]
3. A forward horizontal force of 12 N is used to pull a 240 N cart at constant velocity on a rough horizontal floor. Determine the coefficient of friction.

4. A box rests on the floor of an elevator. Because of static friction a force is required to start the box sliding across the floor when the elevator is (a) stationary, (b) accelerating upward, (c) accelerating downward. Rank the forces required in three situations in ascending order *ie*, from the smallest first. Explain your reasoning.

5. A block whose weight is 45 N rests on a horizontal table. A horizontal force of 36 N is applied to the block. The μ_s and μ_k values are 0.65 and 0.42 respectively. Will the block move under the influence of force and if so, what would be the acceleration of block ? [Take $g = 10\ \text{ms}^{-2}$]

6. A 60 kg block rests on a rough horizontal surface. What horizontal pushing force is required to (a) just start the block moving and (b) slide the block across the surface at constant speed ? (Take $\mu_s = 0.76, \mu_k = 0.41, g = 10\ \text{ms}^{-2}$)

7. For the situation shown in figure determine the smallest value of F so that m doesn't fall down.



8. A 200 kg block is pulled up on an incline by means of a light rope that is parallel to the incline. The incline is making an angle of 30° with the horizontal. Coefficient of friction between the block and incline is 0.75. If the block is moving up with an acceleration of 0.5 ms^{-2}, then determine the tension in rope.

9. A block is resting on the floor of a moving truck. The coefficient of static friction between the block and the truck floor is 0.4. Assuming the truck is travelling on a level road, determine the maximum deceleration that the truck can have without the block slipping forward relative to the truck. [Take $g = 10\ \text{ms}^{-2}$]

10. A train engine weighing 100 ton (1 ton = 1000 kg) is to pull n cars on a level track. Each car mass is 10 ton. The coefficient of friction between the rails (track) and the wheels of engine is 0.2. If the train is capable to have an acceleration of 0.5 ms^{-2} without slipping, how many cars the engine can pull up ?

11. A block rests on an incline which is making an angle θ with the horizontal. The coefficient of static friction between the block and incline is μ_s. Find the maximum value of θ at which the block starts slipping on incline ?

12. A block slides with constant velocity down an inclined plane of slope α. If it is projected up the same plane with an initial velocity v_0, how far up the plane will it move before coming to rest ?

13. A body of mass 0.4 kg slides on a rough horizontal surface. If the frictional force is 3 N, find (a) the angle made by the contact force on the body with the vertical, and (b) the magnitude of the contact force. [Take $g = 10\ \text{ms}^{-2}$]

14. A box of mass 20 kg is pulled on a rough horizontal surface by applying a horizontal force. If the coefficient of friction between the box and surface is 0.25, find the frictional force exerted by the surface on the box. [Take $g = 10\ \text{ms}^{-2}$]

15. In above question if the applied force is (a) making an upward angle with horizontal, (b) making downward angle with horizontal, then can we determine the frictional force from given information. If not what additional information we need ?

16. A wooden block is kept on a wooden plank and the inclination of the plank is gradually increased. It is found that the block starts slipping when the plank makes an angle of 37° with the horizontal. However, once the motion starts, the block can continue with uniform speed if the inclination is reduced to 30°. Find the coefficients of static and kinetic friction between the block and the plank.

17. A block slides down an incline of angle 30° with acceleration $g/4$. Find the coefficient of friction between the incline and block. [Take $g = 10 \text{ ms}^{-2}$]

18. A block of mass 2.5 kg is kept on a rough horizontal surface. It is found that the block does not slide if a horizontal force less than 15 N is applied to it. Also it is found that it takes 5 s to travel first 10 m if a horizontal force of 15 N is applied to it and the block is gently pushed to start the motion. Determine the coefficients of static and kinetic friction between the surface and block. [Take $g = 10 \text{ ms}^{-2}$]

C. Fill in the Blanks

1. Friction opposes .... motion.
2. Kinetic frictional force is always less than .... frictional force.
3. A block of mass m is kept on a rough horizontal table. If the coefficient of friction is μ, then the frictional force acting on the block is ....
4. A body is slipping on a rough horizontal surface with a deceleration of 4 ms^{-2}. The coefficient of kinetic friction between the block and surface is ....
5. Two identical wooden blocks are tied one behind the other and pulled across a rough horizontal surface with the help of a horizontal force. If the force required to pull them at constant speed is F, and if we attach the block one over the other, then the new force required to pull the blocks with constant speed is ....
6. Direction of static friction is in such a way that it opposes the .... motion.
7. When there is relative motion between two rough surfaces, then contact force between two surfaces is always greater than ....

D. True/False

1. Friction can aid the motion of a body.
2. Static frictional force is always greater than the kinetic frictional force.
3. Friction opposes motion between two surfaces.
4. If magnitude of frictional force is zero then it is certain that surfaces are smooth.
5. The equation $f = \mu N$ is a vector equation.
6. The frictional force can exceed the reaction force on the body from the supporting surface.
7. When a body slides on a rough surface, the frictional force always acts opposite to the direction of the applied force.
8. Friction always acts in a direction opposite to the motion of body.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. A brick slides on a horizontal surface. Which of the following will increase the frictional force on it ?
   (a) Putting a second brick on it
   (b) Decreasing the surface area of contact
   (c) Increasing the surface area of contact
   (d) Decreasing the mass of brick

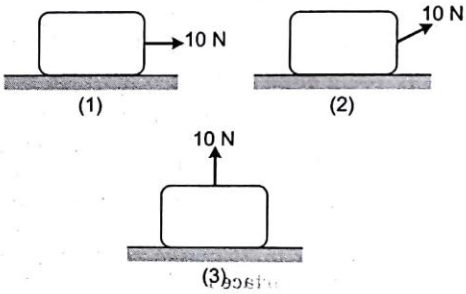
2. The coefficient of kinetic friction
   (a) is in the direction of the frictional force
   (b) is in the direction of the normal force
   (c) is the ratio of force to area
   (d) None of the above

3. When the brakes on an automobile are applied, the road exerts the greatest retarding force
   (a) while the wheels are sliding on the road
   (b) just before the wheels start to slide on the road
   (c) when the automobile is going the fastest
   (d) when the acceleration is the least

4. A 12 N horizontal force is applied to a 40 N block on a rough horizontal surface. The block is initially at rest. If $\mu_s = 0.5$ and $\mu_k = 0.4$, the frictional force on the block is
   (a) 18 N (b) 12 N
   (c) 20 N (d) 30 N

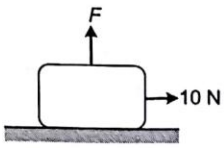
5. A 24 N horizontal force is applied to 40 N block initially at rest on a rough horizontal surface. If $\mu_s = 0.5$ and $\mu_k = 0.4$, the frictional force on the block is
   (a) 8 N (b) 12 N
   (c) 20 N (d) 16 N

6. A crate rests on a horizontal surface and a woman pulls on it with a 10 N force. No matter what the orientation of the force, the crate does not move. Rank the situations shown below according to the magnitude of the frictional force exerted by the surface on the crate, from the least to the greatest.



   (a) 1, 2, 3 (b) 2, 1, 3
   (c) 2, 3, 1 (d) 3, 2, 1

7. A box with weight 50 N rests on a horizontal surface. A person pulls horizontally on it with a force of 10 N and it does not move. To start it moving, a second person pulls vertically upward on the box. If the coefficient of static friction is 0.4, what is the smallest vertical force for which the box moves ?



   (a) 4 N (b) 10 N
   (c) 14 N (d) 25 N

8. A horizontal force of at least 20 N is required to start moving a 800 N crate initially at rest on a horizontal floor. The coefficient of static friction is
   (a) 0.25 (b) 0.125
   (c) 0.50 (d) 4.00

9. A force $\vec{F}$ (larger than the largest possible force of static friction) is applied to the left to

an object moving to the right on a horizontal surface. Then

(a) the object must be moving at constant speed

(b) $\vec{F}$ and the frictional force act in the opposite directions

(c) the object must be slowing down

(d) the object must be speeding up

10. A block rests on a rough horizontal surface ($\mu_s = 0.50$, $\mu_k = 0.40$). A constant horizontal force, just sufficient to start the block in motion, is applied. The acceleration of the block, in ms^{-2}, is

(a) Zero (b) 0.98

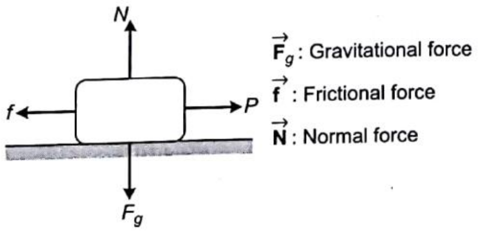
(c) 3.3 (d) 4.5

11. A 12 kg crate rests on a horizontal surface and a boy pulls on it with a force that is 30° above the horizontal. If the coefficient of static friction is 0.40, the minimum magnitude of the force he needs to start the crate moving is

(a) 44 N (b) 47 N

(c) 54 N (d) 56 N

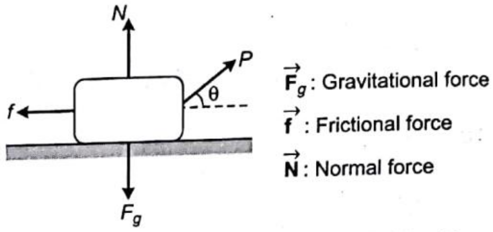
12. A boy pulls a wooden box along a rough horizontal floor at constant speed by means of a force $\vec{P}$ as shown. In the diagram f is the magnitude of the frictional force, N is the magnitude of the normal force, and F_g is the magnitude of the gravitational force. Which of the following must be true ?



(a) $P = f$ and $N = F_g$ (b) $P = f$ and $N > F_g$

(c) $P > f$ and $N < F_g$ (d) $P > f$ and $N = F_g$

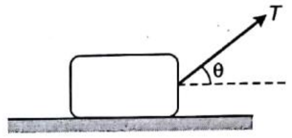
13. A boy pulls a wooden box along a rough horizontal floor at constant speed by means of a force $\vec{P}$ as shown. In the diagram f is the magnitude of the frictional force, N is the magnitude of the normal force, and F_g is the magnitude of the gravitational force. Which of the following must be true ?



(a) $P = f$ and $N = F_g$ (b) $P = f$ and $N > F_g$

(c) $P > f$ and $N < F_g$ (d) $P > f$ and $N = F_g$

14. A block of mass m is pulled at constant velocity along a rough horizontal floor by an applied force $\vec{T}$ as shown. The frictional force is



(a) $T \cos \theta$ (b) $T \sin \theta$

(c) Zero (d) mg

15. A crate resting on a rough horizontal floor is to be moved horizontally. The coefficient of static friction is 0.40. To start the crate moving with the weakest possible applied force, in what direction should the force be applied ?

(a) Horizontal

(b) 24° below the horizontal

(c) 22° above the horizontal

(d) 24° above the horizontal

16. A 50 N force is applied to a crate on a horizontal rough floor, causing it to move horizontally. If the coefficient of kinetic friction is 0.50, in what direction should the force be applied to obtain the greatest acceleration ?

(a) Horizontal

(b) 60° above the horizontal

(c) 30° above the horizontal

(d) 27° above the horizontal

17. A professor holds a eraser against a vertical chalkboard by pushing horizontally on it. He pushes with a force that is much greater that is required to hold the eraser. The force of friction exerted by the board on the eraser increases if he

(a) pushes with slightly greater force

(b) pushes with slightly lesser force

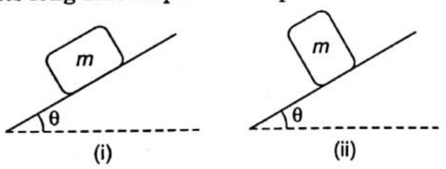
(c) stops pushing
(d) raises his elbow so that the force he exerts is slightly downward but has same magnitude

18. A horizontal force of 12 N pushes 0.5 kg block against a vertical wall. The block is initially at rest. If $\mu_s = 0.8$ and $\mu_k = 0.6$, which of the following is true?
(a) The frictional force is 4.9 N
(b) The frictional force is 7.2 N
(c) The normal force is 4.9 N
(d) The block will start moving and accelerate

19. A horizontal force of 5.0 N pushes 0.50 kg block against a vertical wall. The block is initially at rest. If $\mu_s = 0.8$ and $\mu_k = 0.6$, the frictional force is
(a) Zero (b) 4.9 N
(c) 3.0 N (d) 4.0 N

20. A horizontal force of 12 N pushes a 0.50 kg block against a vertical wall. The block is initially at rest. If $\mu_s = 0.8$ and $\mu_k = 0.6$, the acceleration of the block in ms^{-2} is
(a) Zero (b) 9.4
(c) 9.8 (d) 14.4

21. A block is first placed on its long side and then on its short side on the same inclined plane, as shown. The block slides down the plane on its short side but remains at rest on its long side. A possible explanation is



(a) the shorter side is smoother
(b) the frictional force is less because the contact area is less
(c) the centre of gravity is higher in the second case
(d) the normal force is less in the second case

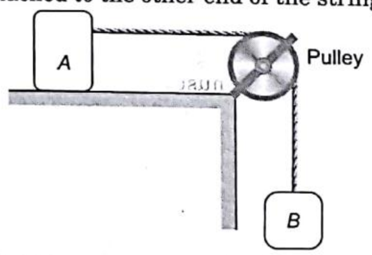
22. A block is placed on a rough wooden plane. It is found that when the plane is tilted 30° to the horizontal, the block will slide down at constant speed. The coefficient of kinetic friction of the block with the plane is
(a) 0.500 (b) 0.577
(c) 1.73 (d) 0.866

23. A 5.0 kg crate is resting on a horizontal plank. The coefficient of static friction is 0.50 and the coefficient of kinetic friction is 0.40. Now one end of the plank is raised so that the plank makes an angle of 30° with the horizontal, the force of friction is
(a) Zero (b) 18 N
(c) 21 N (d) 22 N

24. A 5.0 kg crate is on an incline that makes an angle 30° with the horizontal. If the coefficient of static friction is 0.50, the minimum force that can be applied parallel to the plane to hold the crate at rest is
(a) Zero (b) 3.3 N
(c) 30 N (d) 46 N

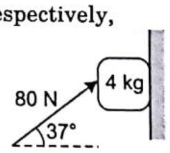
25. A 5.0 kg crate is on an incline that makes an angle of 30° with the horizontal. If the coefficient of static friction is 0.5, the maximum force that can be applied parallel to the plane without moving the plane is
(a) Zero (b) 3.3 N
(c) 30 N (d) 46 N

26. Block A, with a mass of 50 kg, rests on a horizontal table top. The coefficient of static friction is 0.40. A horizontal string is attached to A and passes over a massless, frictionless pulley as shown. The smallest mass m_B that will start A moving when it is attached to the other end of the string is



(a) 20 kg (b) 30 kg
(c) 40 kg (d) 50 kg

27. A block of mass 4 kg is pressed against the wall by a force of 80 N as shown in figure. The values of frictional force and acceleration of block are respectively,



[Take $g = 10\ ms^{-2}$, $\mu_s = 0.2$, $\mu_k = 0.15$]
(a) 8 N, 0 ms^{-2} (b) 32 N, 6 ms^{-2}
(c) 8 N, 6 ms^{-2} (d) 32 N, 2 ms^{-2}

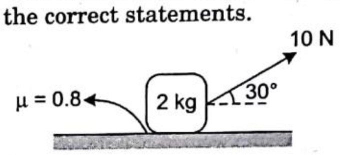
B. More Than One Options Correct

1. Let R, N and f denote the magnitudes of the contact force, normal contact force and frictional force, respectively between a block and a horizontal surface. If all are non-zero, then which of the following are correct ?
   (a) $R > f$
   (b) $R > N$
   (c) $f > N$
   (d) $f < N$

2. If R, N and f denote the magnitudes of the contact force, normal contact force and frictional force, respectively between a block and a horizontal surface, then mark out the correct statements.
   (a) If $R = N$, then surfaces may be smooth
   (b) If $R = N$, then block may be at rest
   (c) If $f = 0$, then surfaces must be smooth
   (d) If $f \neq 0$, then surfaces must be rough

3. Mark out the correct statements.
   (a) Static friction is always equal to $\mu_s N$
   (b) Static friction can be less than $\mu_k N$
   (c) Static friction may be greater than $\mu_k N$
   (d) Static friction can be equal to $\mu_k N$

4. For the situation shown in the figure mark out the correct statements.



   (a) The normal contact force between the block and surface is $2g$
   (b) The normal contact force between the block and surface is less than $2g$
   (c) The frictional force acting on the block is towards left
   (d) The acceleration of the block is zero

C. Assertion & Reason

Directions (Q. Nos. 1 to 3) *Some questions (Assertion-Reason type) are given below. Each question contains* ***Statement I (Assertion)*** *and* ***Statement II (Reason)****. Each question has 4 choices (a), (b), (c) and (d) out of which* ***ONLY ONE*** *is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**
(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**
(c) **Statement I** is True, **Statement II** is False
(d) **Statement I** is False, **Statement II** is True

1. **Statement I** Frictional force can be zero even though the surfaces are rough.
   Statement II Static friction can be zero.
2. **Statement I** Even though there is no relative motion between two surfaces, frictional force can be non-zero between these two surfaces.
   Statement II Static frictional force can be non-zero.
3. **Statement I** If a body is trying to slip over a surface, then the frictional force acting on the body is necessarily equal to the limiting frictional force.
   Statement II Static frictional force can be less than the limiting frictional force.

D. Comprehend the Passage Questions

Passage I

A 50 kg block is on the floor of a truck. The coefficient of friction is 0.25.
(*Take* $g = 10\ ms^{-2}$).

Based on above information, answer the following questions :

1. If the truck is at rest, then frictional force will be

(a) Zero (b) 75 N
(c) 125 N (d) 200 N

2. If the truck is accelerating with an acceleration 1.5 ms^{-2}, then frictional force will be

(a) Zero (b) 75 N
(c) 125 N (d) 200 N

3. If the truck is accelerating with an acceleration 4 ms^{-2}, then frictional force will be

(a) Zero (b) 75 N
(c) 125 N (d) 200 N

Passage II

A block of mass 5 kg rests on a horizontal surface. The coefficient of friction between the block and surfaces are given by $\mu_s = 0.3$ and $\mu_k = 0.25$. The block is acted upon by a variable (time varying) horizontal force P. $P = 2t$ N, where t is time in second. (*Take* $g = 10\ ms^{-2}$).

Based on above information, answer the following questions :

4. At what time, the block will start to move ?

(a) 3 s (b) 7.5 s
(c) 15 s (d) 18 s

5. The value of frictional force at $t = 5$ s is

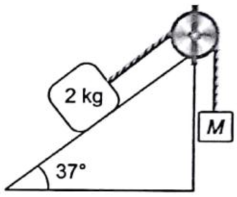
(a) 10 N (b) 12 N
(c) 8 N (d) 15 N

6. The acceleration of the block 8 s after it starts moving is

(a) 6.2 ms^{-2} (b) 3.7 ms^{-2}
(c) 3 ms^{-2} (d) 4.6 ms^{-2}

Passage III

A block of mass 2 kg is connected to a block of mass M with the help of a light, inextensible string as shown in figure. When the system is released from rest, the block of mass 2 kg slides down the incline with an acceleration of 5 ms^{-2}. If the block of mass M is replaced by a block of mass 2 M, then block of mass 2 kg slides up the incline with an acceleration of 5 ms^{-2}.
(*Take* $g = 10\ ms^{-2}$, $\sin 37° = 3/5$, $\cos 37° = 4/5$).



Based on above information, answer the following questions :

7. The value of M is

(a) $\frac{24}{25}$ kg
(b) $\frac{25}{24}$ kg
(c) M can have any value
(d) This situation is not possible for any value of M

8. The coefficient of friction between 2 kg block and incline is

(a) 0.775
(b) 0.865
(c) 0.48
(d) No value of μ exists for this situation

9. The tension in the string when 2 kg block is moving up the incline with an acceleration of 5 ms^{-2} is

(a) 12 N (b) 9.6 N
(c) 10.42 N (d) None of these

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. 62.5 m
2. 3/7
3. 0.05
4. (c), (a), (b)
5. Yes, 3.8 ms^{-2}
6. (a) 456 N, (b) 246 N
7. $\frac{(M+m)g}{\mu}$
8. 2399 N
9. 4 ms^{-2}
10. 30
11. $\tan^{-1}(\mu_s)$
12. $\frac{v_0^2}{4g\sin\theta}$
13. (a) 37°, (b) 5 N
14. 50 N
15. No, magnitude and direction of force
16. $\mu_s = 0.75, \mu_k = 1/\sqrt{3}$
17. $\frac{1}{2\sqrt{3}}$
18. $\mu_s = 0.6, \mu_k = 0.52$

C. Fill in the Blanks

1. Relative
2. Limiting
3. Zero
4. 0.4
5. F
6. Impending
7. Normal contact force or frictional force

D. True/False

1. T **2.** F **3.** F **4.** F **5.** F **6.** F **7.** F **8.** F

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (a) **2.** (d) **3.** (b) **4.** (b) **5.** (d) **6.** (d) **7.** (d) **8.** (a) **9.** (c) **10.** (b)
11. (a) **12.** (a) **13.** (c) **14.** (a) **15.** (c) **16.** (d) **17.** (d) **18.** (a) **19.** (d) **20.** (a)
21. (a) **22.** (b) **23.** (b) **24.** (b) **25.** (d) **26.** (a) **27.** (a)

B. More Than One Options Correct

1. (a, b, d) **2.** (a, b, d) **3.** (b, c, d) **4.** (b, c, d)

C. Assertion & Reason

1. (a) **2.** (a) **3.** (d)

D. Comprehend the Passage Questions

1. (a) **2.** (b) **3.** (c) **4.** (b) **5.** (a) **6.** (b) **7.** (d) **8.** (d) **9.** (d)

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. For the shortest distance, the retardation should be maximum given by

$$a = \mu g = 5\ \text{ms}^{-2}$$

From $\qquad v^2 = u^2 + 2as$

$\Rightarrow \qquad 0 = \left(90 \times \frac{5}{18}\right)^2 - 2 \times (5) \times s$

$\Rightarrow \qquad s = 62.5\ \text{m}$

2. Same concept as Q. No. 1

3. As the object moves with constant velocity, so net force acting on object is zero. So,

$$12 = \mu \times 240$$

$\Rightarrow \qquad \mu = 0.05$

4. If elevator is stationary, $f_L = \mu\, mg$

If elevator is accelerating up,

$$f_L = \mu\, m(g + a)$$

If elevator is accelerating down,

$$f_L = \mu\, m(g - a)$$

The minimum force required to move the block is f_L.

5. $f_L = 0.65 \times 45 = 29.25\ \text{N}$

As $\qquad F > f_L$

So, the block moves with acceleration,

$$a = \frac{F - f_k}{m} = 3.8\ \text{ms}^{-2}$$

6. (a) $F = \mu_s\, mg = 456\ \text{N}$

(b) $F = f_k = \mu_k\, mg = 246\ \text{N}$

7. As there is no relative motion between m and M, both move with same acceleration.

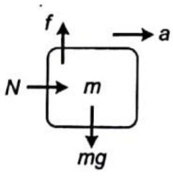
So, $\qquad F = (M + m)\,a$

For small block,

$$N = ma$$

$$f_L = \mu N$$

$$= \mu m \times \frac{F}{M + m}.$$



For no sliding between M and m

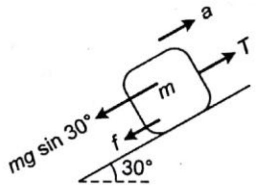
$$f = mg \le f_L$$

$\Rightarrow \qquad mg \le \frac{\mu m F}{M + m}$

$\Rightarrow \qquad F \ge \frac{(M + m)\,g}{\mu}$

8. $T - mg\sin 30° - f = m \times a$

$$f = \mu\, N = \mu\, mg\cos 30°$$



After solving above equation, we get

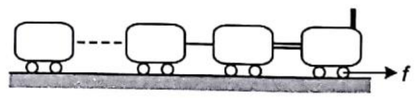
$$T = 2399\ \text{N}$$

9. $f_L = 0.4 \times mg = ma_{max}$

$$a_{max} = 4\ \text{ms}^{-2}$$

10. $f = (M + n \times m)\,a$

$f = \mu\, Mg = (M + nm)\,a$



$\Rightarrow \qquad n = 30$

[$M = 100$ ton, $m = 10$ ton, $a = 0.5\ \text{ms}^{-2}$, $\mu = 0.2$]

11. For block not to slip on incline,

$$f \le f_L$$

As block is at rest,

$$f = mg\sin\theta \le \mu_s\, mg\cos\theta$$

$\Rightarrow \qquad \mu_s \ge \tan\theta$

$\Rightarrow \qquad \theta \le \tan^{-1}(\mu_s)$

12. As the block slides down with constant velocity,

$$mg\sin\theta = \mu\, mg\cos\theta$$

$\Rightarrow \qquad \mu = \tan\theta$

For upward motion, acceleration

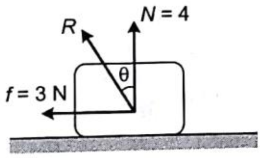
$= -[g(\sin\theta + \mu\cos\theta)] = -2g\sin\theta$

Let s be the required distance, so

$$0 = v_0^2 - 2(2g\sin\theta)\times s$$

$$\Rightarrow \quad s = \frac{v_0^2}{4g\sin\theta}$$

13. $\tan\theta = \frac{f}{N} = \frac{3}{4}$

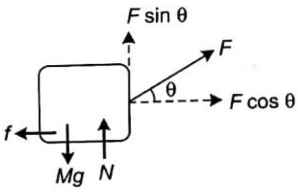


$$R = \sqrt{f^2 + N^2} = 5\text{ N}$$

14. $f = \mu N$

$= 0.25 \times 20 \times 10 = 50\text{ N}$

15. As the direction of applied force changes, the situation is as shown in figure.



$$F\sin\theta + N = Mg$$

$$N = Mg - F\sin\theta$$

$$f = F\cos\theta$$

16. The block starts slipping when,

$$mg\sin\theta = \mu_s\, mg\cos\theta = f_L$$

$$\Rightarrow \quad \mu_s = \tan 37° = 0.75$$

For block to move with constant speed,

$$mg\sin\beta = \mu_k\, mg\cos\beta$$

$$\Rightarrow \quad \mu_k = \tan\beta = \tan 30° = \frac{1}{\sqrt{3}}$$

17. $a = \frac{g}{4} = g\sin 30° - \mu g\cos 30°$

$$\Rightarrow \quad \mu = \frac{1}{2\sqrt{3}}$$

18. $f_L = 15 = \mu_s \times 2.5 \times 10$

$$\Rightarrow \quad \mu_s = 0.6$$

Using $\quad s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad a = \frac{4}{5}\text{ ms}^{-2}$$

$$a = \frac{F - \mu_k mg}{m}$$

$$\Rightarrow \quad \mu_k = 0.52$$

Chapter 7

Work, Energy & Power

The First Steps' Learning

- Work
- Power
- Energy
- Mechanical Energy
- Kinetic Energy
- Potential Energy
- Conservative Forces
- Non-conservative Forces
- Gravitational Potential Energy
- Elastic Potential Energy of Spring
- Law of Conservation of Mechanical Energy

In this chapter, we are going to explore the meaning of terms like work, energy and power. We will also look into the law of conservation of energy, which is the most important principle for entire science whether it is physical, chemical or biological. The terms, work, energy and power are being frequently used in our everyday language. When you study till late night, your parent's may say that you are doing a lot of hard ***work*** *or you need extra* ***energy*** *to do so much work, so take your diet in proper way etc. While playing you may kick the ball very hard and your teammates may wonder what a* ***powerful*** *kick you delivered ? These show that how frequently we use the terms work, energy and power in our daily conversation, but in physics these terms have been defined very precisely.*

In addition to defining these basic terms, we shall also learn about law of conservation of energy in this chapter. It always remains interesting for physicists to find out those physical quantities who don't change as time passes, as this can be very useful in further investigation of science. These physical quantities who don't change ie, remains conserved, constitute the conservation laws. There are many conservation laws like conservation of linear momentum, conservation of energy, conservation of mass, conservation of charge etc. As and when required we will discuss various conservation laws in the book.

Work

In our daily life, we use the term work for all physical or mental exertions. When you are thinking about some questions, then we say you are doing some work, if you hold a book in your hand then you may say that you have done some work in holding the book, etc. But in physics meaning of work is entirely different. In physics work is always associated with a force, work is defined as *the dot product of force with the displacement of the object on which the force is acting**. Consider an object, which is acted upon by force $\vec{\mathbf{F}}$ as shown in figure under the action of which the object is displaced by $\vec{\mathbf{s}}$, then from the definition of work, the work done by this force $\vec{\mathbf{F}}$ on the object over the specified time-interval is

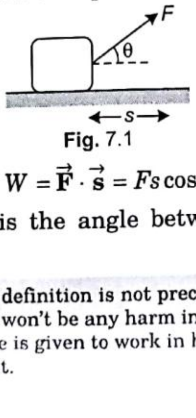


Fig. 7.1

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = Fs\cos\theta$$

where θ is the angle between force and displacement.

Work is a scalar quantity having dimensional formula $[ML^2T^{-2}]$. SI unit of work is joule (J)** and its CGS unit is erg.

It is clear from the definition of work that if the body is free to move and the force acting on the object is more the work done by force on the body is more as compared to when force is having smaller value in the same time-interval.

It has to be kept in mind that above definition of work is valid only for constant forces *ie*, whose magnitude and direction both remain constant *ie*, don't change with time. For work done by a variable force, use of calculus is necessary. Here, we will limit our discussion only to the constant forces.

The definition of work, $W = Fs\cos\theta$ is having one surprising feature. If the displacement of body is zero *ie*, $s = 0$, then work done by the force on the body is zero even though the force is non-zero. For example, you are pushing hard on a wall, quiet obvious it won't move but you will get tired but according to physics (from definition of work) you have not

*Although this definition is not precisely accurate, but it can be considered as appropriate from beginner's point of view and there won't be any harm in learning this.

**The unit joule is given to work in honour of James Joule (1818-1889) who did a lot of 'work' on the nature of work, energy and heat.

done any work on the wall. Really surprising, you get sweats but your physics says that you have not done any work. Then you may think, why I got tired ? If I haven't done any work. We leave the answer to this question for the later chapters.

As far as work is concerned, it is important to keep in mind the following points :

- For analysis of questions involving work, always ask two questions :
  1. Who is doing the work *ie*, which force is doing the work ?
  2. On whom the work has been done ?

     For example, when you are pushing a box kept on the floor, as a result, if it moves on, in this case the force which your hand applied on the box does the work on the box.
- Work can be positive, negative or zero depending on the angle between the force and displacement.

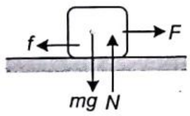


Fig. 7.2

Consider a block moving on a rough horizontal floor under the action of a horizontal force F as shown in figure. If the block gets displaced by s in the direction of F, then the work done by F on block is, $W_1 = Fs$

Work done by friction force f on the block is,

$W_2 = -fs$ as angle between $\vec{\mathbf{s}}$ and $\vec{\mathbf{f}}$ is 180°.

Work done by normal contact force on the block is, $W_3 = 0$ as angle between $\vec{\mathbf{s}}$ and $\vec{\mathbf{N}}$ is $\pi/2$.

Work done by gravity force on the block is,

$W_4 = 0$ as angle between $\vec{\mathbf{s}}$ and $m\vec{\mathbf{g}}$ is $\frac{\pi}{2}$.

- If the force and displacement are perpendicular to each other, then work done by the force on the object is zero even though the force and displacement are non-zero. This is clearly illustrated in previous point. Here keep in mind that work done by only that force is zero which is perpendicular to displacement, other forces may do non-zero work on the object. Like in above illustration $\vec{\mathbf{N}}$ and $m\vec{\mathbf{g}}$ do zero work on block, but work done by $\vec{\mathbf{F}}$ and $\vec{\mathbf{f}}$ on the block is non-zero.
- If displacement of the object is zero, even though the force acting on the object is non-zero, then the work done by force on the object is zero. This we already illustrated in theory discussed above.
- As definition of work involves the displacement, which can have different values in different frames of reference so work done by a force *wrt* different frames of reference in same time interval on same object may be different. In simple words we can say that work done by a force on an object depends on the frame of reference.

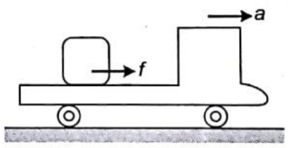


Fig. 7.3

- For example, consider a block lying on the bed of truck (accelerating) as shown in figure. As the block is at rest *wrt* truck, the work done by friction force on block in any time interval in truck frame of reference is zero, while *wrt* ground work done by friction force on block in same time interval is non-zero. So, we summarize. Whenever a question related to work has been asked, then try to answer this question as work is done by which force, on whom work has been done, and with respect to which frame of reference ?
- If a body is moving with constant velocity under the action of certain forces, then total work done on the body is zero, but work done by individual forces on the body may not be zero. Let us consider a block which is moving with constant velocity on a rough floor with the help of an external horizontal constant force F as shown in figure.

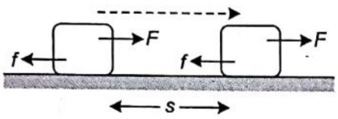


Fig. 7.4

- As the block is moving with constant velocity the net force acting on the block is zero and hence $F - f = 0$ *ie*, $F = f$. For the duration in which the block moves by distance s, the work done by force F on the block is Fs as force and displacement both are in the same direction and work done by friction force f on the block in the same duration is $-fs$ as friction force f and displacement are in opposite direction, while work done by normal contact force and gravity force on block is zero as these forces and displacement are perpendicular to each other, so the total work done on the block is

$$W = Fs - fs = Fs - Fs = 0 \qquad \{\because F = f\}$$

Note that the work done by individual forces on block is non-zero.

- For two-body system, the vector sum of the mutual forces exerted between them is zero in accordance with Newton's third law, but the sum of work done by this mutual interaction forces on two bodies needn't be zero always. However, it may be zero sometimes. Let us consider two blocks of masses m_1 and m_2 placed at certain distance apart, and $\vec{\mathbf{F}}_{12}$ and $\vec{\mathbf{F}}_{21}$ be the mutual interaction forces acting on two blocks as shown in figure. Let in some time t the blocks move by s_1 and s_2 under the action of forces $\vec{\mathbf{F}}_{12}$ and $\vec{\mathbf{F}}_{21}$ respectively as shown in figure.

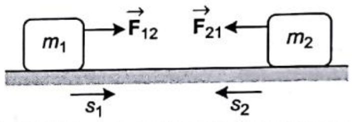


Fig. 7.5 Work done by action reaction pair can be non-zero

Then from Newton's third law,

$$\vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{21} = 0,$$

while work done by these internal forces on the bodies is

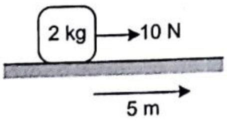
$$W = W_{12} + W_{21}$$
$$= F_{12} \times s_1 + F_{21} \times s_2$$

which is obviously a positive non-zero value.

C-BIs

Concept Building Illustrations

Illustration | 1 *A block of mass* 2 *kg is moving on a smooth horizontal surface under the action of a constant horizontal force of* 10 *N as shown in figure. Determine the work done by this force on block wrt ground in the time duration in which the block moves by* 5 *m.*



- **Solution** Here, the force and displacement are in same direction *ie*, angle between force and displacement is zero. So, from definition of work,

$$W = Fs\cos\theta$$
$$= 10 \times 5 \times \cos 0°$$
$$= 50 \text{ J}$$

Illustration | 2 *In above illustration, determine the work done by force on block in first* 3 s *of application of force?*

Solution Acceleration of the block is,

$$a = \frac{F}{m} = \frac{10}{2} = 5 \text{ ms}^{-2}$$

In 3 s, the block displaces by

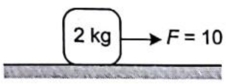
$$s = \frac{1}{2}at^2 = \frac{1}{2} \times 5 \times 3^2 = 22.5 \text{ m}$$

in the direction of force.

So, work done by the force on the block in first 3 s is,

$$W = Fs\cos\theta$$
$$= 10 \times 22.5 \cos 0°$$
$$= 225 \text{ J}$$

Illustration | 3 *A block of mass 2 kg is pulled with the help of a horizontal force of 10 N on a rough horizontal surface as shown in figure. The coefficient of friction between the block and surface is 0.2.*



Determine the

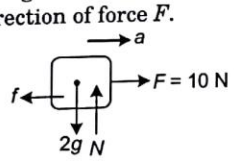
(a) *work done by friction force on the block in first 2 s?*

(b) *work done by force F on the block in first 2 s?*

(c) *work done by the normal contact force on the block in first 2 s?*

(d) *work done by weight of block on the block in first 2 s?*
(Take $g = 10\, ms^{-2}$).

Solution Free body diagram of the block is as shown in figure. From vertical equilibrium of block, $N = 2g = 20$ N, $f = \mu N = 0.2 \times 20 = 4$ N as force applied on the block is greater than limiting friction force, so it means the block is accelerating. Let the acceleration of block be a in the direction of force F.



From Newton's second law equation

$$F - f = ma$$
$$\Rightarrow \quad 10 - 4 = 2a$$
$$\Rightarrow \quad a = 3 \text{ ms}^{-2}$$

The displacement of block in first 2 s is,

$$s = \frac{1}{2} \times at^2 = \frac{1}{2} \times 3 \times 2^2 = 6 \text{ m}$$

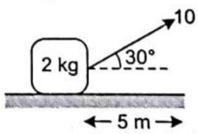
(a) Work done by friction force on block,
$$W_f = fs\cos 180°$$
$$= 4 \times 6 \times (-1) = -24 \text{ J}$$

(b) Work done by force F on block,
$$W_F = Fs\cos 0° = 10 \times 6 \times 1 = 60 \text{ J}$$

(c) Work done by normal contact force N on block,
$$W_N = Ns\cos\frac{\pi}{2} = 0 \text{ J}$$

(d) Work done by weight of block on block is,
$$W_{mg} = mgs\cos\frac{\pi}{2} = 0 \text{ J}$$

Illustration | 4 *A block of mass 2 kg is pulled with the help of a 10 N force on a smooth horizontal surface as shown in figure. Determine the work done by force on the block as the block moves by 5 m along the horizontal surface?*



Solution Here the angle between the force and displacement is 30°, so the work done by force on the block is

$$W = Fs\cos\theta = 10 \times 5 \times \cos 30°$$
$$= 10 \times 5 \times \frac{\sqrt{3}}{2} = 25\sqrt{3} \text{ J}$$

Power

In many practical situations, not only this is important that how much work has been done but it is also important that in how much time the work has been done. The physical quantity which serves this purpose is the "power". In physics, "power is defined as the rate of doing work", or the "rate at which energy is transferred". Let us consider an illustration. If you do a piece of work in 1 month and the same work has been done by your friend in 2 months,

then who would 'possess' more power or who has done the work faster or whose rate of doing work is more ? Obviously your answer would be "myself". This is what, where the importance of power lies in physics as well as in our daily life.

Definition of Power :

$$\text{Power}\,(P_{av}) = \frac{\text{work}}{\text{time}} = \frac{W}{t},$$

Here, we have defined the average power.

Instantaneous power is defined as, $P = \lim\limits_{\Delta t \to 0} \dfrac{\Delta W}{\Delta t}$ where Δt is very small time-interval. As work is always associated with a force so power is also associated with a force and in terms of force power can be defined as dot product of force with the velocity of object, *ie*, $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$.

Power is a scalar quantity and its SI unit is watt. 1 W is equal to 1 Js^{-1}, the dimensional formula of power is $[ML^2T^{-3}]$.

The unit of power is named after **James Watt**, the inventor of steam engine. You may heard about the power of automobiles or motors like 2 Horse Power motor etc. Horse Power is nothing but an another unit of power. 1 HP = 746 W. The more powerful an automobile is, more quickly it can acquire high speeds or we can say quickly it can accelerate, for example, a car having 200 HP engine can acquire a speed of 100 kmh^{-1} from 0 in less time than a car which is having an engine of 150 HP.

Sometimes we may require large unit of power like kilowatt or megawatt. 1 kW = 1000 W and 1 MW = 10^6 W.

C-BIs

Concept Building Illustrations

Illustration | 5 *An engine does 800 J of work on a body in 4 s. Determine the average power of engine assuming total work done by the engine is utilised by the body.*

Solution As the total work done by engine is utilised by body, the rate at which the body is consuming energy is same as the power delivered by the engine.

So, $$P = \frac{W}{t} = \frac{800}{4} = 200\ \text{W}$$

Illustration | 6 *A force of 10 N is acting on an object which is moving with a constant velocity of 2 ms^{-1}. The directions of this force and velocity are the same. Determine the rate at which this force is doing work on the object.*

Solution Here the object is moving with constant velocity, so it means some other force excluding the described force is also acting on the object, but this doesn't concern with the question.

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

$= Fv\cos 0°$ as angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{v}}$ is 0°

$$= 10\ \text{N} \times 2\ \text{ms}^{-1} = 20\ \text{W}.$$

Illustration | 7 *An elevator of total mass 1800 kg is moving up with a constant speed of 2 ms^{-1}. A frictional force of 4000 N opposses the motion of elevator. Determine the power delivered by the motor to the elevator. [Take $g = 10\ ms^{-2}$]*

Solution In this question, the motor has to do the work against the gravity force and the frictional force, so the force which the motor is exerting on the cable of elevator is

$$F = mg + f$$
$$= 1800 \times 10 + 4000$$
$$= 22000\ \text{N}$$
$$\text{Power} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$
$$= 22000 \times 2 = 44\ \text{kW}$$

Illustration | 8 *A spider is climbing a wall at a constant speed of* $0.5\ ms^{-1}$. *The mass of the spider is* 10 *g. Determine the rate at which wall is doing work on the spider.* [*Take* $g = 10\ ms^{-2}$]

Solution As the spider is moving with constant velocity, so the net force acting on it would be zero. For vertical direction, the component of force exerted by wall on spider balances the gravity force and only this component do the work on spider. Let this component be F_y, then

$$F_y = mg = 0.1\ \text{N}$$
$$P = F_y v = 0.1 \times 0.5\ \text{W}$$
$$= 0.05\ \text{W}$$

Energy

In everyday context the term energy is used in different ways, like energy of cell is decreased due to its continual usage, we get more energy by taking proper diet at proper time, proper use of energy is very important for success, the cost of energy what we consume through electricity, fuel etc. However, in physics the energy is defined very precisely just like work has been defined. It is a very common observation that a more energetic person can do more work, and analogously in physics the energy is defined as "The capacity or ability of an object to do work". Means more is the energy of object, more the work it can do, or we can say that energy and work are equivalent in some way, later on we shall consider this point in detail.

Energy is a scalar quantity and its SI unit is joule (J) and has the dimensions $[ML^2T^{-2}]$. The practical (commercial) unit of energy is kilowatt hour (kWh), the relation between kWh and Joule we will be explored later in the chapter.

Energy is present in the universe in various forms like mechanical energy (kinetic energy and potential energy), electrical energy, sound energy, thermal energy, light energy, chemical energy, nuclear energy etc. In the present chapter, we will limit our discussion only to mechanical energy. Before discussing the details of mechanical energy let's have a look on the significance of the definition of energy for better understanding of it. Let us consider some illustrations to understand the relation between work and energy.

***Illustration* I** Let us say you are playing carrom and it's now your chance to strike. You take the possession of the striker and hit it, unfortunately the strike made by you is very slow, as a result the striker stop's just before coming in contact with the coin you want to hit, in this case the striker is not able to displace the coin and hence no work has been done on the coin by the striker. Now, let us assume that you hit the striker in such a way that it strikes the specified coin, and the coin gets displaced from its position, but is not able to reach its desired position. You try again and hit the striker harder and the strike is perfect and coin reaches the hole, if you strike somewhat more harder the coin may bounce off. So in this case we conclude that more is the speed/velocity of striker just before striking the coin, more would be the work done on coin or we can say that more is the energy possessed by a body, it can do more work on other objects. This type of energy which is possessed by a body due to its motion is termed as kinetic energy (In the mechanical energy section we will shall discuss this in detail). From above observation we can also say, that faster the object is moving, more is its capacity to do work.

***Illustration* II** Let us consider that a heavy object (say of iron) is dropped from a height h (say equal to 1 m) on a muddy floor, as the object falls down and strikes the mud, it creates some depression on the floor *ie*, it displaces some mud or we can say the object

does some work on the mud. If we repeat process by increasing h, say to 2 m, then it is quiet obvious that depression in the floor would be larger *ie*, more amount of mud has been displaced *ie*, more work has been done on the mud. Thus, we can conclude that more is the height of object, more the work it can do on mud *ie*, from the definition of energy we can say that height of object from earth's surface can be linked with some form of energy.

***Illustration* III** You may be familiar with catapult, a y-shaped wooden object on which we tie an elastic cord. It is used to throw stones or some small objects like marbles etc. What we do is, we place the small object on the cord and stretch it, and when we release the object it flies away with some velocity to hit its target, more we stretch the elastic cord more is the velocity acquired by the object, and it goes farther. Thus, we can say that some energy is possessed by the elastic cord which does work on the stone (object) to move it. This type of energy is the elastic potential energy, possessed by a body due to its elastic properties.

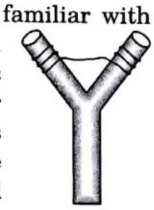


Fig. 7.6

***Illustration* IV** Let us consider a block A connected to a spring as shown in figure and another block B being kept little away from it. Assume the surface to be smooth. Now if we compress the spring and release it, then A may come in contact with B and imparts some velocity to it. It can be easily visualized that more we compress the spring, more the velocity can be imparted to B. Thus, we can conclude that

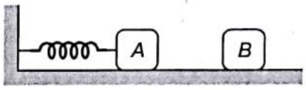


Fig. 7.7

when the compressed spring is released it can do some work, thus a compressed spring is having some energy which we call as elastic potential energy or spring energy. In above discussion we considered only compression of spring, but the conclusion is equally valid for elongation also. You can think some simple situation by yourself.

We hope that with the reading and understanding of above illustrations you are able to understand the definition of energy *ie*, "capacity of doing work is energy". Now, before seeing that how a body having energy does the work, we are discussing the mechanical energy.

Mechanical Energy

The form of energy which we deal in mechanics is the mechanical energy. For example, the output of an electric motor is in the form of mechanical energy, the moving vehicle possesses mechanical energy etc.

Mechanical energy is classified into two categories :

1. Kinetic Energy,
2. Potential Energy.

Kinetic Energy

The energy possessed by a body by virtue of its motion is termed as its kinetic energy. We know that a moving object can do work, this is because of the fact that a moving object possesses kinetic energy. Consider an illustration, during a practice session (cricket game) a bowler practices for bowling. If the bowler is a spinner and the ball hits the wicket, then it is a very common observation that bails of the wicket are not disturbed much and if the same ball is thrown by fast bowler and it hits the same wicket then bails may be castled away completely. From this, we conclude that a fast moving ball can do more work on the wicket or in other words we can say that a fast moving ball possesses more kinetic energy and hence we say that kinetic energy of a moving body depends upon the speed of body.

For a object of mass m moving with speed v, its KE is given by

$$K = \frac{mv^2}{2}$$

As m is always a positive quantity and v^2 is also a positive quantity, so kinetic energy of a body is always a non-negative number, *ie* KE of a body can never be negative. If the body is at rest, then its KE would be zero.

KE is a form of energy and hence its SI unit and dimensional formula are same as that of energy. KE is a quantity which depends on frame of reference. Consider this example : A block A is kept at rest on a railroad car which is moving with constant velocity on a horizontal surface as shown in figure, with respect to ground frame of reference the block is moving with velocity v, while *wrt* car frame of reference the block is at rest, so KE of block *wrt* ground frame of reference is $\frac{mv^2}{2}$ while *wrt* car frame of reference it is zero.

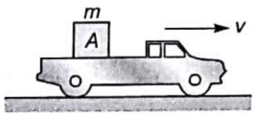


Fig. 7.8

C-BIs

Concept Building Illustrations

Illustration | 9 *A block of mass 2 kg is moving with a constant speed of 5 ms^{-1}. Determine the KE of block.*

Solution From $K = \frac{mv^2}{2}$

$$\Rightarrow \quad K = \frac{2 \times 5^2}{2} = 25\text{ J}$$

Illustration | 10 *The speed of a block of mass 5 kg is changing from 4 ms^{-1} to 8 ms^{-1} due to some force acting on it, in a time span of 5 s.*

Determine the change in KE of block in this time span.

Solution Change in any physical quantity is equal to its final value minus initial value, so

$$\Delta K = K_f - K_i$$

$$K_i = \frac{mv_i^2}{2}$$

$$= \frac{5 \times 4^2}{2} = 40\text{ J}$$

$$K_f = \frac{mv_f^2}{2}$$

$$= \frac{5 \times 8^2}{2} = 160\text{ J}$$

$$\Rightarrow \quad \Delta K = (160 - 40)\text{ J} = 120\text{ J}$$

Whenever a person does some work, he/she wants something in return as a result. Like if you are studying (doing work), you will surely expect good marks as a result of the work you did. In physics, also this type of things happen. In physics whenever a net force does some work on an object, then as a result of this work done on the object its KE changes. In other way, we can say,

Change in KE (ΔK) = Total work done on the object (W).

The above relation between the change in KE and the work done on the object is known as the **work-energy theorem**. This is one of the most important theorems in physics, and law of conservation of energy can be understood very clearly with the help of this theorem. The exact and complete details of this theorem you will study in your higher classes, here we provide you with the derivation of work energy theorem for bodies under the action of a net constant force.

Let us consider an object of mass m, which is under the action of constant force F as a result of which its velocity changes from u to v in time interval t, and in this time interval the object moves by distance s. Then from equations of motion

$$v^2 = u^2 + 2as$$

where a is the acceleration of body

$$\frac{v^2 - u^2}{2} = as$$

Multiplying above equation by m, we get

$$\frac{mv^2}{2} - \frac{mu^2}{2} = ma \times s$$

$$\Rightarrow \quad K_f - K_i = Fs$$

$$\Rightarrow \quad \Delta K = W$$

where $W = Fs$ is the work done by force F on object.

Some points worth mentioning about work-energy theorem are :

- In RHS of work-energy theorem, the work done on the system (W) is the total work done. W includes the work done by internal as well as external forces, thus we can say that
$$\Delta K = W = W_{ext} + W_{int}$$
where W_{ext} and W_{int} represents the work done by external and internal forces, respectively.
- Although both the quantities, KE and work depend upon the frame of reference. The work energy theorem is valid in all frames of reference, whether it is inertial or non-inertial. In non-inertial frame of reference we have to consider work done by a *pseudo force**. The only thing you have to keep in mind is that all the quantities *ie*, initial and final kinetic energies and work done has to be written *wrt* same frame of reference.
- Work energy theorem is valid for variable forces too.
- Work energy theorem is also valid when the object is not moving along a straight line *ie*, the object follows some curved path.

C-BIs

Concept Building Illustrations

Illustration | 11 *The KE of an object changes from 100 J to 120 J under the action of a force. Determine the work done by force on the object in the corresponding duration.*

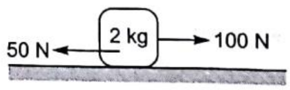
Solution From work energy theorem,

$$\Delta K = W$$

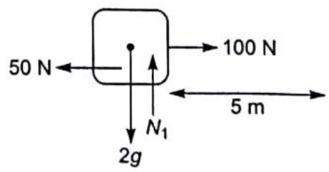
$$\Rightarrow \quad K_f - K_i = W$$

$$\Rightarrow \quad W = (120 - 100)\ \text{J} = 20\ \text{J}$$

Illustration | 12 *A force of 100 N acts on a block of mass 2 kg as shown in figure. The average resistive force acting on the block is 50 N. Determine the change in KE of block as it moves by 5 m. [Take $g = 10\ ms^{-2}$]*



Solution The free body diagram of block is as shown in figure. The forces which are acting on the block are



(a) Externally applied force $F = 100$ N.
(b) Resistive force, $f = 50$ N
(c) Gravity force, mg
(d) Normal contact force N_1

* A *pseudo force* is an imaginary force, which we have to apply on our object if we want to apply Newton's laws of motion in a non-inertial frame of reference.

Among these four forces, N_1 and mg are not doing any work on the block as displacement of block is perpendicular to these forces. So, from work- energy theorem

$$\Delta K = W_f + W_F + W_{N_1} + W_{mg}$$
$$= W_f + W_F$$

$$W_f = -f \times s$$
$$= -50 \times 5 = -250 \text{ J}$$
$$W_F = Fs = 100 \times 5 = 500 \text{ J}$$

So, $\Delta K = -250 + 500 = 250 \text{ J}$

Now in this section we are exploring a very basic concept underlying the definition of energy. We know, that the "ability to do work is the measure of energy", but how a body having energy can do the work ? To make this concept clear, let us consider the Carrom-Board game described above. When the striker hits the coin, it exerts a force on the coin as a result the coin moves (as it is free to move) and acquires some KE. More is the force exerted by striker on coin, more the KE the coin can acquire. Let us consider a case when coin is not allowed to move, then even though the striker exerts a force on coin but it is not doing any work on coin and hence coin doesn't acquire any KE. From here we can also conclude that the KE of striker is not transferred to coin as coin is not allowed to move *ie*, on coin no work has been done. Thus, we can say that for work to be done on any object some medium is necessary which can transfer the energy to the object and for this the medium particles must move.

Potential Energy

The energy possessed by a system due to its configuration is termed as the potential energy of system. Here word configuration is related to the relative position of different parts of the system. For illustration the three particles are initially located at the vertices of an equilateral triangle of side 2 cm and after some time due to some internal forces acting on them the location of particles changes and they acquire the position characterised by vertices of another equilateral triangle say of side 4 cm as shown in figure. Then the triangle ABC represents the initial configuration of the system, and the triangle $A'B'C'$ represents the final configuration of the system. Now, before coming into the details of PE, it is necessary to get familiarized with conservative and non-conservative forces first.

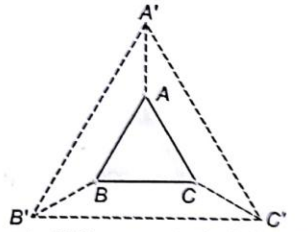


Fig. 7.9 Triangle ABC represents the initial configuration of system of three particles while $A'B'C'$ represents the configuration of system after some time.

Conservative Forces

If the work done by a force over a closed path is zero, then force is said to be conservative in nature, *ie*, work done by a conservative force over a closed path is zero. Gravity force, electrostatic force, spring force are few of the examples of conservative forces. Let us consider that a ball of mass m is thrown upwards from the surface of earth as a result it reaches to height h (say point 2), and then comes back to the initial point of projection (say 1) as shown in the figure.

Fig. 7.10

In this case, the work done by gravity force on the ball as the ball moves from 1 to 2 is, $W_{12} = -mg \times h$, –ve sign is

because of the fact that gravity force and displacement are in opposite directions.

During the descent of the ball, the work done by gravity force on the ball is, $W_{21} = mg \times h$.

The total work done by gravity force on ball over the round trip $1-2-1$ is, $W = W_{12} + W_{21} = -mgh + mgh = 0$, and thus it satisfies the condition of a conservative force.

The definition of a conservative force can be interpreted in another way "Work done by a conservative force between two positions is independent of the path and is depends only on the initial and final positions."

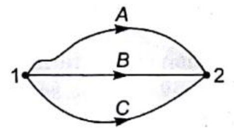


Fig. 7.11 Work done by conservative force is independent of path.

Let us consider an object has to be taken from position 1 to 2 under the action of a conservative force $\vec{F}$, then work done by $\vec{F}$ on the object in taking it from 1 to 2 *via* different paths say A, B and C would be the same.

If you take a ball from position 1 to 2 *via* three different paths A, B and C as shown in figure, then the work done by gravity force on the ball for all three different paths would be same and equal to $-mgh$ where m is the mass of ball.

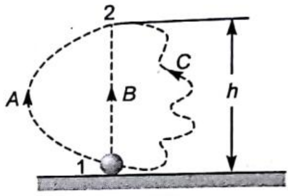
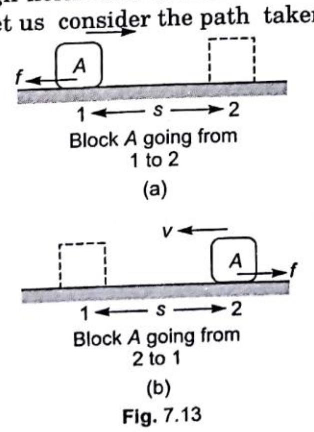


Fig. 7.12

Non-Conservative Forces

If the work done by a force over a closed path is non-zero, then the force is said to be a non-conservative force or in other words we can say that work done by a non-conservative force is path dependent. Examples of non-conservative forces are friction force, viscous force, air friction etc. Consider a block moving on a rough horizontal surface as shown in the figure. Let us consider the path taken by block from 1 to 2 and then back from 2 to 1 under the action of some force as shown in the figure.



Block A going from 1 to 2

(a)

Block A going from 2 to 1

(b)

Fig. 7.13

Work done by friction on block as the block moves from 1 to 2 is, $W_{12} = -fs$ and work done by friction on block as block moves from 2 to 1 is,

$$W_{21} = fs$$

The total work done by friction force on block for complete round trip $1-2-1$ is,

$$\begin{aligned} W &= W_{12} + W_{21} \\ &= -fs - fs \\ &= -2fs \neq 0 \end{aligned}$$

Potential Energy (Continued ......)

Now we come back to our discussion of potential energy. Potential energy is defined as "For any internal conservative force, the change in PE is equal to –ve of the work done by corresponding conservative force."

ie, $$\Delta U = -W_{\text{conservative force}}$$

Now we shall try to understand the basic concept behind this equation. Let us consider a system of two particles A and B whose initial and final configurations are as shown in figure. The configuration of the system changes because of some internal conservative force, the word internal means within the system *ie*, the mutual interaction between the two particles. Let this internal force be $\vec{F}$ and potential energy corresponding to this force $\vec{F}$ in initial and final configurations be U_i and U_f, respectively. Then from definition of potential energy,

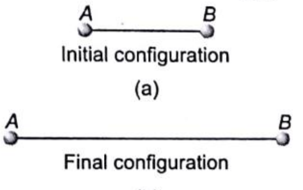


Fig. 7.14

$$\Delta U = U_f - U_i = -W_F$$

where, W_F represents the work done by force $\vec{F}$ on the system in changing the configuration from initial to final.

Unlike kinetic energy, PE can be +ve, –ve or zero, and thus the mechanical energy (sum of kinetic and potential energies) can be +ve, –ve or zero.

It is very clear from the definition that PE is defined only for internal conservative forces *ie*, for conservative forces within a system. It means we can't write PE for an individual part of the system, rather PE is a combined property of system. Although in some approximations we can write the PE of a part of the system. In this book we shall also write the PE of individual parts of the system also. Within a system, there is a possibility that more than one conservative forces are present, in this situation PE corresponding to all conservative forces has to be written separately. For example, consider a block hanging from a spring as shown in figure, in this case spring force and the gravity force are two conservative forces present and hence PE corresponding to both has to be written.

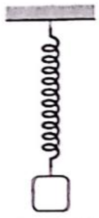


Fig. 7.15

Another important point about PE to be noted is that only change in PE is defined, then how can we write PE for a particular configuration *ie*, absolute PE. For this problem, we consider a particular configuration to be the reference configuration and assume the PE to be zero in this configuration, then absolute PE can be determined *wrt* this reference configuration. For example, consider the two configurations as mentioned above, from definition of PE,

$$U_f - U_i = -W_F$$

If we assume initial configuration as reference configuration, then $U_i = 0$ and we get $U_f = -W_F$ where now W_F represents the work done by the conservative force $\vec{F}$ in changing the configuration of the system from reference to final configuration. Thus, we can define absolute PE of a system as, the negative of the work done by corresponding conservative force on the system in changing its configuration from some reference configuration to the desired configuration.

Even if the above two concepts related to PE to understand clearly you are not able, don't get panicky, you will understand these in time and it won't affect your continuity while going through this book.

Now we are going to discuss the PE corresponding to gravity and spring forces.

Gravitational Potential Energy

If you have gone through the previous chapters you are well aware of the gravity force experienced by any body (having mass) due to earth's gravitational influence. In the preceding discussion we have shown that gravity force is a conservative force and hence, potential energy can be defined corresponding to the gravity force. Let us consider a block of mass m, placed

at the surface of earth, if we want to take it at some height say at height h from the earth's surface, then we have to perform some work against earth's gravity force, now a very simple and conceptual question can arise, where has this work done by us all gone up ? The answer is the work done is stored as gravitational PE in the earth-block system as earth is not moving [a very good approximation for practical purposes] so we can also say that the work done by us is stored as gravitational PE of block, this we also call the PE of an object at a height.

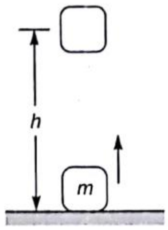


Fig. 7.16

If we take the block from ground level to a height h, then from the definition of PE.

$U(h) - U(0) = (-)$ work done by the gravity force in taking the block from ground level to height h where $U(h)$ represents the gravitational PE of block when it is at height h and $U(0)$ represents the gravitational PE of block when it is on ground.

$$U(h) - U(0) = -[-mg \times h] = mgh$$

If we take ground level as reference configuration (which is usually the standard choice), then $U(0) = 0$, and hence,

$$U(h) = mgh$$

ie, gravitational PE of the block of mass m when it is at height h from earth's surface is mgh.

Thus, we can say that more is the height of object from ground level, the more gravitational PE it possesses. This fact you can easily visualize from the above Illustration II in the discussion of energy.

Remember !

It is very important to keep in mind that the absolute PE of a body depends upon the choice of reference level or zero PE level, if we chose some other level as the reference level, then the PE (absolute) comes out to be different. Although everywhere we will tell you about the standard reference configuration for different types of PE, but you have complete liberty to choose any configuration as the reference configuration, the only thing you have to keep in mind is that in a particular question don't change your reference configuration. Whatever reference configuration you assume in the start, stick to it till you have not solved the question. However, in different questions you can take different reference configurations. If nothing has been mentioned in the question, then we will take up the standard choice for reference configuration.

C-BIs

Concept Building Illustrations

Illustration | 13 *A block of mass 5 kg is at a height of 10 m from earth's surface. Determine the gravitational PE of block wrt earth level. [Take $g = 10\ ms^{-2}$]*

Solution From the expression,

$$U = mgh$$

In this expression we have taken ground level as the reference level so we can use it directly in this question

$$U = 5 \times 10 \times 10\ \text{J}$$
$$= 500\ \text{J}$$

Illustration | 14 *A block of mass 5 kg is taken from a height of 2 m to 10 m measured from earth's surface. Determine the change in its gravitational PE. [Take $g = 10\ ms^{-2}$]. Does your answer depends upon the choice of reference level ?*

Solution Change in PE is independent of the choice of reference level, it is very clear from the definition of PE itself. So,

$$\Delta U = U_f - U_i$$
$$= mgh_f - mgh_i$$
$$= 5 \times 10\,(10 - 2) = 400\ \text{J}$$

Elastic Potential Energy of Spring

While discussing the Newton's laws of motion we found that, to elongate or compress a spring from its equilibrium position we have to exert some force on it. In other words, we say that to elongate or compress the spring from its natural length, we have to perform some work against the spring force, this work done by us is stored as elastic PE in the spring. If the spring of spring constant k is compressed or elongated from its natural length by x_0, then elastic PE stored in the spring is,

$U = \frac{kx_0^2}{2}$, here the natural length of spring is considered as the reference level for PE.

So, if the spring is compressed or elongated by x_0, the PE stored in the spring is same in both the cases, and is equal to $\frac{kx_0^2}{2}$.

C-BIs

Concept Building Illustrations

Illustration | 15 *A spring of spring constant 100 Nm^{-1} is compressed by 10 cm. Find the elastic PE stored in the spring.*

Solution Elastic PE stored in spring,

$$U = \frac{kx_0^2}{2}$$

Here, $k = 100\,\text{Nm}^{-1}$, $x_0 = 0.1\text{ m}$

So, $U = \frac{100 \times (0.1)^2}{2} = 0.5\text{ J}$

Illustration | 16 *A spring of spring constant 100 Nm^{-1} is elongated by 10 cm. Find the elastic PE stored in the spring.*

Solution $U = \frac{kx_0^2}{2}$

$$\Rightarrow \quad U = \frac{100 \times (0.1)^2}{2} = 0.5\text{ J}$$

From these illustrations we see that same PE is stored in spring if the spring is elongated or compressed by same amount.

Law of Conservation of Energy

It has been always the favourite topic for physicists to identify those physical quantities whose values don't change *ie*, remain constant under some conditions or without any condition. This fact makes it possible for physicists to develop conservation laws, these conservation laws are very helpful in analysing various physical situations and also in developing new theory. Here, now we are going to see one such law of conservation, which is one of the most important laws in entire science—**the law of conservation of energy**.

According to this law—**"Energy can neither be created nor be destroyed. It can be only transformed from one form to another."** This means if we consider all forms of energy, then total energy of the universe always remains constant. Law of conservation of energy can also be stated as "For an **isolated system*** the total energy of the system remains constant, even though energy can be transformed from one form to another."

* An isolated system means on which no force is acting from outside, or in other words, we can say that no work has been done on the system from outside.

Now you may ask many questions like—Power plants are generating electricity, which is a form of energy and thus you can say a power plant creates energy. You get energy after having a glass of glucose. This way you can think various situations where it seems to be that energy is created or destroyed but this is not so, it is only our limited knowledge due to which we may not be able to answer the question correctly like in above two illustrations, some other form of energy (like chemical energy, nuclear energy, hydel energy or wind energy etc.) depending on the nature of power plant is converted to electrical energy. Similarly when you think in case of glucose, the chemical energy of glucose is increasing the internal energy of body. Till now, there has been not identified a single event happening in the physical world which violates the energy conservation principle. Physicists have a deep faith in this law, and on the basis of this, they have developed all their theories.

Take a simple example, to explain the transformation of energy from one form to another. Consider a ball of mass m dropped from rest from height h, as shown in figure. Let us neglect the air friction force, so the only force acting on the ball is gravity force. As the ball comes down, its speed increases *ie*, KE increases and this increase in the KE is at the expense of gravitational PE *ie*, as the ball moves down KE increases while PE decreases, when the ball is just going to strike ground its PE is completely converted into KE and when ball strikes the ground, KE acquired by it is converted to other forms of energy like sound energy, heat energy and deformation energy (work used up in deforming the body).

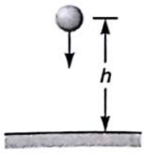


Fig. 7.17

While applying energy conservation principle you have to simply look for all forms of energy at particular instants and also keep in mind that work and energy are equivalent. The major advantage of law of conservation of energy is that, lengthy mathematical calculations including time can be avoided if we use energy conservation principle instead of equation of kinematics.

Law of Conservation of Mechanical Energy

If in a system only conservative forces are present *ie*, no external force and non-conservative forces are present or even if they are present they are not doing any work on system, then work energy theorem takes the form

$$\Delta K = W_{\text{conservative forces}}$$

and from the definition of PE,

$$\Delta U = -W_{\text{conservative forces}}$$

From above two expressions

$$\Delta K = -\Delta U$$

$$\Rightarrow \quad \Delta K + \Delta U = 0$$

$$\Rightarrow \quad \Delta(K + U) = 0$$

$$\Rightarrow \quad \Delta E = 0$$

where $E = K + U$ is the mechanical energy of system

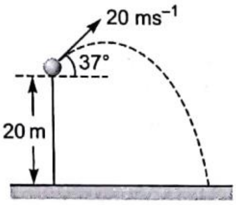
$$\Rightarrow \quad E = \text{constant}$$

Thus, we can say, that in the "absence of external and non-conservative forces the total mechanical energy *ie*, sum of kinetic and potential energy of the system is conserved." This statement is termed as the law of conservation of mechanical energy. Remember this law can be applied only under restriction that work done by external and non-conservative forces on the system is zero, while work energy theorem can be applied in all circumstances. So, it is better to make a habit of using work energy theorem, instead of conservation of mechanical energy and same methodology we will follow in this book.

C-BIs

Concept Building Illustrations

Illustration | 17 *A ball of mass 2 kg is projected with a speed of 20 ms^{-1} from the top of a tower of height 20 m as shown in figure. Determine the speed of ball when the ball is at vertical distance of 10 m below the point of projection.*
[Take $g = 10\ ms^{-2}$].



Solution Here if we go for equations of kinematics, the question becomes somewhat tedious but by using law of conservation of energy we can avoid lengthy calculations. In this question we will use work-energy theorem and the concept that work done by gravity force is independent of path.
From work energy theorem, $\Delta K = W$

$$K_f - K_i = W_{\text{gravity force}}$$

Let v be the required speed of ball, then

$$K_f = \frac{mv^2}{2} = \frac{2 \times v^2}{2} = v^2$$

$$K_i = \frac{mv_i^2}{2} = \frac{2 \times (20)^2}{2} = 400\text{ J}$$

$$W_{\text{gravity force}} = mg \times 10 = 2 \times 10 \times 10 = 200\text{ J}$$

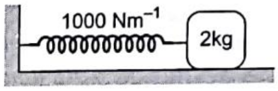
An alternative way to compute the work done by gravity force is,

$$W_{\text{gravity force}} = -\Delta U = -[U_f - U_i]$$
$$= -[mg \times 10 - mg \times 20] \quad [\because U = mgh]$$
$$= mg \times 10 = 200\text{ J}$$

So, $\quad v^2 - 400 = 200$

$\Rightarrow \quad v = \sqrt{600}\text{ ms}^{-1} = 24.5\text{ ms}^{-1}$

Illustration | 18 *A spring-block system is placed on a smooth horizontal surface as shown in figure. Now the block is pushed towards left so that the spring compresses by 10 cm and then the block is released from rest. Determine the velocity of block when the spring acquires its natural length.*



Solution Let v be the required speed, then by using work-energy theorem for initial and final positions we can solve the question

$$\Delta K = W_{\text{spring force}} + W_{mg} + W_N$$

Here three forces are acting—on the block, spring force, gravity force and normal contact force.

$W_N = W_{mg} = 0$ as the displacement of block is perpendicular to these forces.

So, $\quad K_f - K_i = W_{\text{spring force}}$

$\Rightarrow \quad \dfrac{mv^2}{2} - 0 = -\Delta U$

[$\because$ Block is at rest initially]

$\Delta U = U_f - U_i$

$U_f = 0$ as spring is in natural length.

$$U_i = \frac{kx^2}{2} = \frac{1000 \times (0.1)^2}{2} = 5\text{ J}$$

So, $\dfrac{2 \times v^2}{2} = -[0 - 5] \Rightarrow v = \sqrt{5}\text{ ms}^{-1}$

Work done by the spring force can't be computed without using calculus, thus we will use $W_{\text{spring force}} = -\Delta U$ everywhere in this book to compute $W_{\text{spring force}}$. Moreover, it is a better method and we can avoid some common pitfalls in calculations using this concept.

Illustration | 19 *A ball of 5 kg is projected from the surface of earth in vertical upward direction with a speed of 10 ms^{-1}. Determine the maximum height attained by ball by using energy conservation law.*
[Take $g = 10\ ms^{-2}$].

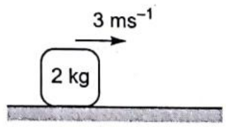
Solution Let h be the maximum height attained by ball and we know at maximum height, speed of the ball is zero. Applying work-energy theorem

$$\Delta K = W_{\text{gravity force}}$$
$$K_f - K_i = W_{\text{gravity force}}$$
$$0 - \frac{5 \times (10)^2}{2} = -5\,g \times h \Rightarrow h = 5\text{ m}.$$

Proficiency in Concepts (PIC)

Problems

Problem 1. *A block of mass 2 kg is moving with a speed of 3 ms^{-1} on a rough horizontal surface. Determine the work done by friction force on the block, when its speed reduces to 1 ms^{-1}. Is coefficient of friction value is needed to solve the questions ?*



Solution Here in this question even without coefficient of friction value we can solve the question by using the work-energy theorem.

$$\Delta K = W_{\text{friction}}$$

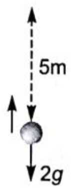
[As other forces are not doing any work on the block]

$$K_f - K_i = W_{\text{friction}}$$

$$\Rightarrow \quad \frac{2 \times 1^2}{2} - \frac{2 \times 3^2}{2} = W_{\text{friction}}$$

$$\Rightarrow \quad W_{\text{friction}} = -8 \text{ J}.$$

Problem 2. *A ball of mass 2 kg is taken slowly* from earth's surface to a height of 5 m. Determine the work done by gravity force on ball and work done by external agent on ball. [Take $g = 10\,ms^{-2}$]*



Solution Here the gravity force and the displacement of ball are in the opposite direction.

$$W_{\text{gravity force}} = 2\,g \times 5 \times (-1)$$
$$= -10\,g = -100 \text{ J}$$

As the ball is moving slowly, so it means the external force is having same magnitude as that of gravity force but is acting in opposite direction to gravity force.

$$W_{\text{external force}} = 2g \times 5$$
$$= 10g = 100 \text{ J}$$

Work done by an external force can also be computed by using work-energy theorem. As the object is moving slowly, $\Delta K = 0$.

So, from work energy theorem

$$\Delta K = W_{\text{gravity}} + W_{\text{external}}.$$
$$0 = -100 + W_{\text{external}}$$
$$\Rightarrow \quad W_{\text{external}} = 100 \text{ J}$$

Problem 3. *A boy pushes a toy car with a force of 50 N. Determine the work done by boy on car, if car displaces by 2 m in the direction of force.*

Solution Work done $= Fs\cos\theta$

Here $F = 50$ N, $s = 2$ m and $\theta = 0°$

So, $\quad W = 50 \times 2 \times \cos 0° = 100$ J

Problem 4. *In above question if the boy takes 50 s to do this task, then what is the power spent by boy ?*

Solution Power $= \dfrac{\text{work}}{\text{time}}$

$$P = \frac{100 \text{ J}}{50 \text{ s}} = 2 \text{ Js}^{-1}$$
$$= 2 \text{ W}$$

Problem 5. *A person of mass 50 kg climbs up a staircase to reach a height of 15 m. Determine the average power spent by him if he reaches from bottom to top in 2 min. [Take $g = 10\,ms^{-2}$]*

Solution In climbing up the stairs, the person has to do work against gravity force. So, the work done by the person in climbing up is

* Slowly means we are not changing the speed of object as it moves *ie*, if the object moves slowly then it means the object is moving with constant velocity.

$$W = mg \times h$$
$$= 50 \times 10 \times 15 = 7500 \text{ J}$$
$$P_{av} = \frac{W}{t} = \frac{7500}{2 \times 60} \text{ Js}^{-1} = 62.5 \text{ W}$$

Problem 6. *A machine is delivering energy to a block at constant rate of 10 W so that the block can move with a constant velocity of 2 ms^{-1}. Determine the force exerted by machine on block.*

Solution $P = Fv$

Here, $P = 10 \text{ W}$ and $v = 2 \text{ ms}^{-1}$

So, $10 = F \times 2$

$\Rightarrow$ $F = 5 \text{ N}$

Problem 7. *A block of mass 2 kg is moving with a speed of 3 ms^{-1}. On its way it collides with another block and comes to rest. Determine the energy given by the first block to second block.*

Solution The entire KE of first block is taken up by second block, so required energy $= \frac{2 \times 3^2}{2} = 9 \text{ J}$.

Problem 8. *An object of mass 3 kg is moving with a constant velocity of 4 ms^{-1}. Determine the KE possessed by the object.*

Solution $\text{KE} = \frac{mv^2}{2} = \frac{3 \times 4^2}{2} = 24 \text{ J}$

Problem 9. *If the KE of an object changes from 10 J to 35 J in 15 s, then determine the total work done on the object in these 15 s and also the average power delivered to the object.*

Solution From work-energy theorem

$$\Delta K = W$$
$$\Rightarrow \quad W = K_f - K_i$$
$$= (35 - 10) \text{ J}$$
$$= 25 \text{ J}$$
$$P_{av} = \frac{W}{t} = \frac{25 \text{ J}}{15 \text{ s}} = \frac{5}{3} \text{ W}$$

Problem 10. *With the help of some external force a spring of spring constant 500 Nm^{-1} is elongated by 10 cm from its natural length. Determine the work done by the external force against the spring force.*

Solution $W_{ext.} = -W_{\text{spring force}}$

$$= -[-\Delta U]$$
$$= \Delta U = U_f - U_i$$
$$= \frac{kx_f^2}{2} - 0$$

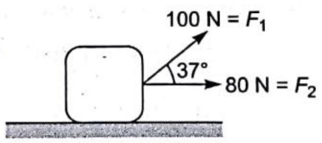
[$\because$ Initially spring is in its natural length]

$$= \frac{500 \times (0.1)^2}{2}$$
$$= 2.5 \text{ J}$$

Towards Proficiency Problems
Exercise 1

A. Subjective Discussions

1. Work done by frictional force acting on an object can be positive. Comment on this statement.
2. Work done by normal contact force on an object can be non-zero. Explain this statement.
3. Can we apply work energy theorem in non-inertial frames of reference ?
4. Is change in PE of a system dependent upon the reference level chosen ?
5. Is absolute PE of a system dependent upon the reference level chosen ?
6. Two forces F_1 and F_2 are acting on a block as shown in figure. Which force does more work on the block ? Examine your answer.

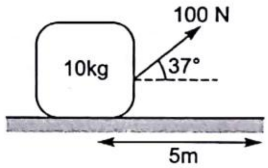


7. A block is moving on a rough horizontal floor under the action of a horizontal force. Determine the sign of work done by applied force, friction force, normal contact force and gravity force on block.
8. A force does positive work on a particle that has a displacement pointing in +ve X-direction. The same force does negative work on a particle that has a displacement pointing in the +ve Y-direction. In which quadrant does the force lie ? Explain your answer.
9. A slow moving car can have more KE than a fast moving motorcycle. Explain this statement.
10. The speed of a particle doubles from its initial value and then again doubles because of the net constant force acting on it. Does the net force do more work during the first or second doubling ?
11. A shopping bag is hanging straight down from your hand as you walk across a horizontal road at constant velocity.
    (a) Does the force your hand exerts on the bag's handle do any work on the bag ?
    (b) Does this force do any work if you are standing in a running lift ? Explain your answer and consider various frames of reference.
12. A net external non-conservative force does positive work on a particle, and both the KE and PE of the particle changes. What, if anything you can conclude about
    (a) the change in the particle's total mechanical energy, and the
    (b) the individual changes in the kinetic and potential energies ?
13. When you lift a box from the floor and put it on an almiraha the PE of the box increases, but there is no change in its KE. Is it a violation of conservation of energy ?

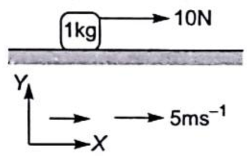
14. In a tug of war, the team that exerts a larger tangential force on the ground wins. Consider the period in which a team is dragging the opposite team by applying a larger tangential force on the ground. List which of the following works are positive, negative or zero ?
    1. Work done by winning team on the losing team.
    2. Work done by the losing team on the winning team.
    3. Work done by the ground on winning team.
    4. Work done by the ground on losing team.
    5. Total external work on the two teams.

B. Numerical Answer Types

1. A force of 10 N acts on a block, as a result of which it moves by a distance of 3 m in the direction of force in 2 s. Determine the work done by this force on block.
2. In above question if resistive force of 3 N is acting on block, and is directed in a direction opposite to motion of block, then determine the work done by resistive force on block. Also, determine the work done by applied force against the resistive force.
3. In question 1 determine the KE of block at $t = 2$ s if at $t = 0$ the block starts from rest.
4. In above question determine the average power delivered to the block by the applied force, and the average power spent by block against the resistive force.
5. A block of mass 5 kg slides down an inclined plane of inclination 37°. Determine the work done by gravity force on the block if incline length is 6 m.
6. A block of mass 10 kg is pulled with the help of a 100 N force on a rough horizontal surface as shown in figure. The coefficient of friction between block and surface is 0.5. For 5 m displacement of block, determine the
   (a) work done by applied force
   (b) work done by gravity force
   (c) work done by friction force
   (d) work done by normal contact force

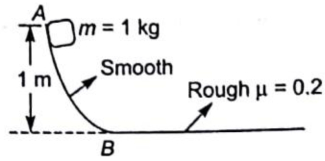


7. A block of mass 1 kg is moving on a smooth horizontal floor under the action of a 10 N force as shown in figure. If the block is at rest at $t = 0$ and the force also starts acting at $t = 0$, then determine the work done by 100 N force on the block in 5 s *wrt* a frame of reference which is moving with constant velocity of 5 ms^{-1} in the same direction as that of force ?



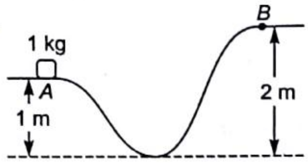
8. A ball of mass 3 kg is changing its velocity from 2 ms^{-1} to 8 ms^{-1} in 5 s. Determine the change in KE of ball and also the power delivered to it.
9. A block of mass 3 kg is moved by 3 m in vertical upward direction in 3 s. Determine the power delivered to block.
10. Two persons A and B of masses 50 kg and 60 kg respectively are both climbing up a ladder so that to reach a height of 12 m, A takes 10 s while B takes 5 s to reach the top. Determine the ratio of power spent by A and B against the gravity force.
11. Two electric bulbs each of 100 W are operated for 10 h daily. Determine the total electrical energy consumed in 2 months (1 month = 30 days).
12. A block of mass 2 kg is moving with a speed of 10 ms^{-1} on a rough horizontal floor till it stops. Determine the work done against the friction.

13. A block of mass 1 kg is released from rest from a height of 1 m on a smooth curved track as shown in figure. The curved track is joined to a rough horizontal track as shown in figure.

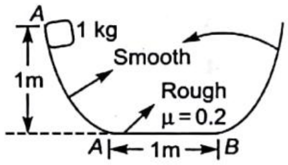


Determine the distance travelled by block as measured from B before it stops. [Take $g = 10\,\text{ms}^{-2}$]

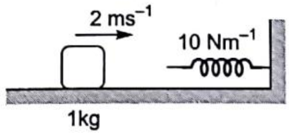
14. A block of mass 1 kg is projected from A with a velocity v_0 on a smooth track as shown in figure. Determine the minimum value of v_0 so that the block reaches B. [Take $g = 10\ \text{ms}^{-2}$]



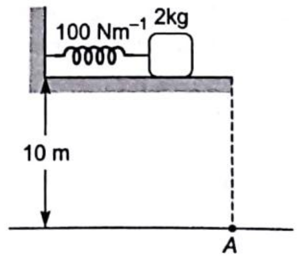
15. A block of mass 1 kg is released from rest from A on a curved track as shown in figure.
The horizontal section AB of length 1 m is rough, remaining all is smooth. Determine the position where the block finally stops.
[Take $g = 10\ \text{ms}^{-2}$]



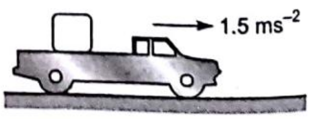
16. A block of mass 1 kg is projected with a velocity of 2 ms^{-1} towards a spring of spring constant 10 Nm^{-1} as shown in figure. Neglect friction everywhere. Determine the maximum compression in the spring.



17. A block of mass 2 kg is compressed against a spring of spring constant 100 Nm^{-1} such that the compression in the spring is 20 cm, from here the block is released from rest as shown in figure. Determine the distance from A where the block falls. [Take $g = 10\ \text{ms}^{-2}$]



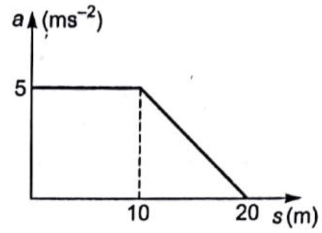
18. Figure shows a 100 kg block kept on the back of a truck that is moving with acceleration 1.5 ms^{-2}. There is no relative motion between the block and truck.
Determine the work done by friction force on block as the truck is displaced by 10 m (a) *wrt* ground frame of reference. (b) *wrt* truck frame of reference.



19. A block of mass M kg is dropped from a height H. It reaches the ground with a speed of $1.2\sqrt{gH}$. Determine the work done by air friction on block.

20. A 1.1×10^3 kg car, starting from rest, accelerates for 5 s. The magnitude of the acceleration is 4.6 ms^{-2}. Determine the average power generated by the net force that accelerates the car.

21. The acceleration *versus* displacement graph of a particle of mass 2 kg is as shown in figure. If the particle displaces by 20 m in 10 s, then determine

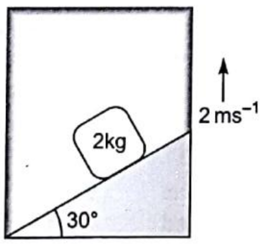


(a) the total work done on particle in 10 s.
(b) the average power delivered to block in 10 s.

22. During a tug-of-war, team *A* pulls on team *B* by applying a force of 1100 N on the rope between them. How much work does team *A* do if they pull team *B* towards them by a distance of 2 m?

23. A 100 kg block is being pulled across a rough horizontal floor by a force *F* that makes an angle of 30° above horizontal. If $\mu_k = 0.2$, then determine the value of *F* so that the net work done on the block is zero.

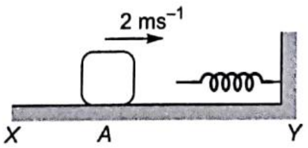
24. A block of mass 2 kg is placed at rest on an inclined plane (rough) fitted in an elevator which is moving up with a speed (constant) of 2 ms^{-1} as shown in figure. Determine the work done by



(a) gravity force on block in 2 s.
(b) friction force on block in 2 s.
(c) normal contact force on block in 2 s.

25. A spring of spring constant 2000 Nm^{-1} is initially having a compression of 20 cm as measured from its natural length. If the spring is stretched so that finally the spring elongates by 20 cm, then determine the work done by spring force for this stretching.

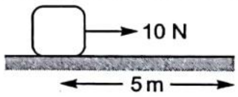
26. A block of mass 2 kg is projected towards a spring with a speed of 2 ms^{-1} as shown in figure. The spring constant of the spring is 10 Nm^{-1}. The portion *AY* of the surface is smooth and *AX* is rough having $\mu = 0.2$. Determine



(a) the maximum compression in spring.
(b) the velocity of block when compression in the spring is half of the maximum value found in part *A*.
(c) the location/instant when block leaves the contact with the spring. Given your answer in analytical ways only.
(d) the location where the block finally stops as measured from *A*.

C. Fill in the Blanks

1. The amount of work required to stop a moving object is equal to ....
2. Work done by a conservative force is .... of path.
3. Law of conservation of mechanical energy is valid only when ....
4. The work done by the external forces on a system equals the change in ....
5. The negative of the work done by the internal conservative forces on a system equals the change in ....
6. The work done by all the forces on a system equals the change in its ....
7. .... of a two particle system depends only on the separation between them.
8. Law of conservation of mechanical energy is a special case of ....
9. A block of mass 2 kg is moving with constant velocity on a rough horizontal surface as shown in figure. The work done by friction when the block moves by 5 m is ....



10. In above question, the work done by 10 N applied force is ....

D. True/False

1. Work done by a force on an object in certain time interval can be different for different observers.
2. A non-zero force acting on a body can do zero work on it.
3. A ball has a speed of 10 ms^{-1}, under the action of a single force acting on it, its speed decreases to 5 ms^{-1}. The work done by this force on ball is –ve.
4. Static friction can't do non-zero work on an object.
5. KE of a system can be increased without applying any external force on the system.
6. KE of a body depends on frame of reference.
7. KE of a heavier body can be less than the KE of lighter body.
8. Work energy theorem is valid in all inertial frames of references.
9. Law of conservation of mechanical energy can be applied to a non-isolated system.
10. As a ball is dropped from some height, the work done by gravity on the ball appears as its KE.

High Skill Questions

Exercise 2

A. Only One Option Correct

1. A boy holds a 40 N weight at arm's length for 10 s. His arm is 1.5 m above the ground, the work done by the boy on the weight while holding it is [Take $g = 10 \text{ ms}^{-2}$]

(a) Zero (b) 6.1 J
(c) 40 J (d) 90 J

2. The work done by gravity during the descent of a projectile is

(a) positive
(b) negative
(c) Zero
(d) Depends upon the sign convention we use

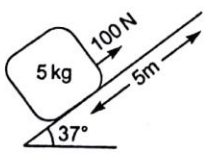
3. The power delivered to a projectile by gravity force at the highest point of trajectory is

(a) non-zero
(b) Zero
(c) positive
(d) Can't say anything

4. A 1 kg block is lifted up in vertical upward direction by 1 m. The work done on the block is about [Take $g = 10 \text{ ms}^{-2}$]

(a) 1 J (b) 5 J
(c) 10 J (d) Zero

5. A block of mass 5 kg is pulled up on a rough incline with the help of 100 N force as shown in figure. The coefficient of friction between the block and incline is 0.3. For the 5 m displacement of block along the incline, the work done by 100 N force on the block is [Take $g = 10 \text{ ms}^{-2}$]



(a) 500 J (b) 150 J
(c) 60 J (d) 290 J

6. In Q. 5, the work done by force of gravity on block is

(a) 500 J (b) –150 J
(c) – 60 J (d) 290 J

7. In Q. 5, the work done by friction force on the block is

(a) 500 J (b) –150 J
(c) – 60 J (d) 290 J

8. In Q. 5, the change in KE of block for the mentioned displacement is

(a) 500 J (b) –150 J
(c) – 60 J (d) 290 J

9. A particle moves by 5 m in the positive x direction while being acted by a constant force $\vec{F} = (4\hat{i} + 2\hat{j} - 4\hat{k})$ N. The work done on the particle by this force is

(a) 10 J (b) 20 J
(c) – 20 J (d) 50 J

10. A block is attached to the end of an ideal spring and moved from coordinate x_i to coordinate x_f. The relaxed position of spring-block system is at $x = 0$. The work done by spring is positive if

(a) $x_i = 2$ cm, and $x_f = 4$ cm
(b) $x_i = -4$ cm, and $x_f = -2$ cm
(c) $x_i = -2$ cm, and $x_f = 4$ cm
(d) $x_i = -2$ cm, and $x_f = -4$ cm

11. An ideal spring is hung vertically from the ceiling. When a 2 kg mass hangs at rest the spring is elongated by 6 cm from its relaxed length. A downward force is now applied on the mass to elongate the spring further by 10 cm. When the spring is elongated by force, the work done by the spring is

(a) 3.6 J (b) – 3.6 J
(c) 4.26 J (d) – 4.26 J

12. Which of the following bodies have the largest KE ?
(a) Mass 3 m and speed v
(b) Mass 3 m and speed $2v$
(c) Mass 2 m and speed $3v$
(d) Mass m and speed $4v$

13. An 8 N block slides down an incline, its initial speed is 7 ms^{-1}. The work done by the resultant force on the block is
(a) 3 J
(b) 6 J
(c) Information insufficient
(d) None of the above

14. A 2 kg ball connected to the end of a spring is pulled out by 0.5 m and then released from rest. The spring constant is 200 Nm^{-1}. When the particle passes the point where the spring force is zero, its speed would be
(a) Zero (b) 0.05 ms^{-1}
(c) 5 ms^{-1} (d) 10 ms^{-1}

15. A kilo-watt hour is a unit of
(a) power (b) energy
(c) work/time (d) None of these

16. A 50 N force acts on a 2 kg block that starts from rest. When the force has been acting for 2 s the rate at which it is doing work is
(a) 75 W (b) 100 W
(c) 1000 W (d) 2500 W

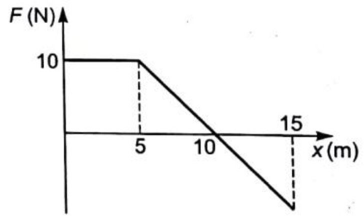
17. A stone is thrown from a cliff of height h with a speed v. The stone will hit the ground with maximum speed if it is thrown
(a) vertically downward
(b) vertically upward
(c) horizontally
(d) The speed will be the same in all the cases

18. An elevator of total mass (elevator + passenger) 1800 kg is moving up with a constant speed of 2 ms^{-1}. A frictional force of 2000 N is opposing its motion. The minimum power delivered by the motor to the elevator is [Take $g = 10\ ms^{-2}$]
(a) 36 kW (b) 4 kW
(c) 40 kW (d) – 40 kW

19. The two bodies (particles) are moving under their mutual gravitational interaction (assume no other force is acting on them), then the total work done on the two bodies in a particular time interval
(a) is always zero
(b) is always positive
(c) is always negative
(d) can be zero sometimes

20. An aeroplane ascends and reaches a velocity of 360 kmh^{-1} at an altitude of 5 km. By how many times is the work performed during the ascent against the force of gravity greater than that performed to increase the velocity of the aeroplane ?
(a) Ten times (b) Nine times
(c) Eleven times (d) Twenty times

21. For a particle moving along a straight line, the force *versus* displacement graph has been given. The work done on the particle in the time- interval in which particle displaced by 15 m, is



(a) 50 J (b) 75 J
(c) –25 J (d) None of these

22. A ball of mass 2 kg is projected vertically up with a speed of 2 ms^{-1}, and it comes back to the ground with a speed of 1 ms^{-1}. The work done by air friction force is
(a) 3 J
(b) –3 J
(c) –20 J
(d) Information insufficient

B. More Than One Options Correct

1. Mark the correct statement(s) related to work.
   (a) Work done by a force can be different as measured by different observers.
   (b) Work done by a force can be zero even though the force is non-zero.
   (c) Work done by a force on a object would be non-zero if the force is not perpendicular to displacement of object.
   (d) None of the above

2. Two particles interact by conservative forces and no other forces act. They complete a round trip, ending at the points where they started. For this round trip mark out the incorrect statement(s).
   (a) The total KE might have a different value at the beginning, and at the end.
   (b) The total PE might have a different value at the beginning, and at the end.
   (c) The total mechanical energy might have a different value at the beginning, and at the end.
   (d) The total mechanical energy might have same value at the beginning, and at the end.

3. In which of the following situations, a non-zero work has been done ?
   (a) A weightlifter lifts up a rod.
   (b) A weightlifter holds a rod in air.
   (c) A person is pushing a block kept on a smooth horizontal floor.
   (d) A person is pushing against a rigid wall.

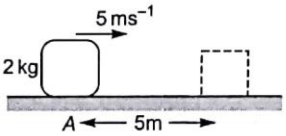
4. In which of the following situations the energy of the body increases ?
   (a) An accelerating car — the energy of car
   (b) A weightlifter lifts up the rod — the energy of rod
   (c) A person pushing a rigid wall — the energy of wall
   (d) A ball kicked by a player — the energy of ball

5. A ball has been dropped from a height, then for this situation mark out the correct statement(s).

   (a) PE of the ball is decreasing.
   (b) Work done by gravity force is positive.
   (c) KE of the ball increases.
   (d) Work done by gravity force is negative.

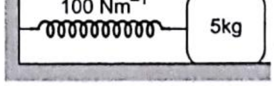
6. Work done by spring force
   (a) can be positive
   (b) can be negative
   (c) is independent of path
   (d) is zero

7. A block of mass 2 kg is moving initially with a speed of 5 ms^{-1} on a rough horizontal floor as shown in figure. After travelling for 5 m the speed of block is 2 ms^{-1}. For this situation mark out the correct statement(s). [Take $g = 10\ ms^{-2}$]



   (a) The work done by friction force on block is – 21 J.
   (b) The work done by friction force on block is 21 J.
   (c) The coefficient of friction between the block and surface is 0.42.
   (d) The work done by contact force (between block and surface) on the block is –21 J.

8. A block of mass 5 kg is connected to a spring of spring constant 100 Nm^{-1} as shown in figure. Initially the spring is in natural length and there is no friction anywhere. Now, the block is imparted a velocity of 4 m^{-1} towards left. For this situation mark out the correct statement(s).



   (a) The maximum compression in the spring is $2/\sqrt{5}$ m.
   (b) The maximum elongation in the spring is $2/\sqrt{5}$ m.

(c) The maximum velocity of the block is $4\ ms^{-1}$.

(d) The elongation in the spring when the velocity of block is maximum is zero.

9. Related to spring mark out the correct statement(s), k is spring constant.

(a) If elongation in spring is x_0, then energy stored in spring is $\frac{kx_0^2}{2}$.

(b) If elongation in spring is x_0, then energy stored in spring is $-\frac{kx_0^2}{2}$.

(c) If compression in spring is x_0, then energy stored in spring is $-\frac{kx_0^2}{2}$.

(d) If compression in spring is x_0, then energy stored in spring is $\frac{kx_0^2}{2}$.

10. Initially the spring is elongated by x_0 from its natural length and then compresses to x_0. The work done by spring force is

(a) Zero

(b) value of spring constant must be known

(c) non-zero

(d) Information insufficient

C. Assertion & Reason

Directions (Q. Nos. 1 to 9) *Some questions (Assertion-Reason type) are given below. Each question contains Statement I (**Assertion**) and Statement II (**Reason**). Each question has 4 choices (a), (b), (c) and (d) out of which ONLY ONE is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**

(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**

(c) **Statement I** is True, **Statement II** is False

(d) **Statement I** is False, **Statement II** is True

1. **Statement I** If a body does work, then it loses some of its energy.
Statement II From work-energy theorem we can say work and energy are equivalent.

2. **Statement I** An object having energy can do the work.
Statement II An object having energy can exert a force on other object(s), and hence can transfer the energy.

3. **Statement I** Absolute PE of a system as measured by two different persons at the same time can be different.
Statement II Value of absolute PE of a system depends upon the reference value chosen.

4. **Statement I** Work done by a force would be always positive.
Statement II Work done by a force $\vec{F}$ is defined as $W = \vec{F} \cdot \vec{s}$.

5. **Statement I** Work done by the static friction force is always zero.
Statement II When there is no relative motion between the two surfaces, then friction is static in nature.

6. **Statement I** KE of a lighter body can be greater than of a heavier body.
Statement II If we do more work on a body, then its KE increases by more amount.

7. **Statement I** Change in PE of a system can have different values for different observers.
Statement II Change in PE of a system is independent of the reference level chosen.

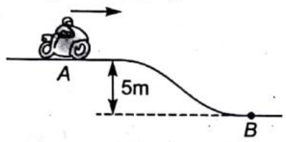
8. **Statement I** Work done by spring force for a closed path is zero.
Statement II Spring force is a conservative force.

9. **Statement I** Work done by normal contact force can be non-zero.
Statement II Normal contact force is always perpendicular to displacement of object. (Here displacement is measured *wrt* frame of reference attached to the two surfaces in contact).

D. Comprehend the Passage Questions

Passage I

A cyclist, together with his bicycle has a total mass of 100 kg, is moving on a horizontal road with constant velocity of 20 ms^{-1}. At A, a slope has come and goes does to reach B as shown in figure. Assume no work has been done against friction. [Take $g = 10\ ms^{-2}$].

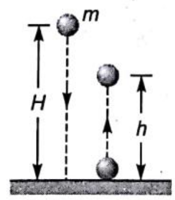


Based on above information, answer the following questions :

1. The initial KE of the cyclist (along with his bicycle is)
   (a) 2000 J (b) 20 kJ
   (c) 4000 J (d) 40 kJ
2. The decrease in gravitational PE of cyclist as it goes from A to B is
   (a) 3000 J (b) 4000 J
   (c) 5000 J (d) 10000 J
3. The speed of the cyclist at B is
   (a) 25 ms^{-1} (b) 22.36 ms^{-1}
   (c) 28.64 ms^{-1} (d) 24.86 ms^{-1}

Passage II

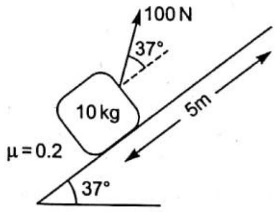
A ball of mass m is dropped from a height H above a level floor as shown in figure. After striking the ground it bounces off back and reaches up to height h. Based on above information, answer the following questions :



4. During the collision, the part of the KE which appears in another forms (other than KE or PE) is (This part of the energy is termed as lost energy as it can't be utilised properly)
   (a) mgH (b) mgh
   (c) $mgH - mgh$ (d) Zero
5. The speed of the ball just after collision is
   (a) $\sqrt{2gH}$ (b) $\sqrt{2gh}$
   (c) $\sqrt{2g(H-h)}$ (d) None of these
6. If the lost energy in the collision is half of the value computed in Q. 4, and $H = \frac{3h}{2}$, then the height attained by the ball after collision is
   (a) $\frac{7h}{4}$ (b) $\frac{3h}{4}$
   (c) $\frac{3h}{2}$ (d) $\frac{9h}{5}$

Passage III

A block of mass 10 kg is pulled (starting from rest) with the help of a 100 N force on a rough inclined plane of inclination 37° as shown in figure. The coefficient of friction between the incline and plane is 0.2. For the displacement of 5 m of block along the incline. Based on above information, answer the following questions : [Take $g = 10\ ms^{-2}$]



7. The work done by 100 N force on the block is
   (a) 40 J (b) 500 J
   (c) 400 J (d) 20 J
8. The work done by friction force on block is
   (a) 40 J (b) –20 J
   (c) –100 J (d) 160 J
9. The final KE of block is
   (a) 40 J
   (b) 20 J
   (c) 80 J
   (d) 160 J

E. Match the Columns

1.

	Column I	Column II
(A)	Kinetic energy can be	(P) Positive
(B)	Potential energy can be	(Q) Negative
(C)	Total mechanical energy can be	(R) Zero
(D)	Work can be	(S) Non-zero

2. In Column I, some incomplete statement(s) related to work done by various forces are given which can be completed by options of Column II. Match the entries of Column I with the entries of Column II. Consider all possibilities while making suitable matches.

	Column I	Column II
(A)	Work done by normal contact force can/must be	(P) Zero
(B)	Work done by static friction can/must be	(Q) Non-zero
(C)	Work done by kinetic friction can/must be	(R) Positive
(D)	Work done by gravity force can/must be	(S) Negative

3. In Column I, some physical situations are given, and in Column II, some statements related to work and energy are given. Match the entries of Column I with the entries of Column II.

	Column I	Column II
(A)	A body is accelerating on a smooth horizontal surface	(P) KE of the body is increasing
(B)	A body is accelerating up on a smooth incline	(Q) Positive work has been done on the body
(C)	A body is moving on a rough horizontal surface and the speed is decreasing	(R) PE of the body is increasing
(D)	A body is dropped from a height	(S) Negative work has been done on the body

4. In Column I, the angle (θ) between force acting on a particle and its displacement are given and in Column II, the statements related to energy and work done are given. Match the entries of Column I with the entries of Column II.

	Column I	Column II
(A)	$\theta = 45°$	(P) KE increases
(B)	$\theta = 90°$	(Q) KE decreases
(C)	$\theta = 135°$	(R) Work is positive
(D)	$\theta = 180°$	(S) Work is zero

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. 30 J **2.** –9 J, 9 J **3.** 30 J **4.** 15 W, 4.5 W **5.** 180 J
6. (a) 400 J, (b) 0, (c) –100 J, (d) 0 **7.** 1000 J **8.** 90 J, 18 W **9.** 30 W
10. $\frac{5}{12}$ **11.** 60 kWh **12.** 100 J **13.** 5 m **14.** $\sqrt{20}$ ms^{-1}
15. At B **16.** 0.63 m **17.** 2 m **18.** (a) 1500 J, (b) 0
19. – 0.28 mgH **20.** 58.19 kW **21.** (a) 150 J, (b) 15 W **22.** 2200 J **23.** 207.04 N
24. (a) – 80 J, (b) 20 J, (c) 60 J **25.** 0
26. (a) 0.89 m, (b) 1.732 ms^{-1}, (c) when spring acquires natural length, (d) 1 m towards left of A

C. Fill in the Blanks

1. Kinetic energy **2.** Independent **3.** Work done by non-conservative and external forces are zero
4. Mechanical energy **5.** Potential energy **6.** Kinetic energy **7.** Potential energy
8. Work-energy theorem **9.** –50 J **10.** 50 J

D. True/False

1. T **2.** T **3.** T **4.** F **5.** T **6.** T **7.** T **8.** T
9. F **10.** T

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (a) **2.** (a) **3.** (b) **4.** (c) **5.** (a) **6.** (b) **7.** (c) **8.** (d) **9.** (b) **10.** (b)
11. (b) **12.** (c) **13.** (c) **14.** (c) **15.** (b) **16.** (d) **17.** (d) **18.** (c) **19.** (b) **20.** (a)
21. (a) **22.** (b)

B. More Than One Options Correct

1. a, b, c **2.** a, b, c **3.** a, c **4.** a, b, d **5.** a, b, c
6. a, b, c **7.** a, c, d **8.** a, b, c, d **9.** a, d **10.** a

C. Assertion & Reason

1. (a) **2.** (a) **3.** (a) **4.** (d) **5.** (d) **6.** (b) **7.** (d) **8.** (a) **9.** (a)

D. Comprehend the Passage Questions

1. (b) **2.** (c) **3.** (b) **4.** (c) **5.** (b) **6.** (a) **7.** (c) **8.** (b) **9.** (c)

E. Match the Columns

1. A → P, R, S; B → P, Q, R, S; C → P, Q, R, S; D → P, Q, R, S
2. A → P, Q, R, S; B → P, Q, R, S; C → P, Q, R, S; D → P, Q, R, S
3. A → P, Q; B → P, Q, R; C → S; D → P, Q
4. A → P, R; B → S; C → Q; D → Q

Explanations

Towards Proficiency Problems

Exercise 1

Numerical Answer Types

1. $W = Fs\cos\theta$

$= 10 \times 3 \times \cos 0 = 30$ J

2. $W_1 = -f \times 3 = -3 \times 3 = -9$ J.

Work done against resistive force

$= -W_1 = 9$ J

3. From work-energy theorem,

$$K_f - K_i = W$$

$$\Rightarrow \quad K_f - 0 = 30$$

$$\Rightarrow \quad K_f = 30 \text{ J}$$

4. Power $= \dfrac{\text{work}}{\text{time}}$.

Power delivered to block by applied force,

$$P_1 = \frac{30}{2} = 15 \text{ W}$$

Power spend against the resistive forces,

$$P_2 = \frac{9}{2} = 4.5 \text{ W}$$

5. Work done by the gravity force,

$$W = mg\sin 37° \times s$$

$$= 5 \times 10 \times \frac{3}{5} \times 6 = 180 \text{ J}$$

6. (a) $W_1 = 100\cos 37° \times 5 = 400$ J

(b) $W_2 = 0$ as angle between displacement and gravity force is $\dfrac{\pi}{2}$.

(c) $W_3 = -fs$

where $f = \mu N = 0.5\,[100 - 100\sin 37°]$

$= 20$ N

$W_3 = -100$ J

(d) $W_4 = 0$ as normal contact force and displacement are perpendicular to each other.

7. With respect to considered frame of reference

$u = -5 \text{ ms}^{-1}, \quad a = 10 \text{ ms}^{-2}$

[Right direction is taken as +ve]

$$s = -5 \times 5 + \frac{1}{2} \times 10 \times 5^2 = +100$$

$$W = Fs = 10 \times 100 = 1000 \text{ J}$$

8. $\Delta K = \dfrac{mv_f^2}{2} - \dfrac{mv_i^2}{2}$

$$= 3\left[\frac{8^2 - 2^2}{2}\right] = 90 \text{ J}$$

$$\text{Power} = \frac{W}{t} = \frac{\Delta K}{t} = \frac{90}{5} = 18 \text{ W}$$

9. Power $= \dfrac{3g \times 3}{3} = 30$ W

10. Power of $A = \dfrac{50 \times 10 \times 12}{10}$

Power of $B = \dfrac{60 \times 10 \times 12}{5}$

$$\frac{P_1}{P_2} = \frac{5}{12}$$

11. Energy consumed daily,

$$E_1 = 100 \times 10 \text{ Wh} = 1 \text{ kWh}$$

Energy consumed in 2 months,

$$E = 60\,E_1 = 60 \text{ kWh}$$

12. Work done against friction = –ve of work done by friction. From work-energy theorem,

$$\Delta K = W_{\text{friction}}$$

Work done against friction,

$$= -\Delta K$$

$$= -[K_f - K_i] = K_i - K_f$$

$$= \frac{mv^2}{2} - 0 = 100 \text{ J}$$

13. Finally let the block comes to rest after travelling a distance of s metre on the rough horizontal surface. So,

$$K_f - K_i = W_{\text{gravity}} + W_{\text{friction}}$$

$$0 - 0 = mg \times 1 - 0.2 \times mg \times s$$

$$\Rightarrow \quad s = 5 \text{ m}$$

14. For minimum value of v_0, the velocity of block at B would be zero. Applying work-energy theorem for A and B,

$$0 - \frac{mv_0^2}{2} = -mg(2 - 1)$$

$$\Rightarrow \quad v_0 = \sqrt{20} \text{ ms}^{-1}$$

15. Here total work done against friction is done by the gravitational PE.

16. At the time of maximum compression in spring, the speed of block would be zero. Applying work-energy theorem for the initial and final compression instants,

$$0-\frac{1\times 2^2}{2}=W_{\text{spring force}}$$

$$=-(U_f-U_i)=-\left[\frac{kx_m^2}{2}-0\right]$$

$\Rightarrow \qquad x_m=0.63\text{ m}$

where x_m is the maximum compression.

17. Let v be the velocity of block, when it leaves the spring, then from energy conservation

$$\frac{mv^2}{2}=\frac{kx^2}{2}$$

where $\qquad x=20\text{ cm}$

$\Rightarrow \qquad v=\sqrt{2}\text{ ms}^{-1}$

Then use kinematics concepts.

18. As there is no relative motion between the block and truck, so friction between them is static in nature and its value is,

$$f=100\times 1.5=150\text{ N}$$

(a) Work done by friction force *wrt* ground $=f\times 10=1500\text{ J}$

(b) Work done by friction force *wrt* truck $=f\times 0=0$

19. Apply work-energy theorem, let the required work be W.

$$\Rightarrow \quad \frac{m(1.2\sqrt{gH})^2}{2}-0=mgH+W$$

$$\Rightarrow \quad W=-0.28\,mgH$$

20. $F=1.1\times 10^3\times 4.6\text{ N}$

$v=at=4.6\times 5\text{ ms}^{-1}$

Power $=Fv=58.19\text{ kW}$.

21. Work done = mass × area under acceleration-displacement graph

(a) $W=2\times\left[5\times 10+\frac{1}{2}\times 5\times 10\right]=150\text{ J}$

(b) Power $=\frac{\text{work}}{\text{time}}=\frac{150}{10}=15\text{ W}$

22. $W=Fs$

$=1100\times 2=2200\text{ J}$

23. Net work done on the block would be zero when the block moves with constant velocity.

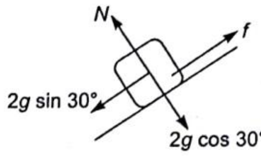
So, $\qquad F\cos 30°=\mu N$

and $\qquad N=mg-F\sin 30°$

After substituting the values, we get

$$F=207.04\text{ N}$$

24. As the block is at rest in elevator it means friction is balancing the component of gravity force along the incline.



In 2 s the displacement of block is 4m in vertical upward direction.

(a) $W_1=-2g\times 4=-80\text{ J}$

(b) $W_2=f\sin 30°\times 4=20\text{ J}$

(c) $W_3=N\cos 30°\times 4=60\text{ J}$

25. $U_i=\frac{kx_i^2}{2}$

$U_f=\frac{kx_f^2}{2}$

where, $x_i=0.2\text{ m}$ and $x_f=0.2\text{ m}$

So, $\qquad W=-\Delta U=0$

26. (a) Let x_m be the maximum compression in spring, at the time of maximum compression the speed of block is zero. From energy conservation,

$$\frac{mv^2}{2}=\frac{kx_m^2}{2}$$

$$\Rightarrow \quad \frac{2\times 2^2}{2}=\frac{10\,x_m^2}{2}$$

$$\Rightarrow \quad x_m=0.89\text{ m}$$

(b) Let v be the velocity of block at the instant when compression in spring is half of the maximum compression

$$\frac{mv^2}{2}-\frac{2\times 2^2}{2}=-\left[\frac{k(x_m/2)^2}{2}\right]$$

$$\Rightarrow \quad v=1.732\text{ ms}^{-1}$$

(c) The block leaves the spring when spring acquires its natural length.

(d) Let block finally stops at a distance s as measured from A,

$$0-\frac{2\times 2^2}{2}=-0.2\times 2\times 10\,s$$

$$s=1\text{ m}.$$

Chapter 8

Linear Momentum, Impulse and Collisions

The First Steps' Learning

- Momentum
- Impulse
- Law of Conservation of Linear Momentum
- Centre of Mass
- Collisions

Have you ever thought that what happens (in terms of contact force between bat and ball) when a ball thrown by a pacer is hit by a batsman or what happens when a football is kicked by a footballer or when two balls (or any other objects) collides. There are many situations in which questions involving forces cannot be answered simply by applying $\vec{F}_{net} = m \times \vec{a}$, ie, Newton's second law. We know that when a ball has been hit by a bat, the ball undergoes a large change in velocity because of the force exerted on it by the bat. But this contact force between bat and ball is varying with time, ie, it is not constant, just before the bat and ball comes in contact it is zero and then increases to a maximum value in a very short duration and then again becomes zero as the ball leaves the bat. Now, as the force in above described situation and in many other situations, is acting for a very small time and varying largely for this small duration, it is extremely difficult to know the exact details of these forces. Remarkably, in this chapter, we will find that we don't require any knowledge of these forces to answer the question related to motion of objects in such situations.

This can be made possible by using two new concepts, impulse and momentum. In addition to this we will also study about conservation of linear momentum or simply conservation of momentum, which is as basic and fundamental law as law of conservation of energy. The law of conservation of momentum is applicable to those situations also where Newton's laws of motion are not applicable (For example, in relativistic physics or for subatomic particles). Then finally in this chapter we will discuss a little about centre of mass and collisions.

Momentum

Suppose you have to either catch a bullet fired from a gun or a ball thrown by a fielder in a cricket match. Which one would you wish to catch ? Surely the ball, because it is easier to get hold of it as it is moving with lesser speed, and it may hurt less. Suppose again, you are being asked to catch a bullet of a gun now thrown by a person, or a cricket ball hit by a batsman. In this case, you would like to catch the bullet, as it is moving with lesser speed than the ball hit by the batsman. The bullet is lighter than the ball, and as it is moving with lesser or equal speed than the ball, so it would be far more convenient to get hold of it. Some other examples based on this reasoning are :

- If you drop two blocks—one heavier than the other from the same height above a sand pile, then the heavier one penetrates more, even though both have the same velocity when they hit the sand pile.
- If you drop two identical blocks from different heights above a sand pile, then the object which is dropped from more height penetrates more, even though both have the same mass.
- You can fearlessly cross a busy road during heavy traffic when all the vehicles on your way are stationary due to red signal, but you will be terrified to cross the road when the vehicles are moving even if most of them are light vehicles.

All these observations lead us to conclude that the force required to stop a moving body not only depends upon its mass but also on its velocity, and hence we get a new term called **momentum** which Newton called as **quantity of motion.**

Linear momentum of a body is defined as the product of its mass and velocity *ie*, $\vec{p} = m\vec{v}$ where $\vec{p}$ is linear momentum or simply momentum of a body of mass m, moving with velocity $\vec{v}$.

Momentum is a vector quantity, its direction is same as that of velocity and its SI unit is N-s or kg-ms^{-1}. Momentum of a body is considered as the measure of its quantity of motion.

C-BIs

Concept Building Illustrations

Illustration | 1 *A body of mass 5 kg is moving with a velocity of 5 ms^{-1} towards east. Determine its momentum.*

Solution Momentum of the body is given by,

$$\vec{p} = m\vec{v}$$

$$\Rightarrow |\vec{p}| = m \times |\vec{v}| = 5 \times 5 \text{ kg-ms}^{-1} = 25 \text{ kg-ms}^{-1}$$

So, momentum of the body is 25 kg-ms^{-1} towards east.

Illustration | 2 *The momentum of a body of mass 3 kg at a particular instant is $(3\hat{i} + 6\hat{j})$ N-s. Determine the velocity of body at this instant.*

Solution From

$$\vec{p} = m\vec{v}$$

$$\Rightarrow \quad \vec{v} = \frac{\vec{p}}{m} = \frac{3\hat{i} + 6\hat{j}}{3} = (\hat{i} + 2\hat{j}) \text{ ms}^{-1}$$

Relation between KE and Momentum

In last chapter, we have seen that kinetic energy of a body of mass m moving with velocity $\vec{v}$ is given by, $K = \frac{mv^2}{2}$. Re-arranging the above expression we can get,

$$K = m \times \frac{mv^2}{2m}$$

$$K = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

as p is the magnitude of momentum of body and equal to mv, so from above expression we can say that if kinetic energy of a body is zero then its momentum would be zero and *vice-versa ie*, a body can't have KE without having the momenta and *vice-versa*.

The above statement is not valid for system of particles or bodies, it is valid only for an individual particle or a body.

Another point to keep in mind is, if we know momentum of a body we can find its KE, but if KE is known then only the magnitude of momentum could be found, and not its direction. Hence, we can conclude as follows :

"If we know momentum of a particle at any instant, we can find its KE at that instant, but the reverse is not true, ie, we cannot find the momentum of a particle completely if we know its KE sat any instant."

C-BIs

Concept Building Illustrations

Illustration | 3 *A ball of mass 3 kg is possessing 405 J of kinetic energy. Determine the magnitude of its momentum.*

Solution From $\quad K = \frac{p^2}{2m}$

$$\Rightarrow \quad p^2 = K \times 2m$$

$$= 405 \times 2 \times 3$$

$$p^2 = 2430$$

$$\Rightarrow \quad p = \sqrt{2430} \text{ N-s}$$

Until now we were talking about individual bodies or individual particles, but if our system consists of two or more bodies or particle, then the situation would be something different and we call this system as system of N-particles in the general case. (If N-particles

are considered in system). Consider a system of two particles A and B having masses m_A and m_B, respectively. Let they are moving towards each other with velocities v_A and v_B, respectively.

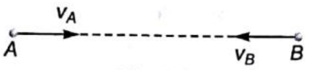


Fig. 8.1

Then, the total momentum of this system of two particles is given by the vector sum of momenta of individual particles, while total KE of this system is equal to sum of individual KEs. So, the total momentum,

$$\vec{\mathbf{p}} = \vec{\mathbf{p}}_A + \vec{\mathbf{p}}_B$$

$$= m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B$$

$$= m_A v_A - m_B v_B$$

Here both the particles are moving along same line so simply we can omit vectors by assigning one direction as positive and other as negative, here we have taken rightward as positive and leftward as negative.

$$\text{Total KE, } K = K_A + K_B$$

$$= \frac{m_A v_A^2}{2} + \frac{m_B v_B^2}{2}$$

Let's consider the situation in which this two- particle system is moving in such a way that total momentum of the system becomes zero *ie*, $m_A v_A = m_B v_B$, then in this situation even though $\vec{\mathbf{p}} = 0$ we have $K \neq 0$, *ie*, for a system of particles at a particular instant $\vec{\mathbf{p}}$ can have a zero value while K is having a non-zero value.

Impulse

As in Newton's laws of motion, we have seen that Newton's second law of motion can be mathematically expressed as

$$\vec{\mathbf{F}}_{\text{net}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t}$$

$$\Rightarrow \quad \Delta \vec{\mathbf{p}} = \vec{\mathbf{F}}_{\text{net}} \times \Delta t$$

ie, change in momentum = Product of force with time interval for which the force acts on the system.

The right hand side term in above equation is termed as impulse ($\vec{\mathbf{J}}$), a vector quantity whose direction is same as that of force *ie*, along the change in momentum vector. Its SI unit is N-s, same as that of momentum. Thus, impulse corresponding to a force $\vec{\mathbf{F}}$ is equal to the product of force $\vec{\mathbf{F}}$ with the time-interval, Δt for which the force is acting.

$$\vec{\mathbf{J}} = \vec{\mathbf{F}} \times \Delta t$$

Thus, we can say, to change the momentum of a body a non-zero impulse must act on the body. Remember that $\vec{\mathbf{J}} = \Delta \vec{\mathbf{p}}$ is a vector equation *ie*, an equation involving vector quantities.

There are various situations as we discussed earlier in which the behaviour of force is very complex (for example, hitting a ball by a bat). In these situations if we want to analyse the motion of an object, then simply we find out the change in momentum of the body and from this we can find the **average force** acting on object.

Impulsive Force

When a very large force is acting for a very small interval of time, then the force said to be **impulsive in** nature. For example, when a ball has been hit by a bat, then a very large contact force acts on the bat and ball during their duration of contact. This force is so large that other forces acting on the bat and ball could be neglected as compared to it.

Depending on the situation, force can be impulsive or non-impulsive in nature.

C-BIs

Concept Building Illustrations

Illustration | 4 *A body of mass 5 kg is moving with a velocity of 3 ms^{-1} towards east, due to an impulse acting on it, its velocity changes to 4 ms^{-1} towards north. Determine*

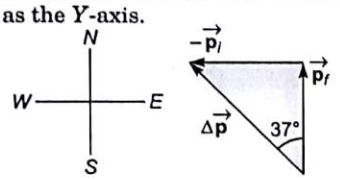
(a) *the change in momentum of body (Both the magnitude and direction).*

(b) *the impulse acting on the object.*

(c) *change in the magnitude of momentum.*

(d) *if the force acts for 0.01 s, then determine the average force acting on the object.*

Solution Let as consider E-W as X-axis, and N-S as the Y-axis.



So, initial momentum,

$$\vec{\mathbf{p}}_i = 5 \times 3\ \hat{\mathbf{i}} = 15\,\hat{\mathbf{i}}\ \text{N-s}$$

Final momentum,

$$\vec{\mathbf{p}}_f = 5 \times 4\hat{\mathbf{j}} = 20\hat{\mathbf{j}}\ \text{N-s.}$$

(a) Change in momentum of body,

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$$

$$\Rightarrow \quad \Delta\vec{\mathbf{p}} = (20\,\hat{\mathbf{j}} - 15\,\hat{\mathbf{i}})\ \text{N-s.}$$

$= 25$ N-s at an angle of 37° with north towards west, as shown clearly in the figure.

(b) Impulse, $\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}} = (20\,\hat{\mathbf{j}} - 15\,\hat{\mathbf{i}})$ N-s

(c) Change in the magnitude of momentum,

$$\Delta|\vec{\mathbf{p}}| = |\vec{\mathbf{p}}_f| - |\vec{\mathbf{p}}_i|$$

$$= 20 - 15 = 5\ \text{N-s}$$

(d) $\vec{\mathbf{J}} = \vec{\mathbf{F}}\,\Delta t$

$$\Rightarrow \quad \vec{\mathbf{F}} = \frac{25}{0.01}\ \text{N-s}$$

$= 2500$ N-s at 37° with north towards west.

In this Illustration just emphasize and observe carefully the difference between parts (a) and (c), in one -direction and magnitude of change in momentum has been asked, and in the other, the change in magnitude of momentum has to be computed. Another point to be kept in mind is that in part (d) we are computing only average value of force and not the exact values which is very difficult to determine theoretically.

Law of Conservation of Linear Momentum

For any system of particles or for a particle, according to Newton's second law of motion $\vec{\mathbf{F}}_{\text{net}} = \dfrac{\Delta\vec{\mathbf{p}}}{\Delta t}$ *ie*, net external force acting on the system is equal to rate of change of momentum.

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}_{\text{net}} \times \Delta t$$

If net external force acting on a system is zero *ie*, if $\vec{\mathbf{F}}_{\text{net}} = 0$, then $\Delta\vec{\mathbf{p}} = 0$ *ie*, change in the momentum of the system is zero or in other words we can say that momentum of the system remains constant. This leads us to the law of conservation of linear momentum, which states that **"if net external force acting on a system remains conserved, then the linear momentum of the system remains constant".**

1. It is worth important to note here that total momentum of the system is constant if $\vec{\mathbf{F}}_{\text{ext}} = 0$, even though the individual momenta of the particles can change. For example, consider a two-particle system comprising A and B which are released

from rest in the position shown in the figure and both are allowed to move freely under their mutual gravitational force, due to which they start moving towards each other and both acquire some velocity and hence momentum, but the total momentum of the system remains zero at all instants equal to initial momentum because $\vec{\mathbf{F}}_{ext} = 0$. If we consider only particle A or B as our system, then its momentum doesn't remain conserved as a net external force is acting on both the particles, the force exerted on one by other.

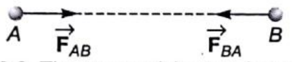


Fig. 8.2 The two particles moving under the influence of each other.

In other words, we can say that for N-particle system if $\vec{\mathbf{F}}_{ext} = 0$, then the vector sum of individual momenta, remain constant *ie*, $\vec{\mathbf{p}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \dots \vec{\mathbf{p}}_N =$ constant, though the individual momenta, *ie*, $\vec{\mathbf{p}}_1, \vec{\mathbf{p}}_2, \dots\dots$ could change, but this change always takes place in such way that total momentum of the system always remains constant.

2. While applying law of conservation of momentum always keeps in mind that momentum is a vector quantity and while adding momentum of different parts of the system we have to use vector addition.
3. If $\vec{\mathbf{F}}_{ext} \neq 0$, then total momentum of the system will change and conservation of momentum is not applicable.
4. If net external force on the system is zero in a particular direction only, then we can apply linear momentum conservation in that particular direction, though, the momenta in other directions may change. For example, two balls A and B of masses m_A and m_B respectively are colliding in air when they are moving horizontally. If we neglect air friction etc, then the only forces acting on the system is in vertical direction, and no net force acts on the system in horizontal direction, and hence momentum in horizontal direction will remain conserved.

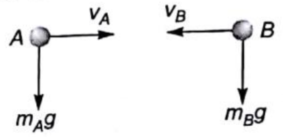


Fig. 8.3

5. Law of conservation of linear momentum is a general principle, which is applicable to all domains *ie*, for subatomic particles as well as for heavenly objects. It is an experimentally verified law, which can be applied to any system.

We can derive law of conservation of momentum by using Newton's laws but this is not correct as conservation of momentum is a general independent principle which is applicable in those situations also where Newton's law of motion are not applicable.

Examples of Momentum Conservation

There are numerous applications of law of conservation of momentum, like in rifle-bullet system, two blocks-spring system, earth-ball system, propulsion of rockets etc.

And above all the most important applications of conservation of momentum is in collision which we shall discuss later on in this chapter. Here we shall discuss one of the basic applications of conservation of momentum.

Rifle-Bullet System

Consider a rifle-bullet system with rifle mass M and bullet mass m. Initially the system is at rest. Now if we trigger the rifle, then the rifle fires the bullet, and the bullet moves speedily in forward direction.

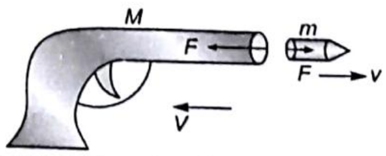


Fig. 8.4 Bullet-rifle system, for this system momentum remains conserved in horizontal direction.

If we consider bullet only as system then due to firing its momentum changes from zero to some non-zero value *ie*, its momentum is changing. Is it a violation of conservation of linear momentum ? If not, then which force is responsible to change the momentum of bullet ? Remember, the law of conservation of momenta is a general principle and holds good in all areas, here the bullet is experiencing a force due to spring/striker embedded in rifle and this external force to system (bullet) is changing the momentum of system. Now we consider rifle + bullet as our system. In this case, our system consists of two parts—rifle and bullet. Before firing, both the rifle as well as the bullet are at rest. Hence before firing momentum of bullet as well as momentum of rifle both equal to zero. Thus, Before firing,

$$p_{\text{system}} = p_{\text{bullet}} + p_{\text{rifle}} = 0 + 0 = 0$$
$$K_{\text{system}} = K_{\text{bullet}} + K_{\text{rifle}} = 0 + 0 = 0$$

As the rifle is triggered, the bullet speeds up in forward direction *ie*, the bullet acquires a non-zero momentum. In this case, the force exerted by rifle on bullet is internal to the system, and in horizontal direction no force is acting on the system, thus the momentum of the system remains conserved in horizontal direction. As due to firing the bullet acquires a momentum $p'_{\text{bullet}} = mv$ in forward direction, so the rifle has to move backward (recoil) to conserve the momentum and the force which is responsible for the recoiling of gun is the force exerted by bullet on gun. Let the rifle recoil with speed V, then the momentum of gun after firing is $p'_{\text{rifle}} = MV$ in backward direction.

From momentum conservation,

$$p_{\text{initial}} = p_{\text{final}}$$
$$0 = \vec{\mathbf{p}}_{\text{bullet}} + \vec{\mathbf{p}}_{\text{rifle}} = mv - MV$$

[Consider forward direction as positive, and the backward direction as negative]

$$\Rightarrow V \text{ (Recoiling speed of rifle)} = \frac{mv}{M}$$

After firing, the kinetic energy of system is,

$$K'_{\text{system}} = K'_{\text{bullet}} + K'_{\text{rifle}}$$
$$= \frac{mv^2}{2} + \frac{MV^2}{2} \neq 0$$

So it means that due to firing the kinetic energy of the system is increasing, this is at the expense of elastic potential energy stored in the spring and chemical energy of gun powder.

From above discussion, we hope that you understand, *why a soldier presses the rifle against his shoulder while firing ?*

Steps to Solve Problems Applying Momentum Conservation Law

Step 1 : Identify the system.

Step 2 : Check whether the net external force acting on the system is zero or not. Also see whether it is zero totally or it is zero only in a particular direction. If it is zero totally, then the total momentum of the system remains conserved and if it is zero only in a particular direction, then the momentum of system is conserved only in this particular direction.

Step 3 : According to the requirement of the question, decide two instants where we will equate the momentum of system.

Step 4 : Write down the momentum of all parts of system with respect to a given frame of reference at two instants chosen in the above step.

Step 5 : Equate the total momentum of the system at the two chosen instants, and get the required variables.

C-BIs

Concept Building Illustrations

Illustration | 5 *A rifle of mass 2 kg fires a bullet of mass 200 g with the speed of 200 ms^{-1} in forward direction. Determine the recoil velocity of rifle, assuming no external force acts on the system in horizontal direction.*

Solution Let the recoil speed of rifle be V, then from momentum conservation in horizontal direction,

$$0 = mv - MV$$

where m = mass of bullet

M = mass of rifle

v = velocity of bullet

V = velocity of rifle

$$\Rightarrow \quad 0 = 0.2 \times 200 - 2 \times V$$

$$\Rightarrow \quad V = 20 \text{ ms}^{-1}$$

Illustration | 6 *Two particles of masses m_A and m_B respectively are released from rest on a smooth horizontal surface when they are at a separation of r_1. They are allowed to move only under their mutual gravitational force. The speed of the particle of mass m_A at a particular instant is v_0. Determine the speed of other particle at this instant.*

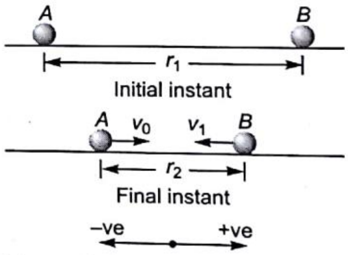
Solution Along horizontal direction, there is no external force acting on the system as the surface is smooth, so no friction force. Due to their mutual gravitational interaction they will approach each other as shown in figure. Let the required speed be v_1. Momentum of the system in horizontal direction remains conserved.

At initial instant,

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_A + \vec{\mathbf{p}}_B = 0$$

At the final instant,

$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_A + \vec{\mathbf{p}}_B = m_A v_0 - m_B v_1$$



[Taking rightward as +ve]

From conservation of momentum,

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$$

$$\Rightarrow \quad 0 = m_A v_0 - m_B v_1$$

$$\Rightarrow \quad v_1 = \frac{m_A}{m_B} \times v_0$$

Centre of Mass

You may have seen in your daily life that some people hold big and heavy objects (say notebooks, boxes etc) on the tip of their finger and the object remains balanced. If you would have the curiosity to know the reason of this happening, then this can be answered and explained with the help of the concept of centre of mass.

Every physical system has associated with it, a certain point having remarkable properties, this point is termed as the centre of mass. The motion of centre of mass of a system characterizes the motion of the system as a whole. When the system moves under some external forces, then this point moves as if the entire mass of the system is concentrated at this point and also the external force is applied at this point. In this book we are not going to disscuss the concept of centre of mass in detail.

Collisions

Collision is an event in which an impulsive force acts between two or more bodies (or particles) for a short time, which is responsible for their change in momenta. Or we can say, in a collision, bodies (or particles) exert a very large interaction force on each other for a very small duration. Example of collision are—hitting a cricket ball by bat, collision of two cars moving at high speeds, a ball rebounding after striking a wall, collision between two billiard balls, collision of striker with a coin in carrom, etc. In all these examples, the bodies are exerting a large force on each other, which lasts only for small duration. For illustration, consider the collision between two billiards balls.

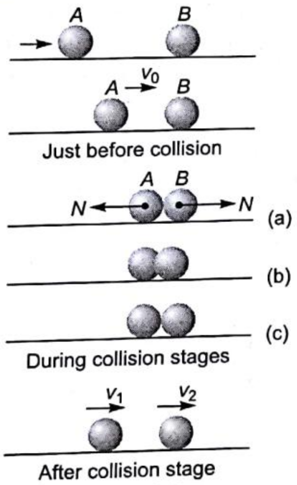


Fig. 8.5

Consider two billiard balls A and B as shown in figure, let us say ball A is striked by cue (a long bar) as a result it approaches B and collides with B. As the ball A comes in contact with ball B, a large contact force starts acting on both the balls which is along the normal to the contact surfaces. This contact force between the balls is varying with time and determination of its exact nature is next to impossible. The duration for which the balls remain in contact or we can say for which the contact force exists is termed as collision duration and this stage is termed as *during collision stage*. Due to the contact forces acting on balls their individual momenta change, and finally the balls get separated.

Thus, we can say that a collision event can be divided in three stages, in which the interaction force acts for during collision stage, and its duration is very small. These are

(a) Before collision
(b) During collision, and
(c) After collision

Now here we mention some of the important points related to collision :

1. The interaction force which comes into existence during the collision stage is impulsive in nature *ie*, of very large magnitude and hence other external forces (non-impulsive) can be neglected and hence won't be taken into consideration.

2. The interaction force between the bodies is acting along the line drawn normal to the surfaces in contact, and this line is termed as the **line of impact**. Now the question arises how to draw the line of impact, simply you draw a line along the normal to the surfaces of the bodies where they come into contact, this line itself is termed as the line of impact. The line of impact in various situations are as shown in the figure.

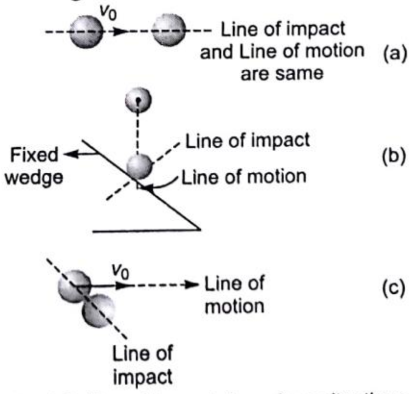


Fig. 8.6 Line of impacts in various situations

3. During collision transfer of momentum takes place from one body to another along the line of impact.
4. In the process of collision, the momentum of the system remains conserved, this is due to the fact that interaction force is very large and other non-impulsive external forces are negligible as compared to interaction force and moreover the interaction force is an internal force, so the net external force acting on system is zero and hence momentum of the system remains conserved in a collision process.

   Momentum of the system just before collision

   = momentum of the system just after collision
5. In a collision process, total energy of the system also remain conserved. Total energy includes energy of all forms.

Types of Collisions

Collisions are classified on the basis of direction of line of impact and line of motion and also on the basis of energy.

On the basis of line of impact : On this basis, the collisions are classified as

(a) Head on collision

(b) Oblique collision

Head on collision : When the velocities of all objects before and after the collision are along the same line (line of impact), then the collision is said to be head on collision. Example of this type is as we discussed earlier in the illustration of collision between two billiard balls. Figure 8.6 (a) is corresponding to head on collision.

Oblique collision : If one or more of the velocities of the colliding bodies before and after the collision are along different lines, then the collision is said to be an oblique collision. In other words, we can say if direction of motion of any object before or after collision is not along line of impact, then the collision is said to be an oblique collision. Fig. 8.6 (b) and (c) show oblique collision.

On the basis of energy : On the basis of relation between kinetic energy before and after the collision, the collisions are classified into three categories :

(a) Perfectly elastic collision or elastic collision

(b) Perfectly inelastic collision

(c) Inelastic collision.

Whenever we exert some force on a body, then the body gets deformed and depending upon the nature of body the extent of deformation depends (you can deform sponge more and the steel ball less). Actually what happens when we apply some force on the body? The applied force does some work on the body, and this work done is used in deforming the body, and is stored as the deformation energy in the body. When we remove the applied force, the body tries to acquire its original shape and size and the extent of restoration again depends upon the nature of the material of body. If the body acquires its original shape and size completely on the removal of force, then the body is said to be completely elastic, while if the body has no tendency to acquire its original shape and size, then the body is said to be perfectly inelastic. All other bodies come into the category of inelastic bodies *ie*, bodies which acquire its original shape and size partially are termed as inelastic bodies. Actually, no real body is perfectly elastic or perfectly inelastic.

Perfectly Elastic Collision : In an elastic collision, the colliding bodies regain their shape and size completely after collisions. In this type of collision as the two bodies collide, the part of initial kinetic energy of the two bodies converted into deformation energy during collision and as they separate out, total deformation energy is again converted back into KE. Thus, we can say that in an elastic collision the kinetic energy of the system before collision is equal to the kinetic energy of the system after collision.

Inelastic Collision : In an inelastic collision, the bodies don't regain their original shapes and sizes completely and some deformation would be left in the bodies. In this

type of collision the kinetic energy before collision is not equal to the kinetic energy after collision.

Perfectly Inelastic Collision: A collision is said to be perfectly inelastic if both the particles are having some velocity along the line of impact after the collision. In simple words, for head on-perfectly inelastic collision, the two particles stick together after collision.

Here, we will limit our discussion only to the head on collisions.

Head on Elastic Collision

Let us consider two balls of masses m_1 and m_2, moving with velocity u_1 and u_2 respectively as shown in the figure. For collision to take place $u_1 > u_2$ and let us say after collision their velocities becomes v_1 and v_2 respectively.

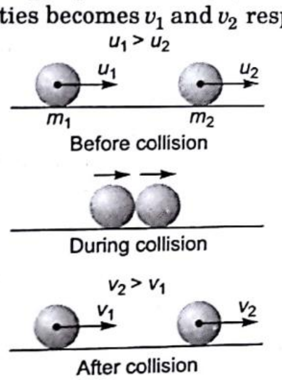


Fig. 8.7 Head on elastic collision

From momentum conservation along line of impact (line of motion and line of impact are same here)

$$\vec{\mathbf{p}}_{\text{Before coll.}} = \vec{\mathbf{p}}_{\text{After coll.}}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{...(i)}$$

For elastic collision $KE_{BC} = KE_{AC}$

$$\frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2} = \frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2} \quad \text{...(ii)}$$

Rewriting the Eqs. (i) and (ii), we get

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{...(iii)}$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$\Rightarrow \quad m_1(u_1 - v_1)(u_1 + v_1)$$
$$= m_2(v_2 - u_2)(v_2 + u_2) \quad \text{...(iv)}$$

Divide Eq. (iv) by Eq. (iii), we get

$$u_1 + v_1 = v_2 + u_2$$

$$\Rightarrow \quad u_1 - u_2 = v_2 - v_1 \quad \text{...(v)}$$

The left hand term of above equation represents the velocity with which two balls approach before collision *ie*, if one ball is made stationary, then with what velocity the other ball is approaching it, while the right side term of same equation represents the velocity with which two balls are getting separated after collision *ie*, if one ball is made stationary then with what velocity the other ball is getting separated from it. Now we are defining a term, **coefficient of restitution** (e) which is very important to solve the questions of collision.

Coefficient of restitution is defined as the ratio of velocity of separation along line of impact after collision to velocity of approach along line of impact before collision.

$$e = \frac{\text{Velocity of separation after collision along line of impact}}{\text{Velocity of approach before collision along line of impact}}$$

For elastic collision *ie*, in present case,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

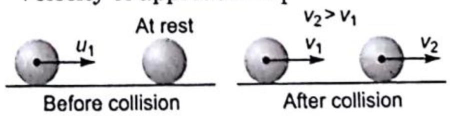
From Eq. (v), $v_2 - v_1 = u_1 - u_2$ so, $e = 1$ *ie*, for elastic collision, the value of coefficient of restitution is 1.

For perfectly inelastic collision $e = 0$ and for inelastic collision value of e lies between 0 and 1. Coefficient of restitution is defined along line of impact only, as we are discussing only head on collisions so there won't be any problem in writing equation for e. In your higher classes you will learn how to write equations for coefficient of restitution for oblique collisions.

Now before coming back to the discussion of elastic collision, here we are giving you various cases for which we shall write velocities of separation and velocity of approach.

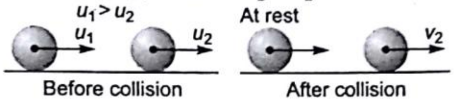
1. Velocity of separation $= v_2 - v_1$

   Velocity of approach $= u_1$



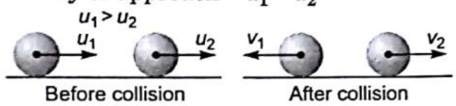
(a)

2. Velocity of separation $= v_2$
   Velocity of approach $= u_1 - u_2$



(b)

3. Velocity of separation $= v_2 + v_1$
   Velocity of approach $= u_1 - u_2$



(c)

Fig. 8.8

Now again we are coming back to our discussion of elastic collision, from Eq. (v), we have

$$u_1 - u_2 = v_2 - v_1$$

Multiply above equation by m_2 and subtract from Eq. (i)

$$m_2u_1 - m_2u_2 = m_2v_2 - m_2v_1 \quad \text{...(vi)}$$

$$m_1u_1 + m_2u_2 = m_2v_2 + m_1v_1$$

...[Eq. (i) rewritten]

Subtracting Eq. (vi) from Eq. (i), we get

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad \text{...(a)}$$

Substituting the value of v_1 in Eq. (v), we get

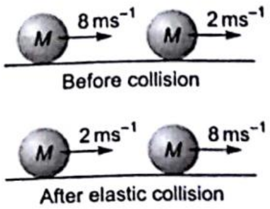
$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \quad \text{...(b)}$$

The Eqs. (a) and (b) provide us the velocity of two balls after collision.

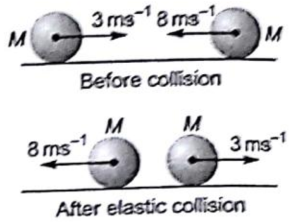
Now let's discuss some special cases :

1. If $m_1 = m_2$, then $v_1 = u_2$,
   and $v_2 = u_1$
   ie, if two identical balls collide elastically then after collision their velocities get interchanged.

Example 1



Example 2



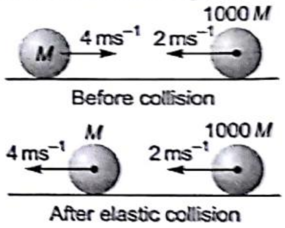
2. If $m_2 >> m_1$ *ie*, mass of 2nd body is very large as compared to 1st body, then

$$v_1 = -u_1 + 0$$

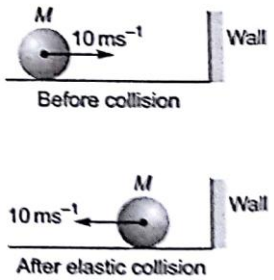
$$v_2 = 0 + u_2$$

ie, velocity of lighter body gets reversed in direction without change in magnitude while velocity of the heavier body remains unaffected. In other words, we can say when a ball collides elastically with a very massive object, then the ball rebounds with the same speed while velocity of the massive object remains unchanged.

Example 1



Example 2



Head on Inelastic Collision

In an inelastic collision the kinetic energy of the system before collision is not equal to kinetic energy of system after collision and the value of coefficient of restitution lies between 0 and 1. In this type of collision a part of kinetic energy appears as deformation in the colliding bodies.

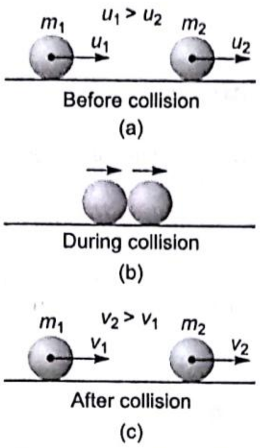


Fig. 8.9 Inelastic collision

Just like for elastic collision, consider two balls which are going to collide inelastically with coefficient of restitution e. The situation is clearly shown in the Fig. 8.9.

From momentum conservation,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \ldots\text{(i)}$$

From coefficient of restitution equation,

$$e = \frac{v_2 - v_1}{u_1 - u_2} \quad \ldots\text{(ii)}$$

Rewriting the Eq. (ii), we get

$$v_2 - v_1 = e(u_1 - u_2)$$

Multiply Eq. (ii) by m_1 and then add it with Eq. (i), we get

$$v_2 = \left(\frac{m_1 + em_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 \quad \ldots\text{(a)}$$

Substituting this value of v_2 in Eq. (ii), then we get

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 + em_2}{m_1 + m_2}\right)u_2 \quad \ldots\text{(b)}$$

From Eqs. (a) and (b), we can get the velocity of two balls after collision. Here for inelastic collision, $\text{KE}_{BC} \neq \text{KE}_{AC}$

$$\text{KE}_{BC} = \frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2}$$

$$\text{KE}_{AC} = \frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2}$$

Head on Perfectly Inelastic Collision

In a perfectly inelastic head on collision the two bodies stick together* and move with same velocity after collision. For perfectly inelastic collision the value of e is zero as velocity of separation after collision is zero. [$\because$ both the bodies are moving with same velocity]

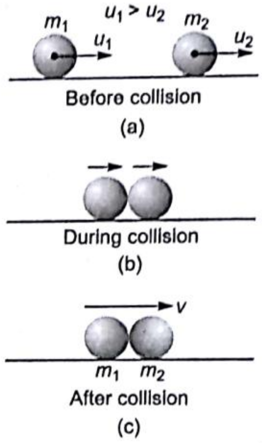


Fig. 8.10 Perfectly head on inelastic collision

Let us consider two balls of masses m_1 and m_2 moving with velocities u_1 and u_2, which undergoes a perfectly inelastic collision as shown in figure. Let the common velocity of two balls after collision is v.

From momentum conservation

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$\Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

The above equation gives the common velocity of two balls after collision.

Here $\quad \text{KE}_{BC} \neq \text{KE}_{AC}$

$$\text{KE}_{BC} = \frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2}$$

$$\text{KE}_{AC} = \frac{(m_1 + m_2)v^2}{2}$$

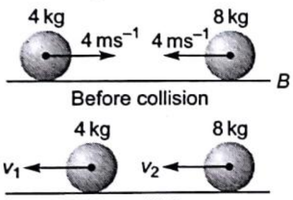
* **Stick together doesn't mean that something like glue made them stick, the thing is that both the balls acquire some velocity after collision and hence, are moving together which seems to be that two bodies are actually stuck.**

C-BIs

Concept Building Illustrations

Illustration | 7 *The two balls of masses 4 kg and 8 kg are moving towards each other each with a speed of 4 ms^{-1} and undergo an elastic collision. Determine the velocity of two balls after collision.*

Solution The situation is shown in the figure. Consider the rightward direction as positive.



From momentum conservation,

$$4 \times 4 - 8 \times 4 = -4v_1 - 8v_2$$

$$\Rightarrow \quad v_1 + 2v_2 = 4 \qquad \text{...(i)}$$

From coefficient of restitution equation,

$$e = 1 = \frac{v_1 - v_2}{4 + 4}$$

$$\Rightarrow \quad v_1 - v_2 = 8 \qquad \text{...(ii)}$$

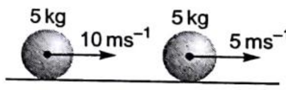
Solving above two equations, we get

$$v_1 = \frac{20}{3}\ \text{ms}^{-1},$$

and $$v_2 = -\frac{4}{3}\ \text{ms}^{-1}$$

Negative value of v_2 says that our assumed direction of v_2 is wrong and the ball of 8 kg mass is moving towards right with $\frac{4}{3}$ ms^{-1} after collision.

Illustration | 8 *Two balls each of mass 5 kg are approaching each other as shown in the figure. These two balls undergo a perfectly inelastic collision. Determine the common velocity of two balls after collision and also find out the loss in kinetic energy due to collision.*



Solution Let v be the common speed of two balls just after collision, then from momentum conservation

$$5 \times 10 + 5 \times 5 = (5 + 5)v$$

$$\Rightarrow \quad v = 7.5\ \text{ms}^{-1}$$

Final kinetic energy *ie*, kinetic energy after collision

$$= \frac{(5+5)(7.5)^2}{2} = 281.25\ \text{J}$$

Initial kinetic energy *ie*, kinetic energy before collision

$$= \frac{5 \times 10^2}{2} + \frac{5 \times 5^2}{2} = 312.5\ \text{J}$$

Loss in energy,

$$\Delta K = K_f - K_i$$

$$= (281.25 - 312.5)\ \text{J}$$

$$= -31.25\ \text{J}$$

ie, 31.25 J of kinetic energy is lost is collision which appears in the form of other energies.

Proficiency in Concepts (PIC)

Problems

Problem | 1 *Determine the momentum of a person of mass 50 kg when he is moving with a speed of 10 ms^{-1}.*

Solution Momentum, $p = mv$

$$\Rightarrow \quad p = 50 \times 10 \text{ kg-ms}^{-1} = 500 \text{ kg-ms}^{-1}$$

Problem | 2 *An object of mass 3 kg is moving with a velocity of $(2\hat{i} + 8\hat{j})\ ms^{-1}$. Determine the momentum of object.*

Solution Momentum $\vec{p} = m\vec{v}$

$$\vec{p} = 3 \times (2\hat{i} + 8\hat{j}) \text{ kg-ms}^{-1}$$
$$= (6\hat{i} + 24\hat{j}) \text{ kg-ms}^{-1}$$

Problem | 3 *A ball of mass 5 kg is changing its velocity from $2\hat{i}\ ms^{-1}$ to $(5\hat{i} + 6\hat{j})\ ms^{-1}$ due to application of some force on it. Determine the change in momentum of the ball.*

Solution Initial momentum,

$$\vec{p}_i = m\vec{v}_i$$
$$= 5(2\hat{i}) \text{ N-s}$$
$$= 10\hat{i} \text{ N-s}$$

Final momentum,

$$\vec{p}_f = m\vec{v}_f$$
$$= 5(5\hat{i} + 6\hat{j}) \text{ N-s}$$
$$= (25\hat{i} + 30\hat{j}) \text{ N-s}$$

Change in momentum,

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$
$$= (25\hat{i} + 30\hat{j}) - 10\hat{i} \text{ N-s}$$
$$= (15\hat{i} + 30\hat{j}) \text{ N-s}$$

Problem | 4 *In previous example, determine the magnitude of impulse which is responsible for change in its momentum.*

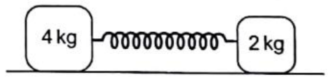
Solution We know,

Impulse = Change in momentum

ie, $\quad \vec{J} = \Delta\vec{p}$

$$= (15\hat{i} + 30\hat{j}) \text{ N-s}$$
$$|\vec{J}| = \sqrt{(15)^2 + (30)^2}$$
$$= 15\sqrt{5} \text{ N-s}$$

Problem | 5 *Two blocks of masses 4 kg and 2 kg are connected with the help of a spring as shown in figure. Initially the spring is stretched by some amount and the blocks are released from rest. After sometime the 4 kg block is moving towards right with a speed of 2 ms^{-1}. At this instant determine the velocity of 2 kg block. Neglect friction everywhere.*



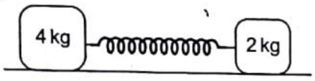
Solution As the surfaces are smooth, so no friction force is there on the blocks. In horizontal direction no external force is acting on the blocks-spring system, and hence momentum of the system remains conserved in horizontal direction. Initially the system is released from rest so initial momentum of the system is zero. Consider left direction as negative and the one as right. Let at the given instant the velocity of 2 kg block is v towards right. Then from momentum conservation,

$$0 = 4 \times 2 + 2 \times v$$
$$\Rightarrow \quad v = -4 \text{ ms}^{-1}$$

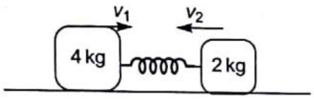
Thus, the 2 kg block is moving with 4 ms^{-1} towards left at given instant.

Problem | 6 *In previous question if initially the spring is stretched by 20 cm and spring constant is $k = 1000\ Nm^{-1}$, then determine the velocity of two blocks when the spring acquires its natural length.*

Solution The situation is shown clearly in the figure.



Initial position



Final specified position when spring is in natural length

From the concept described in last example,

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$$

$$\Rightarrow \quad 0 = 4v_1 - 2v_2$$

$$\Rightarrow \quad v_2 = 2v_1 \quad \text{...(i)}$$

From energy conservation

$$K_f - K_i = -(U_f - U_i)$$

$$\left(\frac{4 \times v_1^2}{2} + \frac{2 \times v_2^2}{2}\right) - 0 = -\left[0 - \frac{kx_0^2}{2}\right]$$

where $x_0 = 0.2$ m.

$$\Rightarrow \quad 2v_1^2 + v_2^2 = 20 \quad \text{...(ii)}$$

Substitute value of v_2 from Eq. (i) in Eq. (ii), we get

$$2v_1^2 + (2v_1)^2 = 20$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{10}{3}} = 1.83\ \text{ms}^{-1}$$

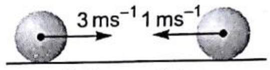
So, $\quad v_2 = 2v_1 = 3.66\ \text{ms}^{-1}$

Problem 7. *A ball moving with 3 ms^{-1} collides elastically with a wall (fixed). Determine the velocity of ball after collision.*

Solution In elastic collision if a lighter body collides with a massive body then the velocity of lighter body reverses its direction without any change in its magnitude. The velocity of ball after collision in above case is 3 ms^{-1} away from the wall.

Problem 8. *The diagram below shows the velocities of two balls before and after collision. Determine the coefficient of restitution for this collision and state which type of collision it is.*

Solution From before collision diagram, velocity of approach $= 3 + 1 = 4\ \text{ms}^{-1}$



Before collision

After collision

From after collision diagram, the velocity of separation

$$= 2 + 1 = 3\ \text{ms}^{-1}$$

From definition of coefficient of restitution,

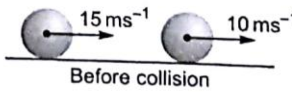
$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$$e = \frac{3}{4}$$

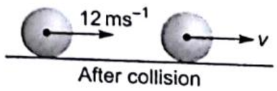
As value of e is lying between 0 and 1, so it is an inelastic collision.

Problem 9. *A ball of mass 10 kg moving with a velocity of 15 ms^{-1} collides head on with another ball of mass 10 kg moving with a speed of 10 ms^{-1} in the same direction. If after the collision, the velocity of first ball is 12 ms^{-1} in the same direction as of its initial motion, then determine the velocity of second ball after collision and also determine the coefficient of restitution.*

Solution The situation is shown clearly in the figure.



Before collision



After collision

Let the required velocity be v, using momentum conservation

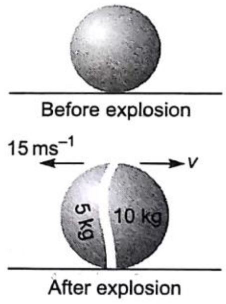
$$10 \times 15 + 10 \times 10 = 10 \times 12 + 10 \times v$$

$$\Rightarrow v = \frac{250 - 120}{10} \text{ ms}^{-1} = 13 \text{ ms}^{-1}$$

$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}} = \frac{13 - 12}{15 - 10} = \frac{1}{5}$$

Problem 10. *An explosive body of mass 15 kg is kept at rest on a smooth horizontal surface. Suddenly it explodes into two pieces of masses 5 kg and 10 kg due to some internal forces, as a result the 5 kg piece starts moving back with a speed of 15 ms^{-1}, then determine the velocity of 10 kg piece.*

Solution As no external force was acting on the system and the explosion takes place because of internal forces, the momentum of the system remains conserved.



Before explosion the system is at rest and hence its momentum is zero. After explosion, let the 10 kg piece moves towards right with speed v, then $p_f = 10v - 5 \times 15$ [Right side direction is considered as positive]

$$10v - 5 \times 15 = 0$$

[From momentum conservation]

$$\Rightarrow \quad v = 7.5 \text{ ms}^{-1}$$

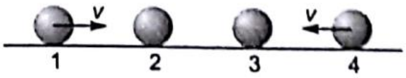
Towards Proficiency Problems

Exercise 1

A. Subjective

Discussions

1. Two identical cars have the same speed, one travelling east and the other travelling west. Do these cars have the same momentum ? Explain.
2. Many identical particles are moving randomly in a confined space. What is the total linear momentum of the system consisting of all particles at any instant ? What is the momentum of individual particles at the same instant ? Is it zero or non-zero ? Explain your reasoning.
3. Two objects of different masses have the same momentum. Do the velocities of these objects necessarily have the (a) same direction, (b) same magnitudes ?
4. Can a single object have kinetic energy but no momentum ? Can a system of two or more particles have kinetic energy but no momentum ? Explain your answer.
5. The momentum of a ball changes from $\vec{\mathbf{p}}_1 = (3\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}} - 2\,\hat{\mathbf{k}})$ N-s to $\vec{\mathbf{p}}_2 = (3\,\hat{\mathbf{i}} + 8\,\hat{\mathbf{j}} - 2\,\hat{\mathbf{k}})$ N-s. In which direction the force is acting on ball ? In which direction the impulse is acting on the ball ?
6. What do you mean by an impulsive force ? Can gravity force be considered as impulsive force ? Explain your answer.
7. Which one you will prefer to catch—a ball having mass of 5 kg and moving with a speed of 2 ms^{-1} or a ball having mass of 2.5 kg and moving with a speed of 4 ms^{-1} ? Or are both equally comfortable to catch ?
8. An object is sliding on a rough horizontal surface of a fixed table, after some time it stops and hence the momentum of block changes. Is it a violation of conservation of energy ? If not explain your answer. Discuss the case when you consider table-plus-object as your system.
9. Four identical balls 1, 2, 3 and 4 are placed on a smooth horizontal surface as shown in figure. The balls 1 and 4 are given velocities as shown in figure. Assuming all collisions to be elastic, determine the velocities of all 4 balls after all the collisions have taken place ? Justify your answer with proper reasoning.



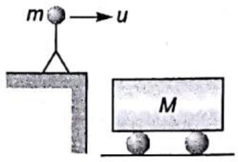
10. Without knowing the exact nature of the force between two bodies while colliding, we can determine the motion of two bodies after collision. Comment on this statement.
11. When a stationary rifle fires a bullet, the KE of rifle + bullet system increases to conserve the momentum. From where this energy is coming from ? Or in this situation does the energy conservation law don't hold good.

B. Numerical Answer Types

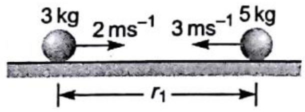
1. A particle of mass 2 kg is moving with a velocity of 2 ms^{-1} along east. Determine its kinetic energy and momentum.
2. A ball of mass 5 kg starts moving from rest with a constant acceleration of 3 ms^{-2}. Determine the magnitude of momentum of the ball at $t = 3$ s.
3. If in above question the initial velocity of ball is 9 ms^{-1} in a direction opposite to acceleration, then determine the magnitude and direction of momentum of the ball at $t = 2$ s, $t = 3$ s, $t = 5$ s.
4. A ball of mass 1 kg is having a momentum of $(2\hat{i} + 3\hat{j} - 6\hat{k})$ kg-ms^{-1}. Determine the velocity of ball at this instant.
5. A ball of mass 5 kg is having a kinetic energy of 50 J. Determine the magnitude and direction of momentum of ball.
6. A ball is having a momentum of $(3\hat{i} + 5\hat{j} + \sqrt{15}\hat{k})$ N-s, and is having a mass of $\frac{7}{4}$ kg. Determine the kinetic energy of ball.
7. A ball of mass 1.4 kg has an initial velocity of 38 ms^{-1} as it approaches a bat. On being hit by the bat the ball rebounds back will a velocity of 58 ms^{-1}. Assume that the bat exerts a force on ball which is much larger than the weight of ball.
   (a) Determine the magnitude and direction of impulse exerted on ball by the bat and by the ball on bat.
   (b) If the contact between ball and bat is lost for 0.02 s, then determine the magnitude of average force exerted by bat on ball.
8. Two men push a stationary car (on smooth horizontal surface) with a force of 720 N for 8 s. Determine the magnitude of momentum of car at $t = 8$ s.
9. Three particles of masses 1 kg, 2 kg and 3 kg are moving with velocities $\vec{v}_1 = (3\hat{i} + 2\hat{j})\ ms^{-1}$, $\vec{v}_2 = (6\hat{i})\ ms^{-1}$ and $\vec{v}_3 = (4\hat{k})\ ms^{-1}$ respectively. Determine the momentum of this three-particle system.
10. A person of mass 50 kg is standing on a fixed ice platform. He pushes a fixed structure mounted on ice, as a result he moves back with a speed of 2 ms^{-1}. Determine the magnitude and direction of impulse exerted on wall (fixed structure) by the person. If the person remains in contact with the wall for 0.05 s, then determine the average force exerted on the man by wall.
11. A 0.5 kg ball is dropped from rest from a height of 5 m above the floor. The ball rebounds straight upwards and reaches to a height of 1.25 m above the floor. Determine the magnitude and direction of impulse of the net force applied to the ball during the collision with the floor.
12. A ball of mass 0.5 kg in moving with a velocity of 2 ms^{-1} along the positive x-axis. A force is applied on the ball as a result its velocity changes to 4 ms^{-1} along positive y-axis. Determine the direction of the force acting on the ball.
13. A person of mass 50 kg holds a ball of mass 5 kg, and is standing on a smooth horizontal surface. He throws the ball in forward direction with a speed of 10 ms^{-1}. Determine the velocity with which the person moves backward. Also determine the impulse of the net force acting on the person due to throwing of ball.
14. Two friends Ram and Rahim have a combined mass of 160 kg. They are standing on a smooth horizontal surface holding each other. Now they push each other, as a result, Ram moves backwards with a speed of 2 ms^{-1} while Rahim moves in opposite direction with a speed of 3 m^{-1}. Determine the masses of both Ram and Rahim.

15. A person of mass 50 kg jumps from a bridge on a boat of mass 100 kg which is initially moving with a speed of 5 ms^{-1}. The boat is vertically below the bridge when the person lands on it. Determine the final velocity of boat.

16. A boy of mass m kg jumps with a speed of u ms^{-1} on a cart of mass M kg which is kept stationary as shown in the figure. Determine the velocity of cart + boy, when boy lands in cart. Neglect friction between cart and ground.

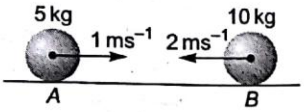


17. Two particles of masses 3 kg and 5 kg are initially at a separation of r_1 metre and moving with velocity of 2 ms^{-1} and 3 ms^{-1} as shown in the figure. They are approaching each other due to their mutual gravitational force. After some time when the separation between them reduces to r_2 the velocity of the 3 kg particle is 3 ms^{-1}. Determine the velocity of 5 kg particle at this instant. Neglect friction.



18. A shell is fired from a cannon with a speed of 100 ms^{-1} at an angle of 60° with the horizontal (positive x-direction). At the highest point of its trajectory, the shell explodes into two equal fragments. One of the pieces moves along negative x-direction with a speed of 50 ms^{-1}. Determine the speed of other piece at the time of explosion.

19. Two balls A and B of masses 5 kg and 10 kg respectively are moving on a smooth horizontal surface as shown in the figure. They undergo an elastic collision. Determine the velocities of two balls after the collision.



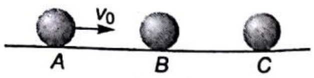
20. In the above question if collision is inelastic with $e = 2/5$, then repeat the above question and also determine the loss in kinetic energy due to collision.

21. Two identical balls A and B are approaching each other with velocity 3 ms^{-1} and 7 ms^{-1} respectively and undergo an elastic collision. Determine the velocities of two balls after the collision.

22. In previous question if collision is inelastic with $e = 2/5$, then repeat the above questions.

23. A 5 kg ball moving with a velocity of 2 ms^{-1} collides head on with a stationary ball of mass 2.5 kg. The balls undergo a completely inelastic collision. Determine the final velocities of two balls after the collision.

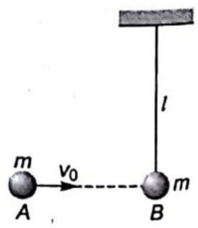
24. Three identical balls A, B and C are placed on a smooth table as shown in figure. Now, ball A is imparted a velocity v_0 as shown. All the collisions are elastic in nature. Determine the final velocities of all the balls.



25. A ball B of mass m is suspended freely from a string of length l as shown in figure. Another identical ball A moving horizontally with speed $v_0 (< \sqrt{2gl})$ strikes B elastically. Determine

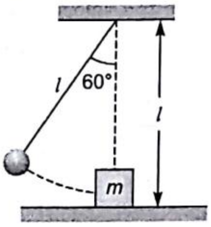
(a) the speed of two balls just after collision.

(b) the maximum height with respect to initial reference level attained by B.



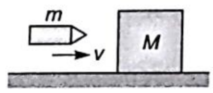
26. On a frictionless surface, a ball of mass m moving at a speed v makes an head on inelastic collision with an identical ball at rest. The kinetic energy of the balls after collision is 3/4th of the total kinetic energy before collision. Determine the coefficient of restitution.

27. A ball of mass m attached to a string of length l is released from rest when the string makes an angle of 60° with the horizontal as shown in the figure. A block of mass m is kept at rest on a rough horizontal surface. The coefficient of friction between the block and surface is μ. The collision between ball and block is an inelastic one having coefficient of restitution e.

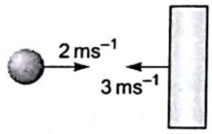


Determine
(a) the velocity of block just after the collision.
(b) the rebound velocity of the ball and the angle which the string makes with the vertical when the ball comes to rest instantaneously for the first time after collision.
(c) the distance moved by the block on surface before it comes to rest.

28. A bullet of mass m moving horizontally with speed v, gets embedded into a stationary block of mass M, kept on a rough horizontal surface. Determine the velocity of block + pulley system just after the bullet embeds into the block. If coefficient of friction between the block and surface is μ, then determine the distance travelled by the block before it stops.



29. A ball is moving with velocity 2 ms^{-1} towards a very heavy object moving towards the ball with a speed of 3 ms^{-1} as shown in figure. The ball collides elastically with the heavy object. Determine the velocity of ball immediately after the collision.



30. In above question if collision is inelastic having value of e as 4/5, then repeat the above questions.

31. Three guns are aimed at the centre of a circle. They are mounted on the circle 120° apart. They fire in a timed sequence such that the three bullets collide simultaneously at the centre and combine to form a stationary lump of three bullets. Two of the bullets have masses of 4.5 g each, and are moving with speeds v_1 and v_2 while the third bullet is having a mass of 2.5 g and moving with a speed of 575 ms^{-1}. Determine the values of v_1 and v_2.

Analyse the situation if all the three bullets are having same mass ? Same momentum ?

32. A 50 kg skater is travelling due east at a speed of 3 ms^{-1}. Another 70 kg skater is moving due south at a speed of 7 ms^{-1}. They collide and hold on to each other after the collision. Determine the magnitude and direction of their combined velocity after collision.

33. A ball is released from rest from the top of a 10 m tall building. The ball falls straight downward, strikes the floor and rebounds to a height of 8 m after 1st collision. Again it falls and strikes the floor and rebounds. Determine the coefficient of restitution for the collision. After 5th collision, what is the height to which the ball raises ?

34. Two balls of masses 2 kg and 4 kg are moving towards each other with velocities 4 ms^{-1} and 2 ms^{-1} respectively on a frictonless surface. After collision the 2 kg ball returns back with a velocity of 2 ms^{-1}. Determine
(a) the velocity of 4 kg block after collision.
(b) the loss in kinetic energy in collision.
(c) coefficient of restitution.

C. Fill in the Blanks

1. Quantity of motion of a moving object is used to represent its ....
2. Direction of impulse acting on a body is same as that of ....
3. In an ...... collision, the kinetic energy before collision is equal to kinetic energy after collision.
4. On the basis of direction of motion of objects and line of impact the collisions are classified as ...... and ...... collisions.
5. In a collision, the momentum remains ....
6. If two identical objects undergo elastic collision, then the velocities of the objects get ...... after collision.
7. In a perfectly head on inelastic collision the two bodies ...... to each other after collision.
8. Value of e is always lying between ....

D. True/False

1. A particle can have non-zero kinetic energy and zero momentum.
2. A system of two particles can have non-zero kinetic energy and zero momentum.
3. If $F_{ext} = 0$, then the momentum of individual parts of the system must remain constant.
4. If $F_{ext} = 0$, then for a system, the momentum of different parts of the system changes in such a way that total momentum of the system remains constant.
5. The conservation of linear momentum can be applied in one direction while at the same time it can't be applied in other directions.
6. Law of conservation of linear momentum is a consequence of Newton's third law.
7. The collision duration is very small as compared to other time intervals for which observation is made.
8. The interaction force (in collision) acts only for a small time-interval.
9. For a perfectly inelastic collision $e = 0$.
10. For a perfectly elastic collision $e = 0$.

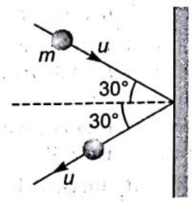
High Skill Questions

Exercise 2

A. Only One Option Correct

1. A particle of mass 5 kg is moving with a velocity of $10\hat{i}$ ms^{-1}. The momentum of this particle from a frame of reference which is moving with a velocity is $2\hat{i}$ ms^{-1}, is
(a) $50\hat{i}$ N-s (b) $40\hat{i}$ N-s
(c) $45\hat{i}$ N-s (d) None of these

2. A billiard ball collides elastically with a rigid wall as shown in the figure. The direction of the force exerted by the ball on the wall is



(a) along the normal to the wall
(b) inclined at 30° to the normal to wall
(c) inclined at 60° to the normal to wall
(d) inclined at 45° to the normal to wall

3. In above question the magnitude of the impulse imparted to the ball is
(a) mu (b) $2\,mu$
(c) $\sqrt{3}\,mu$ (d) $\frac{mu}{2}$

4. A steel ball is dropped on a hard surface from a height of 1 m and rebounds to a height of 0.64 m. The maximum height attained by the ball after n^{th} bounce is
(a) 1 m (b) $(0.8)^{2n}$
(c) $(0.5)^{2n}$ (d) $(0.8)^n$

5. A 2 kg ball moving straight down strikes the floor at 8 ms^{-1}. It rebounds upwards at 6 ms^{-1}. The change in the magnitude of momentum is
(a) 28 kg-ms^{-1} (b) 4 kg-ms^{-1}
(c) 2 kg-ms^{-1} (d) 14 kg-ms^{-1}

6. In above question the magnitude of change in momentum is
(a) 28 kg-ms^{-1} (b) 4 kg-ms^{-1}
(c) 2 kg-ms^{-1} (d) 14 kg-ms^{-1}

7. If the kinetic energy of a body becomes four times its original value, then the new momentum will be more than its initial momentum by
(a) 50% (b) 100%
(c) 125% (d) 150%

8. A man of mass m climbs up on a rope of length L suspended below a balloon of mass M. The balloon is stationary with respect to ground. If the man begins to climb up the rope at a speed v_{rel} (relative to rope). In what direction and with what speed (relative to ground) will the balloon move ?
(a) Downwards, $\frac{mv_{rel}}{m+M}$
(b) Upwards, $\frac{Mv_{rel}}{M+m}$
(c) Downwards, $\frac{mv_{rel}}{M}$
(d) Upwards, $\frac{(M+m)v_{rel}}{M}$

9. A shell is fired from a rifle with a velocity V at an angle θ to the horizontal direction. At the highest points on its path, it explodes into two pieces of equal masses. One of the pieces retraces its path to the rifle. The speed of the other piece immediately after the explosion is
(a) $3V\cos\theta$ (b) $2V\cos\theta$
(c) $\frac{3}{2}V\cos\theta$ (d) $V\cos\theta$

10. A ball of mass 50 g is dropped from a height $h = 10$ m. It rebounds losing 75% of its KE. If it remains in contact with the ground for $\Delta t = 0.01$ s, the impulse of the impact force is

(a) 1.3 N-s (b) 1.05 N-s
(c) 1300 N-s (d) 105 N-s

11. A man, standing at rest on a horizontal frictionless floor, might get himself moving by
(a) walking
(b) rolling
(c) crawling slowly
(d) throwing a shoe horizontally

12. A rifle of mass M is initially at rest but free to recoil. It fires a bullet of mass m and velocity v (relative to ground). After firing, the velocity of the rifle with respect to ground is
(a) $-mv$ (b) $-\frac{Mv}{m}$
(c) $-\frac{mv}{M}$ (d) $-v$

13. The physical quantity impulse has the same dimensions as that of
(a) force (b) power
(c) energy (d) momentum

14. A sphere X of mass 2 kg is moving to the right at 10 ms^{-1}. Sphere Y of mass 4 kg is moving to the left at 10 ms^{-1}. The two spheres collide head-on. The ratio of the magnitude of the impulse exerted by X on Y to that exerted by Y on X is
(a) 1/4 (b) 1/2
(c) 1 (d) 2/1

15. Two bodies of unequal masses, placed at rest on a frictionless surface, are acted upon by equal horizontal forces for equal times. Just after these forces are removed, the body of greater mass will have
(a) greater speed
(b) the greater acceleration
(c) the greater momentum
(d) the same momentum as the other body

16. A completely head on inelastic collision between two objects is a collision in which
(a) the momenta of the objects are the same after the collision
(b) the momenta of each object is same after the collision as it was before
(c) the momenta of each object reverses its direction
(d) the velocities of the objects are same after the collision

17. A very massive object travelling at 10 ms^{-1} strikes a very light object, initially at rest and the light object moves off in the direction of travel of the heavy object. If the collision is elastic, the speed of the lighter object is
(a) 5 ms^{-1} (b) 10 ms^{-1}
(c) 15 ms^{-1} (d) 20 ms^{-1}

18. Blocks A and B are moving towards each other along the x-axis. A has a mass of 2 kg and a velocity of $50\hat{i}$ ms^{-1}, while B has a mass of 4 kg and a velocity of $-25\hat{i}$ ms^{-1}. They suffer an elastic collision and move off along the x-axis. After the collision the velocities of both A and B, respectively are
(a) $-50\hat{i}$ ms^{-1} and $25\hat{i}$ ms^{-1}
(b) $50\hat{i}$ ms^{-1} and $-25\hat{i}$ ms^{-1}
(c) $-25\hat{i}$ ms^{-1} and $50\hat{i}$ ms^{-1}
(d) $25\hat{i}$ ms^{-1} and $-50\hat{i}$ ms^{-1}

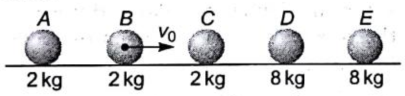
19. A man sits at the back of a long boat in still water. He then moves to the front of the boat and sits there. Then [*Neglect friction*]
(a) the boat first moves forward and then comes to rest
(b) the boat first moves backward and then comes to rest
(c) the boat moves forward continuously
(d) the boat moves backward continuously

20. If the total momentum of a system is changing, then
(a) the particles of system must exert force on each other
(b) the system must be under the influence of gravity
(c) a net external force must act on the system
(d) None of the above

B. More Than One Options Correct

1. Mark the correct option(s) related to impulse.
(a) It is a vector quantity.
(b) It is equal to change in momentum vector.
(c) Its unit is same as that of momentum.
(d) If time interval for which force is acting, decreases keeping force the same, then the impulse decreases.

2. Five balls are placed one after the other along a straight line as shown in figure. Initially all the balls are at rest, now the ball B has been imparted a velocity v_0 towards right as shown.



Assume all collisions to be head on and elastic. For this situation, mark out the correct statement(s).
(a) Total number of collisions in the process is 5
(b) Velocity of separation between the balls A and B after last possible collision is v_0
(c) Finally ball C remains stationary
(d) Finally ball D remains stationary

3. A block moving in air explodes into two parts, then just after explosion
(a) the total momentum must be conserved
(b) the total kinetic energy must be conserved
(c) the total momentum must change
(d) the total kinetic energy must increase

4. In an elastic collision in absence of external force, which of the following is/are correct ?
(a) The linear momentum is conserved.
(b) The PE is conserved in collision.
(c) The final KE is less than the initial KE.
(d) The final KE is equal to the initial KE.

5. A body is moving towards a finite body which is initially at rest, and collides with it. In the absence of any external impulsive force, it is possible that
(a) both the bodies come to rest
(b) both the bodies move after collision
(c) the moving body comes to rest and the stationary body starts moving
(d) the moving body changes its velocity and the stationary body remains stationary

6. In head on elastic collision of two bodies of equal masses
(a) the velocities are interchanged
(b) the speeds are interchanged
(c) the momenta are interchanged
(d) the faster body slows down and the slower body speeds up

7. A ball hits a floor and rebounds after an inelastic collision. In this case
(a) the momentum of ball just after collision is same as that of just before collision
(b) the mechanical energy of the ball remains the same during the collision
(c) the total momentum of the ball and earth is conserved
(d) the total energy of the ball and earth remains the same

8. Whenever an object strikes a stationary object of equal mass,
(a) the two objects stick together
(b) the collision may be elastic
(c) momentum is necessarily conserved
(d) total energy is necessarily conserved

9. In an elastic collision of two bodies
(a) total momentum of the system remains conserved
(b) total energy of the system remains conserved
(c) the kinetic energy before collision is equal to kinetic energy after collision
(d) the kinetic energy changes during collision stage

C. Assertion & Reason

Directions (Q. Nos. 1 to 7) *Some questions (Assertion-Reason type) are given below. Each question contains **Statement I (Assertion)** and **Statement II (Reason)**. Each question has 4 choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**

(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**

(c) **Statement I** is True, **Statement II** is False

(d) **Statement I** is False, **Statement II** is True

1. **Statement I** A single particle is having non-zero kinetic energy then its momentum can be zero.
   Statement II Kinetic energy is a scalar quantity and is always positive, while momentum is a vector quantity.
2. **Statement I** A system of two particles is having non-zero total kinetic energy, but can have total momentum zero.
   Statement II Kinetic energy is a scalar quantity and is always positive, while momentum is a vector quantity. Further vector sum of two vectors can be zero.
3. **Statement I** If net external force acting on a system of particles is zero, then the total momentum of system remains conserved but momentum of individual particles may change.
   Statement II Internal forces acting within different parts of a system may transfer the momentum from one part of system to the other.
4. **Statement I** If the momentum of a system is changing then it is necessary that some non-zero net force is acting on system.
   Statement II Law of conservation of momentum holds good in all domains of physics.
5. **Statement I** In an inelastic collision of two balls a part of kinetic energy is lost and appears as deformation potential energy of balls.
   Statement II In an inelastic collision the value of e is lying between 0 and 1.
6. **Statement I** In a collision the momentum transfer takes place along the line of impact.
   Statement II The large interaction force in collision acts along the line of impact.
7. **Statement I** Two objects are having momentum of 25 N-s along east and 25 N-s along south respectively. The total momenta of this two- object system is 50 N-s in magnitude.
   Statement II Momentum is a vector quantity.

D. Comprehend the Passage Questions

Passage I

A car of mass 1050 kg is moving towards east with a speed of 15 ms^{-1}. A truck is moving towards west. The mass of the truck is 6320 kg. Both the vehicles collide head on and remain locked together after the collision.

Based on above information, answer the following questions :

1. If the truck is moving at 10 ms^{-1} before collision then the common velocity of the two vehicles after collision is
   (a) 6.44 ms^{-1} towards west
   (b) 10.7 ms^{-1} towards east
   (c) 6.44 ms^{-1} towards east
   (d) 10.7 ms^{-1} towards west

2. The speed of truck before the collision so that the vehicles comes to rest after collision is

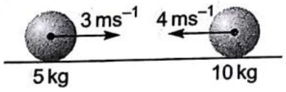
(a) 6 ms^{-1} (b) 2.5 ms^{-1}
(c) 7.5 ms^{-1} (d) 5 ms^{-1}

3. The loss in kinetic energy of the two-vehicle system due to collision in above question is

(a) 118125 J (b) 137875 J
(c) 19750 J (d) 98375 J

Passage II

Two balls of masses 5 kg and 10 kg are approaching each other as shown in figure. The 5 kg ball is moving with a velocity of 3 ms^{-1} and other is moving at 4 ms^{-1}. The two balls under an inelastic collision with $e = \frac{3}{7}$.



Based on above information, answer these questions :

4. The velocity of 5 kg ball after collision is

(a) 11/3 ms^{-1} towards left
(b) 11/3 ms^{-1} towards right
(c) 2/3 ms^{-1} towards left
(d) 2/3 ms^{-1} towards right

5. The velocity of 10 kg ball after collision is

(a) 11/3 ms^{-1} towards left
(b) 11/3 ms^{-1} towards right
(c) 2/3 ms^{-1} towards left
(d) 2/3 ms^{-1} towards right

6. The impulse acting on 10 kg block during collision is

(a) $\frac{100}{3}$ N-s, leftward
(b) $\frac{100}{3}$ N-s, rightward
(c) $\frac{10}{3}$ N-s, leftward
(d) None of the above

7. If collision lasts for 0.01 s, then the magnitude of impulsive force is

(a) $\frac{10^4}{3}$ N
(b) $\frac{10^3}{3}$ N
(c) 1000 N
(d) None of the above

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. 4J, 4N-s
2. 45 N-s
3. 15 N-s, zero, −30 N-s, +ve direction is taken in the direction of initial velocity
4. $(2\hat{i} + 3\hat{j} - 6\hat{k})\ ms^{-1}$
5. $10\sqrt{5}$ N-s. direction can't be found
6. 14 J
7. (a) 134.4 N-s opposite to initial velocity, (b) 6720 N
8. 5760 N-s
9. $(15\hat{i} + 2\hat{j} + 12\hat{k})$
10. 100 N-s, opposite to direction of the motion of person, 2000 N
11. 7.5 N-s in vertical upward direction
12. At an angle of $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{2}\right)$ with +ve x-axis.
13. $1\ ms^{-1}$, 50 N-s
14. 96 kg and 64 kg
15. $\frac{10}{3}\ ms^{-1}$
16. $\frac{mu}{M+m}$
17. $3.6\ ms^{-1}$
18. $150\ ms^{-1}$
19. $v_A = 3\ ms^{-1}$ towards left, $v_B = 0$
20. $v_A = \frac{9}{5}$ towards left, $v_B = \frac{3}{5}\ ms^{-1}$ towards left, $\Delta K = 12.6$ J
21. $7\ ms^{-1}$ and $3\ ms^{-1}$, and recedes away from each other.
22. $v_A = 4\ ms^{-1}$, $v_B = 0$
23. $\frac{4}{3}\ ms^{-1}$
24. $v_A = v_B = 0$, $v_C = v_0$
25. (a) $v_A = 0$, $v_B = v_0$, (b) $\frac{v_0^2}{2g}$
26. $\frac{1}{\sqrt{2}}$
27. (a) $\left(\frac{1+e}{2}\right)\sqrt{gl}$, (b) $\left(\frac{1-e}{2}\right)\sqrt{gl}$, $2l\,(1-\cos\theta) = \left(\frac{1-e}{2}\right)^2$, (c) $\frac{1}{2\mu}\left[\frac{1+e}{2}\right]^2$
28. $\frac{mv}{M+m}, \left(\frac{mv}{M+m}\right)^2 \times \frac{1}{2\mu g}$
29. $8\ ms^{-1}$
30. $7\ ms^{-1}$
31. $v_1 = 319.4\ ms^{-1}$, $v_2 = 319.4\ ms^{-1}$
32. $4.27\ ms^{-1}$ at an angle of $\tan^{-1}\left(\frac{15}{49}\right)$ east to south
33. 0.895, 3.2768 m
34. (a) $1\ ms^{-1}$, (b) 18 J, (c) $\frac{1}{2}$

C. Fill in the Blanks

1. Momentum
2. Change in momentum
3. Elastic
4. Head on, oblique
5. Conserved
6. Interchanged
7. Stick
8. $0 \le e \le 1$

D. True/False

1. F
2. T
3. F
4. T
5. T
6. F
7. T
8. T
9. T
10. F

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (b)	2. (a)	3. (c)	4. (b)	5. (b)	6. (a)	7. (b)	8. (a)	9. (a)	10. (b)
11. (d)	12. (c)	13. (d)	14. (c)	15. (d)	16. (d)	17. (d)	18. (a)	19. (b)	20. (c)

B. More Than One Options Correct

1. (a, b, c, d)	2. (a, b, c, d)	3. (a, d)	4. (a, d)	5. (b, d)
6. (a, b, c, d)	7. (c, d)	8. (a, b, c, d)	9. (a, b, c, d)	

C. Assertion & Reason

1. (d)	2. (a)	3. (a)	4. (b)	5. (b)	6. (a)	7. (d)

D. Comprehend the Passage Questions

1. (a)	2. (b)	3. (b)	4. (a)	5. (c)	6. (a)	7. (a)

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. $K = \dfrac{mv^2}{2} = \dfrac{2 \times 2^2}{2} = 4\ \text{J}$

$p = mv = 4\ \text{kg-ms}^{-1}$

2. $v = 0 + 3 \times 3 = 9\ \text{ms}^{-1}$

$p = mv = 5 \times 9 = 45\ \text{N-s}$

3. Proceed same as in above question.

4. $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

$\Rightarrow \quad (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}) = 1 \times \vec{\mathbf{v}}$

$\Rightarrow \quad \vec{\mathbf{v}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}})\ \text{ms}^{-1}$

5. $K = \dfrac{p^2}{2m}$

$p = \sqrt{2mK}$

$= \sqrt{2 \times 5 \times 50} = 10\sqrt{5}\ \text{N-s}$

Direction of momentum can't be found from the given data.

7. Let the initial direction of motion of ball be along +ve x-axis.

$\vec{\mathbf{p}}_i = 1.4 \times 38\ \hat{\mathbf{i}}\ \text{N-s}$

and $\quad \vec{\mathbf{p}}_f = -1.4 \times 58\ \hat{\mathbf{i}}\ \text{N-s}$

(a) $\quad \vec{\mathbf{J}} = \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$

$= -134.4\ \hat{\mathbf{i}}\ \text{N-s}$

(b) $\quad F\Delta t = 134.4$

$\Rightarrow \quad \dfrac{134.4}{0.02} = 6720\ \text{N}$

8. $\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = F\Delta t$

$\vec{\mathbf{p}}_f - 0 = 720 \times 8\ \text{N-s} = 5760\ \text{N-s}$

9. $\vec{\mathbf{p}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_3$

$= m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + m_3\vec{\mathbf{v}}_3$

$= 1(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) + 2(6\hat{\mathbf{i}}) + 3(4\hat{\mathbf{k}})$

$= (15\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 12\hat{\mathbf{k}})\ \text{kg-ms}^{-1}$

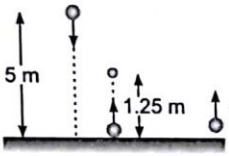
10. Impulse on man,

$$J = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$$

$$J = 50 \times 2 - 0 = 100 \text{ N-s}$$

11. The velocity with which the ball strikes the ground is,

$$v_1 = \sqrt{2 \times 10 \times 5} = 10 \text{ ms}^{-1}$$



Velocity with which the ball rebounds is,

$$v_2 = \sqrt{2 \times 10 \times 1.25} = 5 \text{ ms}^{-1}$$

$$\Delta \vec{\mathbf{p}} = mv_2\,\hat{\mathbf{j}} - (-mv_1\,\hat{\mathbf{j}})$$

$$= 7.5\,\hat{\mathbf{j}} \text{ N-s}$$

12. $(\Delta t)\vec{\mathbf{F}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i$

$$\vec{\mathbf{F}} = \frac{1}{2\Delta t}[4\,\hat{\mathbf{j}} - 2\,\hat{\mathbf{i}}] = \frac{2\,\hat{\mathbf{j}} - \hat{\mathbf{i}}}{\Delta t}$$

So, the required angle is $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{2}\right)$ with +ve x-axis.

13. As no external force is acting on system (person + ball), the momentum of the system remains conserved. Initial momentum is zero, so after throwing also the final momentum of system is zero.

$$\vec{\mathbf{p}}_m + \vec{\mathbf{p}}_{\text{ball}} = 0$$

$\Rightarrow \quad -50 \times v + 5 \times 10 = 0$

$\Rightarrow \quad v = 1 \text{ ms}^{-1}$

$J = \Delta p$ of person

$= 50 \times 1 - 0 = 50$ N-s

14. Let m_1 and m_2 be masses of Ram and Rahim respectively

$$m_1 + m_2 = 160$$

From momentum conservation,

$$m_1 \times 2 = m_2 \times 3$$

After solving above equations, we get

$$m_1 = 96 \text{ kg} \quad \text{and} \quad m_2 = 64 \text{ kg}$$

15. From momentum conservation,

$$(50 + 100)\,v = 100 \times 5$$

$\Rightarrow \quad v = \frac{10}{3} \text{ ms}^{-1}$

16. Initial momentum of *man + cart* system,

$$p_i = mu + 0$$

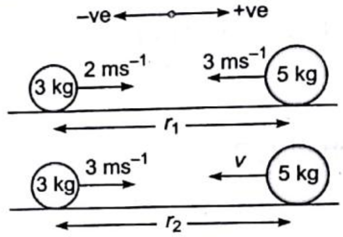
Let the final velocity of system be v.

Then $\quad p_f = (M + m)v$

$\Rightarrow \quad p_i = p_f$

$\Rightarrow \quad v = \frac{mu}{M + m}$

17. Let v be the required velocity. From momentum conservation,



$$3 \times 2 - 3 \times 5 = 3 \times 3 - 5\,v$$

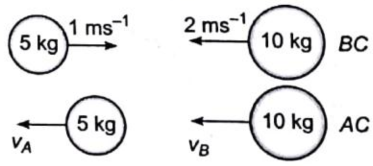
$\Rightarrow \quad v = 3.6 \text{ ms}^{-1}$

18. At the topmost point the shell is moving horizontally and as no external force is acting on the shell in horizontal direction, so the momentum of shell before explosion is momentum of two pieces of the shell after explosion,

$$m \times 100 \cos 60° = -\frac{m}{2} \times 50 + \frac{m}{2} \times v$$

$\Rightarrow \quad v = 150 \text{ ms}^{-1}$

19. From momentum conservation,



$$5 \times 1 - 10 \times 2 = -5v_A - 10v_B$$

$$e = 1 = \frac{v_A - v_B}{3}$$

Solving above equations, we get

$$v_A = 3 \text{ ms}^{-1}, \text{ and } v_B = 0 \text{ ms}^{-1}.$$

20. First equation remains same as in above question and

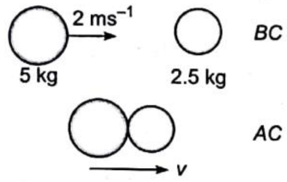
$$e = \frac{2}{5}$$

$$\Delta K = \left[\frac{m_A v_A^2}{2} + \frac{m_B v_B^2}{2}\right] - \left[\frac{m_A \times 1^2}{2} + \frac{m_B \times 2^2}{2}\right]$$

$$= 12.6 \text{ J}$$

21. Proceed similar to Q. No. 20.

23. From momentum conservation,



$$5 \times 2 + 0 = 7.5\,v$$

$$\Rightarrow \quad v = \frac{4}{3}\ \text{ms}^{-1}$$

24. Use the fact that in a head on elastic collision of two identical balls the velocities get interchanged after collision.

25. Using the concepts used in previous question,

v_A after collision $= 0$

v_B after collision $= v_0$

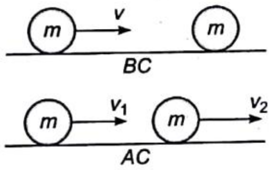
The maximum height attained by B is,

$$0 - \frac{mv_0^2}{2} = -mgh$$

$$\Rightarrow \quad h = \frac{v_0^2}{2g}$$

26. From momentum conservation,

$$mv = mv_1 + mv_2$$

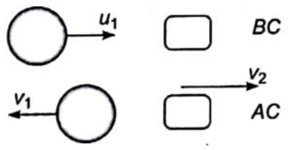


From KE condition,

$$\frac{mv_1^2}{2} + \frac{mv_2^2}{2} = \frac{3}{4} \times \frac{mv^2}{2}$$

$$e = \frac{v_2 - v_1}{v} = \frac{1}{\sqrt{2}}$$

27. The velocity of ball just before collision is



$$v_1 = \sqrt{gl}$$

$$mu_1 = -mv_1 + mv_2$$

$$e = \frac{v_1 + v_2}{u_1}$$

Solving above equations, we get

$$v_1 = \left(\frac{1-e}{2}\right)\sqrt{gl},$$

and $$v_2 = \left(\frac{1+e}{2}\right)\sqrt{gl}$$

The height attained by ball after collision,

$$h = \frac{v_1^2}{2g}$$

The distance moved by the block is given by,

$$\frac{mv_2^2}{2} = \mu\, mgs$$

28. Let v_1 be the required velocity, then

$$mv = (M + m)v_1$$

$$\Rightarrow \quad v_1 = \frac{mv}{M + m}$$

Let s be the distance travelled by block before it stops.

$$\Rightarrow \quad \frac{(m + M)v_1^2}{2} = \mu\,(M + m)\,gs$$

$$\Rightarrow \quad s = \frac{1}{2\mu g} \times \left(\frac{mv}{M + m}\right)^2$$

29. Let v be the velocity of ball immediately after the collision.

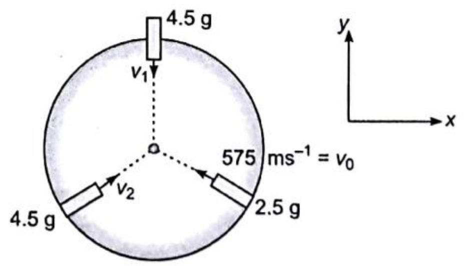
$$\Rightarrow \quad e = 1 = \frac{v - 3}{5}$$

$$\Rightarrow \quad v = 8\ \text{ms}^{-1}$$

30. $e = \dfrac{4}{5} = \dfrac{v - 3}{5}$

$$\Rightarrow \quad v = 7\ \text{ms}^{-1}$$

31. After collision the lump of mass is stationary so it implies that,



$$\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_3 = 0$$

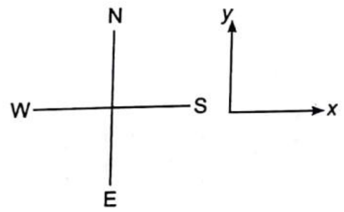
$$m_1 \vec{\mathbf{v}}_1 + m_2 \vec{\mathbf{v}}_2 + m_3 \vec{\mathbf{v}}_0 = 0$$

where $\quad m_1 = m_2 = 4.5 \text{ g}$

and $\quad m_0 = 2.5 \text{ g}$

32. $\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \vec{\mathbf{p}}$

$$50\,(3\,\hat{\mathbf{i}}) + 70\,(-7\,\hat{\mathbf{j}}) = 120\,\vec{\mathbf{v}}$$



$$\Rightarrow \qquad \vec{\mathbf{v}} = \frac{150\,\hat{\mathbf{i}} - 490\,\hat{\mathbf{j}}}{120}$$

33. Velocity just before first collision,
$v_1 = \sqrt{2g \times 10}$

Velocity just after first collision,
$v_2 = \sqrt{2g \times 8}$

$$e = \frac{v_2}{v_1}$$

$$= \sqrt{\frac{8}{10}} = 0.895$$

Velocity after second collision $= ev_2$

Velocity after third collision $= e^2 v_2$

Velocity after fourth collision $= e^3 v_2$

Velocity after fifth collision $= e^4 v_2 = e^5 v_1$

Required height,

$$h = \frac{(e^5 v_1)^2}{2g}.$$

Chapter 9

Circular & Rotational Motion

The First Steps' Learning

- Circular Motion
- Uniform Circular Motion
- Angular Variables
- Angular Displacement
- Angular Velocity
- Angular Acceleration
- Relation between Linear & Angular Variables
- Tangential and Radial Acceleration
- Time Period of Circular Motion
- Non-Uniform Circular Motion
- Rotational Motion
- Torque
- Moment of Inertia

Until now, we discussed various aspects of translational/rectilinear motion. In this chapter we are going to analyse and study a new type of motion which we call as **circular** *and* **rotational motion***. Here we will discuss circular motion in detail, but detailed analysis of rotational kinematics requires some preparatory training to grasp and analyse the situations. Here, we shall only discuss some terms (not all) used in rotational mechanics so that you are well-grounded to understand the rotational motion later on. When you take up your actual preparation for IIT JEE.*

Circular Motion

When a particle moves along a circular path, then it is said to perform a circular motion. For example, the motion of artificial satellite around the earth, motion of earth around the sun, motion of moon around the earth, motion of minute and hour hands of the watch, motion of a vehicle along a circular track, motion of a stone tied to an end of string and the string is whirled in a horizontal or vertical circle with other end at, centre of circular path. All these are classic examples of the circular motion.

Let us see how the motion of a particle takes place when it is moving along a circular path. Let us consider a particle moving along a circular path in X-Y plane as shown in the figure. Suppose the particle is at A at $t = 0$ and after a small time- interval it is at B, then to go from A to B, the velocity of the particle has to be along the tangent to circle at A. If the particle is at P at some time and it travels PP' in the small time-interval, then the direction of velocity of the particle at P is along the tangent drawn to circle at P. So we can say direction of velocity of particle is changing continuously as the particle moves along a circular path, *ie*, we can say that velocity of particle is changing continuously in circular motion and hence the circular motion is an accelerated motion (as velocity is varying), and the direction of velocity of particle undergoing circular motion is always along the tangent to the circular path.

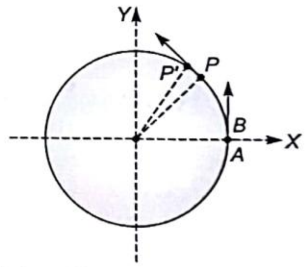


Fig 9.1 A particle moving along a circular path.

Types of Circular Motion

The circular motion of a particle can be of two types :

(a) Uniform circular motion.

(b) Non-uniform circular motion.

In uniform circular motion, the speed (magnitude of velocity) of particle undergoing circular motion remains constant, while in non-uniform circular motion, the speed of particle is always changing.

Uniform Circular Motion

When the speed of a particle undergoing circular motion is constant (not changing) then the circular motion is said to be uniform circular motion. In uniform circular motion the speed of particle is constant, but what about its velocity ? Is the velocity of particle also constant ? As we

have seen that direction of velocity of particle is always changing in circular motion, which is true for uniform circular motion also. So in uniform circular motion, the speed of particle is constant but velocity is not constant as direction of motion of particle is continuously changing.

In uniform rectilinear motion we have seen that velocity of particle is constant and generally we have the notion that uniform motion means velocity of particle during the motion is remaining constant, but in uniform circular motion the word uniform is used with reference to constant speed, and not the constant velocity. As the direction of velocity of particle in uniform circular motion is changing, we can say that velocity of particle is not constant and hence the particle is having some acceleration and as acceleration is produced due to a net force so some force must act on the particle. So we can conclude that in uniform circular motion the particle is experiencing some force as a result of which the particle is accelerated and hence its direction of velocity changes. Now the question arises, what is the direction of acceleration in uniform circular motion ? Is it along the velocity or in some other direction ? Is acceleration constant ? Does acceleration depend on the speed of particle ? Before answering these questions and to discuss the dynamics of uniform circular motion, let's first have a look into the angular variables, just like we have discussed parameters of rectilinear motion in the third chapter. Throughout this chapter we will make an analogy of circular and rotational motion with the rectilinear motion.

Angular Variables

Suppose a particle is moving along a circle of radius R in X-Y plane as shown in figure. Let us say at any time t, the particle is at P, so that

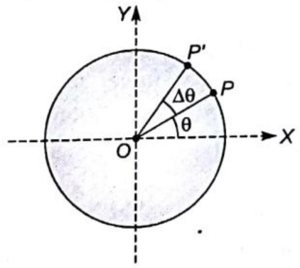


Fig. 9.2

OP is making an angle θ with the X-axis. In circular motion, the term **angular position** would be used to describe the location of particle. Angular position is the angle subtended by OP (the line joining particle and centre of circle) with some reference line (here we have taken it to be OX). Here, θ represents the angular position of particle at time t.

Position of particle in rectilinear motion	Angular position of particle in circular motion
The location of particle *wrt* some reference point, termed as origin.	The angle subtended by particle at centre with some reference line.
Denoted by $x(\vec{r})$ and SI unit is metre.	Denoted by θ and SI unit is radian.

Angular Displacement

If the particle moves along the circle, then the angular position θ, of the particle changes, suppose the particle reaches P' to some nearby point of P such that $\angle P'OP$ is $\Delta\theta$ in time Δt, then angular displacement of the particle in time Δt is $\Delta\theta$.

Angular displacement,

$$\Delta\theta = \theta_f - \theta_i$$

Displacement of particle in rectilinear motion	Angular displacement of particle in circular motion
Displacement of particle is defined as the change in position vector, *ie*, $\Delta \vec{r} = \vec{r}_f - \vec{r}_i$ or $\Delta x = x_f - x_i$ whose SI unit is metre.	Angular displacement of particle is defined as change in its angular position, *ie*, $\Delta\theta = \theta_f - \theta_i$. Its SI unit is radian.

Angular Velocity

If the angular position of a particle is changing with time, then it is said to possess angular velocity. It is denoted by ω (pronounced as 'omega'). Average angular velocity of a particle is defined as the angular displacement of particle in a given time-interval, just like the average velocity in translational motion.

$$\omega_{av} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Same way as we have defined instantaneous velocity or simply velocity in rectilinear motion, we can define instantaneous angular velocity or simply angular velocity as the ratio of angular displacement ($\Delta\theta$) in a time interval Δt, to the time-interval where Δt is very very small.

$$\omega = \frac{\Delta\theta}{\Delta t} \text{ where } \Delta t \text{ approaches zero.}$$

Angular velocity of a particle is different about different points *ie*, angular velocity of same object at same instant can be measured differently about different points, just like linear velocity of a particle at same instant can be different *wrt* different frames of reference.

Angular velocity is a vector quantity and its SI unit is rads^{-1}. Now the question arises, in which direction angular velocity points to ?

There are two possibilities regarding direction of motion of particle, when it is moving along a circle which is clearly shown in figure. Fig.1 shows that particle is moving in clockwise manner (the work clockwise is taken in reference to motion of hands of watch), while Fig. 2 shows that particle is moving in anticlockwise direction or counterclockwise direction *ie*, direction opposite to motion of hands of watch.

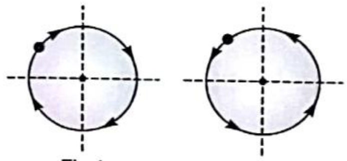


Fig. 1 Clockwise

Fig. 2 Anticlockwise

Fig. 9.3

If you consider clockwise direction as positive, then you have to consider the anti-clockwise direction as negative.

Linear Velocity	Angular Velocity
$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$	$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{\Delta t}$
$v = \frac{\Delta x}{\Delta t}$ where Δt is very small	$\omega = \frac{\Delta\theta}{\Delta t}$ where $\Delta\theta$ is very small
Vector quantity	Vector quantity
SI unit is ms^{-1}	SI unit is rad s^{-1}

Angular Acceleration

If the angular velocity of a particle undergoing circular motion is changing with time then it is said to possess angular acceleration. It is denoted by 'α' (pronounced as 'alpha'). Average angular acceleration of a particle is defined as the change in angular velocity per unit time.

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

While angular acceleration at any instant is defined as, $\alpha = \frac{\Delta\omega}{\Delta t}$ where $\Delta\omega$ is the change in angular velocity in a very small time-interval Δt. Angular acceleration is a vector quantity and its SI unit is rad s^{-2}.

Linear Acceleration (a)	Angular Acceleration (α)
$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$	$\alpha_{av} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i}$
$a = \frac{\Delta v}{\Delta t}$, Δt is very small	$\alpha = \frac{\Delta\omega}{\Delta t}$, Δt is very small
Vector quantity SI unit is ms^{-2}	Vector quantity SI unit is rad s^{-2}

Equations of Circular Motion

Just like in rectilinear motion, we have the equations of motion for a particle moving with constant acceleration in the circular motion. Here also, we have similar types of equations.

Let us consider a particle moving along a circle with constant angular acceleration α. Let the initial angular velocity of particle be ω_0 and the particle is on reference line at $t = 0$. If ω be the angular velocity of particle at any time t, and θ the angular displacement of particle in time t, then the equations

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

are termed as equations of motion of circular motion with constant acceleration.

It is very clear that above equations are almost similar to equations of motion for constant acceleration as discussed earlier in translational motion.

Translational Motion	Circular Motion
$v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$	$\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha\theta$
u : initial velocity.	ω_0 : initial angular velocity
a : constant acceleration	α : constant angular acceleration
v : velocity at any time t	ω : angular velocity at any time t
s : displacement in time t	θ : angular displacement in time t
Proper sign convention has to be used while solving the questions.	Proper sign convention has to be used while solving the questions.

Whatever concepts we discussed in chapter 3, would be applicable here after making a proper analogy.

C-BIs

Concept Building Illustrations

Illustration | 1 *A particle is moving along a circle in anticlockwise direction as shown in figure. At $t = 0$, the particle is at P and in 5 s it moves to P′ such that $\angle P'OP = 45°$. Determine*

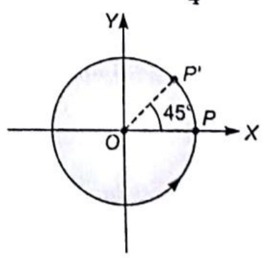
(a) *the angular displacement of particle in 5 s.*

(b) *the average angular velocity of particle for this 5 s interval.*

(c) *the angle made by velocity vector of particle with +ve X-axis at $t = 5$ s.*

Solution (a) At $t = 0, \theta_i = 0°$

At $t = 5$ s, $\theta_f = 45° = \frac{\pi}{4}$ rad



So, angular displacement in 5 s is,

$$\Delta\theta = \theta_f - \theta_i = \frac{\pi}{4} - 0 = \frac{\pi}{4} \text{ rad}$$

Direction of angular displacement is in anticlockwise direction.

(b) The average angular velocity in this 5 s interval is given by

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\pi/4}{5} = \frac{\pi}{20} \text{ rad s}^{-1},$$

along anticlockwise direction.

(c) The velocity vector at $t = 5$ s is along the tangent drawn at point P' as shown in figure.

The required angle is 135°.

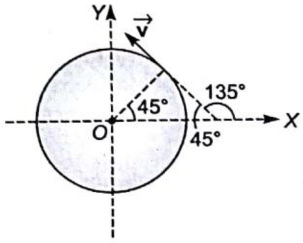


Illustration | 2 *If the angular velocity of the particle mentioned in above question at* P and P' *are 2 rad* s^{-1} *and 5 rad* s^{-1} *respectively, then determine the average angular acceleration of particle for 5 s interval.*

Solution $\alpha_{av} = \dfrac{\omega_f - \omega_i}{\Delta t}$

$$= \frac{5-2}{5} = \frac{3}{5} \text{ rad s}^{-2}$$

$$= 0.6 \text{ rad s}^{-2}, \text{ anticlockwise}$$

Illustration | 3 *If in Illustration 1 the angular velocity of particle at* P and P' *are 5 rad* s^{-1} *and 2 rad* s^{-1} *respectively, then determine the average angular acceleration of particle for 5 s interval.*

Solution Here the angular velocity of particle is decreasing as the time increases, that means, direction of angular velocity and angular acceleration are opposite to each other.

$$\alpha_{av} = \frac{\omega_f - \omega_i}{\Delta t}$$

$$= \frac{2-5}{5} = \frac{-3}{5} \text{ rad s}^{-2}$$

$$= -0.6 \text{ rad s}^{-2}$$

The negative sign shows that direction of angular acceleration is in clockwise direction.

Illustration | 4 *A particle starts from rest to move along a circle, with constant angular acceleration of 5 rad* s^{-2} *in clockwise direction. Determine*

(a) the angular velocity of particle at $t = 3$ *s.*

(b) the angle by which the particle rotates in 3 s.

(c) the number of revolutions made by particle in 3 s.

Solution Here, the angular acceleration is constant, so we can use the equations of motion for circular motion.

Here $\omega_0 = 0$, as the particle starts from rest.

(a) From, $\omega = \omega_0 + \alpha t$

$\Rightarrow \quad \omega = 0 + 5 \times 3$

$= 15 \text{ rad s}^{-1}$; clockwise

(b) From, $\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$

$$= 0 + \frac{1}{2} \times 5 \times 3^2$$

$$= \frac{45}{2} = 22.5 \text{ rad; clockwise}$$

(c) In one complete revolution, the particle rotates by an angle of 360° or 2π radian. So, number of revolutions made by particle in 3 s is,

$$n = \frac{\theta}{2\pi} = \frac{22.5}{2\pi} = \frac{11.25}{\pi}$$

Illustration | 5 *The angular speed of particle moving along a circular path is 300 rev/min. Determine the angular velocity in rad* s^{-1}.

Solution $1 \text{ rev/min} = \dfrac{1 \text{ revolution}}{1 \text{ min}}$

$$= \frac{2\pi \text{ rad}}{60 \text{ s}} = \frac{\pi}{30} \text{ rad s}^{-1}$$

Given angular velocity is 300 rev/min which is equal to

$$300 \times \frac{\pi}{30} \text{ rad s}^{-1} = 10\pi \text{ rad s}^{-1}.$$

Illustration | 6 *A particle is moving along a circle with angular velocity 10 rad s^{-1} in anticlockwise direction. At $t = 0$, a constant angular acceleration of 2 rad s^{-2} is in clockwise direction. Determine*

(a) the instant at which angular velocity become zero.

(b) the angular velocity at $t = 8$ s.

Solution Here in this case, the initial angular velocity and angular acceleration are in opposite directions, so the magnitude of angular velocity first decreases, becomes zero, and then it increases in opposite direction [just like many questions we discussed in chapter 3 where initial velocity and acceleration were in opposite directions]. Let us consider the anticlockwise direction as positive and clockwise direction negative, then

$$\omega_0 = 10 \text{ rad s}^{-1}, \quad \text{and} \quad \alpha = -2 \text{ rad s}^{-2}.$$

Let angular velocity becomes zero at $t = t_0$ sec. Then from

$$\omega = \omega_0 + \alpha t$$

$$\Rightarrow \quad 0 = 10 - 2 \times t_0$$

$$\Rightarrow \quad t_0 = 5 \text{ s}$$

So, angular velocity becomes zero at $t = 5$ s.

Let ω be the angular velocity of particle at $t = 8$ s, then

$$\omega = \omega_0 + \alpha t$$

$$= 10 - 2 \times 8$$

$$= -6 \text{ rad s}^{-1}$$

Negative sign shows that angular velocity is in clockwise direction at $t = 8$ s.

Relation between Linear and Angular Variables

When a particle is moving along a circular path, then angular variables are needed to characterize/explain its motion as we already discussed in previous sections. But in addition, as the particle is moving it also possesses linear velocity and linear acceleration or we can say has linear variables. In this section we are telling you about relation between linear and angular variables for a particle undergoing circular motion. Here, we are telling you only the relation between linear and angular variables without deriving them as the derivation requires use of methods of calculus.

Consider a particle moving in a circle of radius R as shown in figure. Let us consider the instant when the angular velocity and angular acceleration of particle are ω and α respectively, while linear velocity and linear acceleration of particle are v and a, respectively. Then,

$$v = R\omega,$$

and

$$a = R\alpha$$

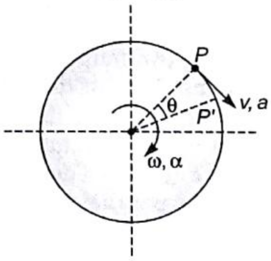


Fig. 9.4 Particle performing circular motion is associate with linear variables

Here, the acceleration of the particle mentioned in above equation is the tangential component of the acceleration (regarding tangential acceleration in detail we will study

in next section), which is responsible for change in magnitude of velocity.

If the particle rotates by an angle θ in any time, then the distance travelled by particle in this duration is given by,

$s = R\theta$ [we hope you have understood this as it is derived directly from the expression-

Arc length = radius × angle subtendend by arc at centre].

So, the relation between linear and angular variables can be summarized as

$$s = R\theta$$
$$v = R\omega$$
$$a = R\alpha$$

C-BIs

Concept Building Illustrations

Illustration | 7 *A particle moves in a circle of radius 5 m and it rotates (displaces) by an angle $\pi/4$ in 3 s. Determine the distance travelled by particle in this 3 s duration.*

Solution From $s = R\theta$

$$\Rightarrow \quad s = 5 \times \frac{\pi}{4} \text{ m}$$
$$= 3.93 \text{ m}$$

Illustration | 8 *A particle moves in a circle of radius 2 m with a constant linear speed of 10 ms^{-1}. Determine the angular velocity of particle.*

Solution The relation between linear and angular velocities is

$$v = R\omega$$
$$\Rightarrow \quad 10 = 2\omega$$
$$\Rightarrow \quad \omega = 5 \text{ rad s}^{-1}.$$

Tangential and Radial Acceleration

If the acceleration of a particle is not along the direction of motion of particle *ie*, angle between velocity vector and acceleration is not equal to 0° or 180°, then acceleration of particle can be resolved into two mutual perpendicular directions, one along the direction of velocity or opposite to it and other in a direction perpendicular to velocity. This situation is shown clearly in Fig. 9.5. The component of acceleration which is parallel or antiparallel to velocity is termed as tangential acceleration (a_t) and is responsible for change in magnitude of

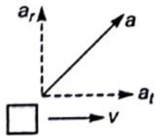


Fig. 9.5

velocity. This component is termed as tangential acceleration as it is along the tangent drawn to path. If tangential acceleration is along the velocity, then the speed of particle increases and if it is opposite to velocity then speed of particle decreases. While the component perpendicular to velocity is termed as radial or normal acceleration (a_r) and is responsible for change in direction of velocity. This component is termed as normal acceleration as it is along the normal (perpendicular) to the motion of particle. Let's have a detailed look at this.

These two components of acceleration work independently as if the other is not present. Let us first consider that only tangential component of acceleration is acting

on particle, then as already we have seen in chapter 3, that for any small time interval it changes the speed of particle only and direction of motion doesn't change. And what happens if only normal component of acceleration is only acting on particle, then it is quiet obvious that speed of particle doesn't change, this we can easily conclude from work-energy theorem as work done by a force acting perpendicular to direction of motion (displacement) does zero work on particle and hence kinetic energy remains constant, and thus the speed of particle. But this component of acceleration affects the motion in some way and that is by changing the direction of motion of particle *ie*, velocity.

So, the combined effect is that magnitude as well as direction of velocity changes when acceleration and velocity vector are not along the same straight line. This situation is clearly shown in Fig. 9.6. Here, initially the particle is

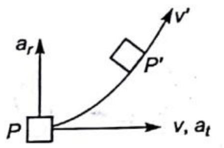


Fig. 9.6

at P and moving towards right and due to acceleration (whose tangential and radial components are a_t and a_r, respectively) after a small time-interval it reaches P' where its velocity changes from v to v'. Thus, we can say that the particle describes a curved path when $\vec{v}$ and $\vec{a}$ are not along some straight line or we can say that both a_r and a_t are required by the particle to follow a curved path in which the speed of particle is changing. In other words, we can say that a_r is necessary by a particle if it follows some curved path while a_t is also required if speed of particle is also changing while following a curved path.

So, if the particle is moving along a circular path (which is obviously a curved path) radial acceleration is necessary which can change the direction of velocity of particle. In uniform circular motion, the speed of the particle is constant and hence $a_t = 0$, while in non-uniform circular motion speed of particle is changing and hence $a_t \neq 0$. In circular motion, the radial acceleration is also known as centripetal acceleration, the direction of centripetal acceleration is always towards the centre of the circle.

From previous topic, $a_t = R\alpha$ and directly we are giving you the expression for radial acceleration.

$$a_R = \frac{v^2}{R} = \frac{(\omega R)^2}{R} = \omega^2 R$$

C-BIs

Concept Building Illustrations

Illustration | 9 *A particle is moving along a circle of radius* 2 m *with a constant speed of* 5 ms^{-1}. *Determine the acceleration of particle.*

Solution As particle is undergoing uniform circular motion, it will experience only centripetal acceleration given by

$$a = \frac{v^2}{R} = \frac{5^2}{2} = 12.5 \text{ ms}^{-2}$$

which is always directed towards the centre.

Time Period of Circular Motion

Earlier we have discussed that speed of a particle is constant in uniform circular motion and hence for uniform circular motion $a_t = 0$. As $a_t = R\alpha$, so we can say in uniform circular motion $\alpha = 0$ and hence the angular velocity of particle is constant. If the particle is moving in a circle of radius R, with constant speed v, then the magnitude of centripetal acceleration required for circular motion is, $a = \dfrac{v^2}{R} = \omega^2 R$ where ω is the angular velocity of particle.

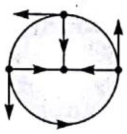


Fig. 9.7

In uniform circular motion, the speed of particle is constant but velocity is not, and magnitude of acceleration is also constant but direction is always changing as it is always towards the centre of the circle and there are infinite directions which are pointing towards the centre. Thus, we can say that in uniform circular motion, the velocity and acceleration of a particle are not constant. Fig. 9.7 shows the direction of velocity and acceleration at three different instants.

As the speed of particle in uniform circular motion is constant, the particle crosses a particular position after a regular (same) interval of time and hence the uniform circular motion is periodic* in nature. Let us consider a particle is performing a circular motion of radius R with constant speed v. If the particle is at point P at any instant, then it will take time T given by $2\pi R = vT$, to again come at same point. Thus, $T = \dfrac{2\pi R}{v}$, is the time-interval after which particle comes to same position and hence T is the time period of uniform circular motion.

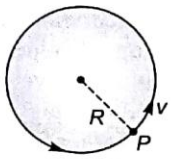


Fig. 9.8

$$\text{Time period} = T = \frac{2\pi R}{v}$$

C-BIs

Concept Building Illustrations

Illustration | 10 *Determine the time period of uniform circular motion of the particle mentioned in illustration 9.*

Solution Time period,

$$T = \frac{2\pi R}{v}$$

$$T = \frac{2\pi \times 2}{5}$$

$$= \frac{4\pi}{5}$$

$$= 2.513 \text{ s}$$

*A periodic motion is one which repeats itself (in position as well as in velocity) after a regular interval of time. For illustration, Motion of earth about its own axis, motion of earth around the sun etc. The minimum time-interval in which particle acquires its original position is the time period of motion. Time periods of the earth's rotation around its own axis is nearly 24, and around the sun is about 365 days. Illustrations time periods are 24 h and 365 days respectively.

As the particle undergoing uniform circular motion is an accelerated one, *ie*, it is having an acceleration equal to $\frac{v^2}{R}$ always directed towards the centre. From Newton's second law, we know that a particle having some acceleration a will experience a net force, $F = ma$ [where m is the mass of particle] in the direction of acceleration. So, it means some force must act on the particle. Which is undergoing uniform circular motion and the magnitude of this force must be equal to, $F =$ mass of particle $\times$ centripetal acceleration *ie*, $F = \frac{mv^2}{R} = m\omega^2 R$, and the direction of this force is always towards the centre. This force we call by the name centripetal force. Thus, we can say a particle of mass m performing uniform circular motion of radius R with speed v will experience a net force equal to $\frac{mv^2}{R}$ which is always directed towards the centre.

Centripetal force is not a different type of force, it is just like downward or upward force *ie*, it is only the name given to the net force acting on the particle towards the centre, performing circular motion. Centripetal force is a necessary requirement for a particle performing circular motion, and in different situations it is provided by different forces like friction, tension in string, gravitational force, magnetic force, electrostatic force etc.

Whenever you solve the question related to circular motion, resolve all the forces acting on the particle in the radial and tangential direction of circle and then find the resultant force acting on the particle towards the centre and along the tangent, let it be F_r and F_t, respectively.

For uniform circular motion, equate

$$F_r = \frac{mv^2}{R},$$

and $$F_t = 0.$$

For non-uniform circular motion, equate,

$$F_r = \frac{mv^2}{R}*$$

and $$F_t = mR\alpha$$

The above equations of motion are equations of dynamics of circular motion. Illustrations of uniform circular motion :

(a) A vehicle moving in a horizontal circle with constant speed, in this case the necessary centripetal force required for circular motion to take place is provided by friction force between the road and tires of vehicle.

(b) A particle connected to end of a string is moving in a circular path on a smooth horizontal surface with constant speed with other end (kept fixed) of the string as centre. Here the necessary centripetal force is provided by the tension in the string.

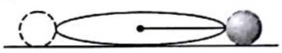


Fig. 9.9

(c) The motion of satellites around the earth in circular orbit, here the necessary centripetal force is provided by gravitational force between the satellite and earth.

Banking of Rails and Roads

Whenever a vehicle is taking a turn on road, it can be considered that vehicle is moving along a circular path. In this case two situations are possible : (I) the road is not banked *ie*, road is horizontal and not elevated up on turning, and (II) the road is banked.

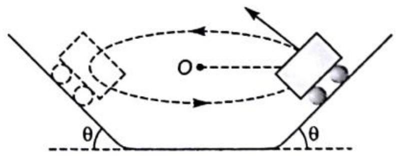


Fig. 9.10 Dotted circle here represents the circle in horizontal plane, along which the vehicle moves.

*Detailed analysis of non-uniform circular motion would be in concerned topic.

Banking of rails and roads means that road is elevated up from the edges as you may easily observe in bicycle racing track in stadium. Two dimensional view of the track is as shown in figure. Now the question arises, why the banking has been done ? The thing is, when the vehicle is moving on a circular path, some force must act on the vehicle towards the centre of circle, which provides the necessary centripetal force, this force can be provided by horizontal component of normal contact force if the road is elevated up from the edges *ie*, road is banked. The free body diagram of the vehicle moving in a horizontal circular path on the banked road is as shown in figure.

If the vehicle of total mass M is taking a turn on a road banked at an angle θ (*ie*, the road is making an angle θ with horizontal) with speed (constant) v, then the free body diagram of vehicle would be like as shown in figure. Let us say the radius of circular turn be R, now in this case we have to resolve the force along radial and tangential direction *ie*, along the horizontal direction and vertical direction. The centre of the circle is at O. After resolving N along horizontal and vertical directions, it is clear from the figure that horizontal component of normal contact force provides the necessary centripetal force and its vertical component balances be the gravity force.

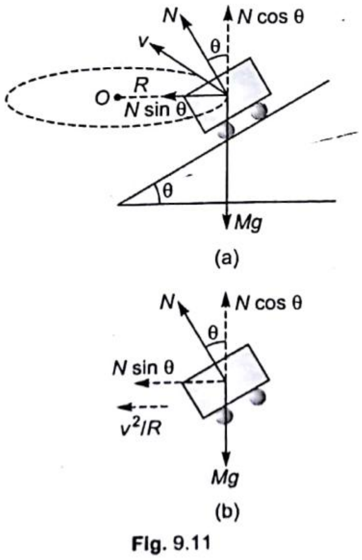


Fig. 9.11

From equation of dynamics of circular motion,

$$N \sin \theta = \frac{Mv^2}{R}$$

For vertical equilibrium,

$$N \cos \theta = Mg$$

Dividing above two equations, we get

$$\frac{N \sin \theta}{N \cos \theta} = \frac{Mv^2/R}{Mg}$$

$$\Rightarrow \qquad \tan \theta = \frac{v^2}{Rg}$$

$$\Rightarrow \qquad v = \sqrt{Rg \tan \theta}$$

So if a vehicle is taking a circular turn of radius R on a road banked at an angle θ, then the vehicle can take the turn safely if $v = \sqrt{Rg \tan \theta}$. So from the above discussion it is clear that a component of normal contact force provides the necessary centripetal force if the vehicle is taking a circular turn on a banked road.

In the above discussion we have not considered the friction present between the road and the tyres of vehicle, which is always present in practical situations. In general, friction and banking both contribute in centripetal force, but the situation becomes somewhat more complex and that's why we are not describing that situation here, but we will tell you the things related to the case when vehicle is taking a turn on a rough horizontal road.

Circular Turn of a Vehicle on a Horizontal Road

If the vehicle is taking a turn on a horizontal road, then the normal contact force would be in vertical upward direction and won't be able to provide necessary centripetal force as its component along horizontal direction (the radial direction) would be zero, then in this case friction provides the necessary centripetal force required for circular motion to take place. The free body diagram of vehicle in this situation is as shown in figure.

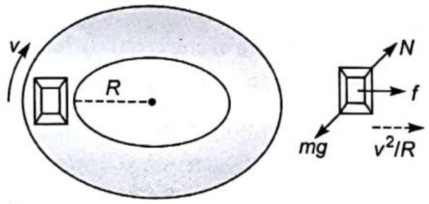


Fig. 9.12 Top view of the vehicle moving in a circular path in a horizontal plane.

The three forces act on the vehicle :

1. The gravity force, mg into the plane of paper.
2. Normal contact force, N out of the plane of paper.
3. Friction force, f directed towards the centre.

Normal contact force balances the gravity force, while friction force provides the necessary centripetal force.

$$N = mg \qquad \text{[For vertical equilibrium]}$$

$$f = \frac{mv^2}{R}$$

[Circular motion dynamics equation]

Here friction is static in nature as there is no relative motion between the vehicle and ground (road) along the radial direction. If relative motion between vehicle and road would be there then it means vehicle is skidding towards the centre of circular path or away from centre of circular path, and friction becomes kinetic in nature and this is not desirable for safe driving.

If μ is the coefficient of friction between the tyres and road, then limiting friction force is, $f_L = \mu N$.

$$\Rightarrow \qquad f_L = \mu mg$$

And, for safe driving,

$$f \le f_L$$

[As friction has to be static in nature for safe driving]

$$\Rightarrow \quad \frac{mv^2}{R} \le \mu mg \quad \Rightarrow \quad v \le \sqrt{\mu Rg}$$

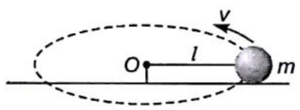
Thus, for a vehicle to take turn safely on a horizontal road of radius R, its speed has to be $\le \sqrt{\mu Rg}$.

If the speed of vehicle is less than $\sqrt{\mu Rg}$, then vehicle will take the turn safely and if speed of vehicle is greater than $\sqrt{\mu Rg}$, then vehicle will skid outwards *ie*, away from the centre of circle.

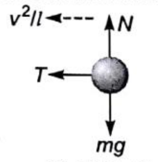
You have to always remember that in circular motion we have to resolve all the forces along radial and tangential directions.

A Ball Connected to a String Moving in a Horizontal Circle

Consider a ball of mass m connected to a string of length l, whose other end is fixed as shown in figure. If the ball is performing circular motion with constant speed v (neglecting friction), then here the necessary centripetal force required for circular motion is provided by the tension in the string. The free body diagram of the ball would be as shown in figure.



(a) Front view of a ball connected to a string moving is a horizontal circular path.



(b) FBD of ball

Fig. 9.13

Here $\qquad N = mg$

[$\because$ ball is in vertical equilibrium]

and $\qquad T = m \times \dfrac{v^2}{l}$

[From circular motion dynamics]

Conical Pendulum

Consider a ball of mass m connected to a string of length l whose other end is fixed to some support as shown in figure. If the ball is whirled in horizontal circle with constant speed v, then string-ball system can be considered to

move along a cone and thus the system is termed as conical pendulum. In this case, the horizontal component of tension in the string is providing necessary centripetal force. The free body diagram of ball is as shown in figure.

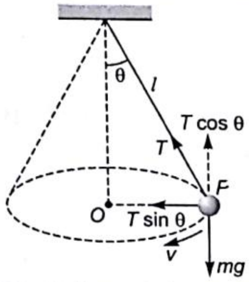


Fig. 9.14 FBD of ball constituting conical pendulum

Here the radius of the circular path is OP,

$$OP = R = l \sin\theta$$

For vertical equilibrium of ball,

$$T\cos\theta = mg$$

From circular motion dynamics equation,

$$T\sin\theta = \frac{mv^2}{R} = \frac{mv^2}{l\sin\theta}$$

On dividing above two equations, we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mv^2}{l\sin\theta \times mg}$$

$$\Rightarrow \qquad v = \sqrt{\frac{lg\sin^2\theta}{\cos\theta}}$$

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Concept Building Illustrations

Illustration | 11 *A particle of mass* 2 *kg is moving along a circle of radius* 1 *m with constant speed of* 3 *ms*$^{-1}$. *Determine the net force acting on particle towards the centre of the circle and also find the net force acting on particle.*

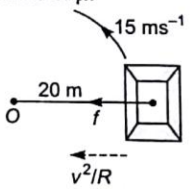
Solution The magnitude of net force acting on the particle towards the centre of the circle is equal to the centripetal force. So, net force towards the centre is,

$$F_r = m \times \frac{v^2}{R} = 2 \times \frac{3^2}{1} = 18\text{ N}$$

As the particle is moving with constant speed, so it means that the tangential component of the force acting on particle is zero and hence the net force acting on particle is equal to the net force towards the centre.

Illustration | 12 *A vehicle of total mass* 200 *kg is taking a turn of radius* 20 *m on a horizontal road with constant speed* 15 *ms*$^{-1}$. *Determine the value of friction force which provides the necessary centripetal force. Assume that there is no skidding. Is the value of* μ *required to solve the question?*

Solution As there is no skidding, it means friction is static in nature and hence there is no need of value of μ.



From circular motion dynamics equation

$$f = m \times \frac{v^2}{R} = 200 \times \frac{(15)^2}{20} = 2250\text{ N}$$

Illustration | 13 *In above illustration, if* $\mu = 0.25$, *then determine the maximum safe speed for turning. [Take* $g = 10\ ms^{-2}$*]*

Solution For safe turning friction force has to be less than the limiting friction force

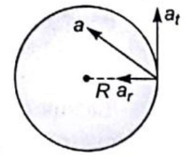
$$f_L = \mu N = \mu mg$$

as $\quad N = mg = 0.25 \times 200 \times 10 = 500\text{ N}$

So, $\quad f \le f_L \Rightarrow \frac{mv^2}{R} \le 500$

$$\Rightarrow \qquad v \le \sqrt{\frac{500 \times 20}{200}} \le \sqrt{50}\ \text{ms}^{-1}$$

Non-Uniform Circular Motion



If the speed of a particle performing circular motion doesn't remain constant, then the motion of particle is said to be non-uniform circular motion. As the speed of particle performing non-uniform circular motion is not constant, in this case, the tangential acceleration of particle is non-zero *ie*, $a_t = R\alpha \neq 0$, and hence $\alpha \neq 0$ *ie*, angular acceleration of particle is non-zero and hence angular velocity of particle, is not constant. In this case both the components of acceleration *ie*, radial and tangential are non-zero. Consider a particle of mass m moving along a circle of radius R such that its speed is not constant, if at any instant v be the speed of particle and α is its angular acceleration (can be constant or varying), then at this instant,

$a_r = \frac{v^2}{R}$ (directed towards the centre), and

$a_t = R\alpha$ (along the tangential direction).

The total acceleration of particle in this case is the resultant of a_r and a_t, which is neither along radial direction nor along tangential direction. The magnitude of acceleration in non-uniform circular motion is given by,

$$a = \sqrt{a_r^2 + a_t^2}$$

[From vector algebra, the resultant of two mutual perpendicular vectors $\vec{A}$ and $\vec{B}$ is given by $\sqrt{A^2 + B^2}$]

Illustration of non-uniform circular motion are numerous just like uniform circular motion. The most common illustration of non-uniform circular motion is the motion of an object in vertical circle under the effect of gravity force and some other force. Other illustrations of non-uniform circular motion are—Motion of a vehicle moving with varying speed along a circular path, motion of conical pendulum in the presence of air friction (here the radius of circular path keeps on decreasing), motion of a block along the inner surface of a hemispherical bowl etc.

Rotational Motion

Motion of door when we close or open it, motion of bicycle wheel running on road, motion of a switched ON fan, motion of the earth about its own axis, motion of flying disk thrown by your friend towards you, motion of phonogram record and many more. All these motions may be familiar to you and you can observe one thing common in all these motions, that is different particles of the object are moving along different and/or same circles whose centres are either at same point or on same straight line. This type of motion is termed as the rotational motion. Let us discuss two illustrations of rotational motion in detail.

Motion of Door

Consider an ordinary door which is attached to the wall (or some other part of building) with the help of two hinges as shown in figure. As the door is fixed to wall, it can't be moved to some other place *ie*, it will remain at its place only. If you apply some force on the door, then the door moves (rotates) from position 1 to position 2, quiet obviously this motion is not a translatory motion but this is also a fact that position of constituent particles of the door changes. If you imagine a little, then you would be able to find out that different particles of the door are moving along different

circles. The motion of three particles of the door is shown in figure, you can also observe that centre of all these circles are lying on a straight vertical line passing through hinges, this line about which circular motion of different particles is taking place is termed as the axis of rotation. In the present case axis of rotation is fixed and the rotational motion of the body about fixed axis of rotation is termed as the pure rotational motion.

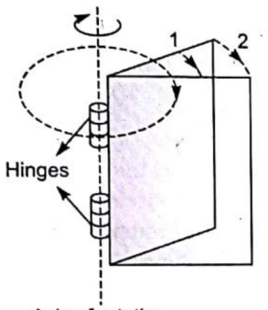


Fig. 9.16 Motion of door is an illustration of rotational motion in which axis of rotation is fixed

Motion of a Cycle Wheel

If you consider the motion of a cycle wheel, very easily you can visualize that the wheel of cycle is moving translationally as well as rotating too.

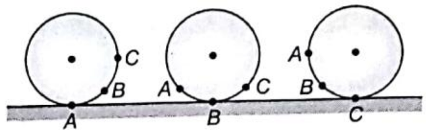


Fig. 9.17

Let us consider a wheel which is rolling (moving translationally as well as rotating) on a horizontal surface as shown in figure. Consider three points A, B and C on the periphery of the wheel, initially point A is in contact with horizontal surface while B and C are in air, after sometime B is in contact with surface while A and C are in air and in the third diagram it is shown that C is in contact with surface while A and B are in air. So we can conclude that in this type of motion the point on the wheel which is in contact with surface is changing as the wheel rolls. In other way, we can interpret it as that body is undergoing combined rotational and translational motion *ie*, axis of rotation is moving and not fixed.

This type of rotational motion in which axis of rotation is not fixed is not a pure rotational motion, as in this case body is undergoing translational motion also, and this type of motion is generally termed as combined rotational and translational motion.

Now let's have a look into different physical quantities used in rotational motion.

Torque

If we ask a question from you—What causes the translational motion of a body ? your answer will come-force (net external) causes the motion of the body. Now you can ask yourself, what causes the rotational motion of the body ? Is it the force or something else, which causes the rotational motion of the body ? The answer to your curiosity is—Torque (net external) acting on a body causes the motion of the body. Now the question arises—What is torque ? Is it someway related to force ?

Torque is defined as "moment of force". In simpler and clear words torque is defined as cross product of the position vector of **point of**

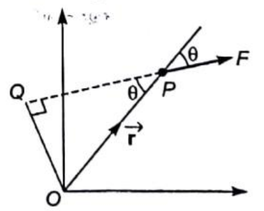


Fig. 9.18 Torgue of a force $\vec{F}$ about O.

application* of force with the force vector. For the situation shown in figure,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where, $\vec{\tau}$ is the symbolic representation of torque.

You can ask that position vector of point of application is different about different points and hence the torque of same force about different points are different, is it correct ? What you are thinking is absolutely correct, as torque is always defined about a point, the torque of same force at the same time about a point can be zero and about any other point can be non-zero. Torque is a vector quantity and its SI unit is N-m. Its dimensional formula is $[ML^2T^{-2}]$, same as that of work or energy but the unit Joule can't be used for torque, as it is specifically reserved for work and energy.

From, $\quad \vec{\tau} = \vec{r} \times \vec{F}$

Magnitude of torque can be written as, $\tau = |\vec{\tau}| = |\vec{r} \times \vec{F}| = rF \sin\theta$, where θ is the angle between $\vec{r}$ and $\vec{F}$ as shown in figure.

$$\tau = rF \sin\theta$$
$$= (r \sin\theta)F$$

From the figure it is clear that $r \sin\theta = OQ$ which is the perpendicular distance from O to the line of action of force, this is termed as the lever arm or moment arm. So torque, $\tau = r_\perp \times F$ where $r_\perp$ is the lever arm.

From $\tau = rF \sin\theta$ it is clear that

(a) If $\theta = 0°$ or π *ie*, $\vec{r}$ and $\vec{F}$ are along same line then $\tau = 0$ *ie*, torque of a force about any point on its line of action is zero.

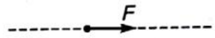


Fig. 9.19 Torque about any point on dotted line due to $\vec{F}$ is zero.

(b) If $\theta = \frac{\pi}{2}$, then torque is maximum about the given point.

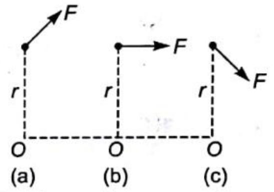


Fig. 9.20 Torque due to same force about a point O when F is acting in three different directions. In case (b) torque is maximum.

(c) For given θ and F, larger is the value of r, larger would be the torque produced.

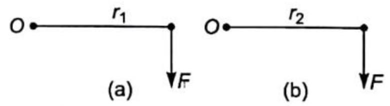


Fig. 9.21 $r_1 > r_2$, torque due to F about O is more in case (b).

* *Point of application of force means the point at which force is assumed to be acting.

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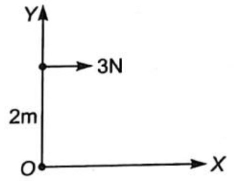
Concept Building Illustrations

Illustration | 14 *Determine the torque of 3 N force about O which is as shown in the figure.*



Solution Here angle between $\vec{r}$ and $\vec{F}$ is $\frac{\pi}{2}$.

So, $\tau = rF \sin \theta$

$= rF \sin 90°$

$= rF = 2 \times 3$

$= 6$ N-s

Moment of Inertia

It is a measure of rotational inertia and plays the same role in rotational mechanics as mass in translational motion. Moment of inertia is denoted by I, its SI unit is kg-m^2, and the dimensional formula is $[ML^2]$. It has a specific axis of rotation and its value depends upon the location of axis of rotation, shape and size of body, distribution of mass etc.

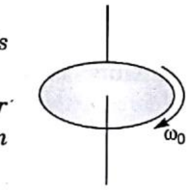
Proficiency in Concepts (PIC)

Problems

Problem 1. *A wheel is initially rotating with angular velocity 20 rad s^{-1} about its own axis in clockwise direction as shown in figure. Now at $t = 0$ a constant angular acceleration of 5 rad s^{-2} is applied to it in anticlockwise direction.*
Determine

(*a*) *the time at which its angular velocity is zero.*

(*b*) *the time at which angular velocity is 20 rad s^{-1} in anticlockwise direction.*



Solution Here initial angular velocity and angular acceleration are in different directions, so angular velocity of the wheel is decreasing.

Here, $\omega_0 = 20$ rad s^{-1},

and $\alpha = -5$ rad s^{-2}.

(a) From $\omega = \omega_0 + \alpha t$

Let angular velocity of wheel be zero at $t = t_0$, then

$$0 = 20 + (-5)t_0 \Rightarrow t_0 = 4 \text{ s}$$

(b) Let angular velocity be 20 rad s^{-1} in anticlockwise direction at $t = t_1$, then substitute $\omega = -20$ rad s^{-1}, $\omega_0 = 20$ rad s^{-1}, $\alpha = -5$ rad s^{-2} and $t = t_1$ in

$$\omega = \omega_0 + \alpha t \Rightarrow -20 = 20 + (-5) \times t_1$$

$$\Rightarrow \quad t_1 = 8 \text{ s}$$

Problem 2. *A particle is moving along a circular path of radius 10 m with a constant speed of 5 ms^{-1}. Determine the time period of circular motion.*

Solution For uniform circular motion,

$$T = \frac{2\pi R}{v}$$

$$= \frac{2\pi \times 10}{5} = 4\pi$$

$$= 12.56 \text{ s}$$

Problem 3. *A cyclist goes around a circular track of radius 70 m in such a way that in 3 min, he completes two turns. Assuming his speed to be constant, determine his speed.*

Solution In 3 min, he completed two turns, so it means to complete 1 turn he takes 1.5 min *ie*, 90 s. So, the time period of circular motion is 90 s.

$$T = \frac{2\pi R}{v} \text{ where } v \text{ is the required speed.}$$

$$90 = \frac{2\pi \times 70}{v}$$

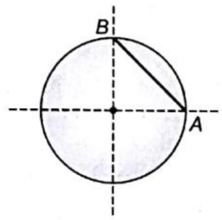
$$\Rightarrow \quad v = \frac{2\pi \times 70}{90} = 4.9 \text{ ms}^{-1}$$

Problem 4. *A particle moving along a circle of radius 5 m with constant speed 2 ms^{-1}. Determine the average velocity of the particle as it moves one-fourth of the circular path.*

Solution The situation is shown clearly in figure, as the particle covers one-fourth of the circle the particle's displacement is,

$$AB = \sqrt{2}\,R = 5\sqrt{2} \text{ m.}$$

For uniform circular motion, the time period is,



$$T = \frac{2\pi R}{v}$$

Time taken by the particle to move from A to B is,

$$t = \frac{T}{4} = \frac{2\pi R}{4v}$$

So, average velocity for the time-interval in which the particle moves from A to B, is

$$v_{av} = \frac{AB}{t} = \frac{5\sqrt{2}}{\frac{2\pi \times 5}{4 \times 2}}$$

$$= \frac{4\sqrt{2}}{\pi} \text{ ms}^{-1}$$

Problem 5. *The constant angular velocity of a particle moving along a circle of radius 3 m is 2 rad s^{-1}. Determine the linear velocity and acceleration of particle.*

Solution The relation between linear velocity and angular velocity is $v = R\omega$

$$\Rightarrow \quad v = 3 \times 2 = 6 \text{ ms}^{-1}$$

The acceleration of particle is towards the centre, and is given by, $a = \omega^2 R$

$$a = 2^2 \times 3 = 12 \text{ ms}^{-2}$$

Problem 6. *A particle is moving along a circle of radius 3 m with constant angular acceleration of 2 rad s^{-2}. Determine the tangential acceleration of particle.*

Solution Tangential acceleration of particle is given by,

$$a_t = R\alpha$$

$$\Rightarrow \quad a_t = 3 \times 2 = 6 \text{ ms}^{-2}$$

Problem 7. *In the above problem if angular velocity of particle at any instant is 6 rad s^{-1}, then determine the acceleration of particle at this instant.*

Solution In this case, the particle is having both components of acceleration *ie*, radial as well as tangential.

$$a_r = \omega^2 R = 6^2 \times 3 = 108 \text{ ms}^{-2}$$

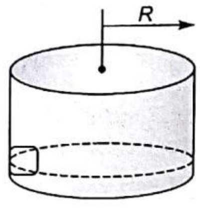
From previous solution, $a_t = 6 \text{ ms}^{-2}$

As we know a_r and a_t are perpendicular to each other, so resultant acceleration of particle is given by

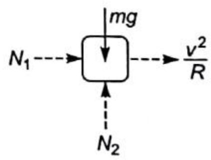
$$a = \sqrt{a_r^2 + a_t^2}$$

$$= \sqrt{(108)^2 + (6)^2} = 108.16 \text{ ms}^{-2}.$$

Problem 8. *A small block of mass 2 kg moves with constant speed in a horizontal circle along the wall of the cylinder as shown in figure. The radius of the cylinder is 1 m. If the speed of particle is 3 ms^{-1}, then determine the normal contact force exerted by the walls of the cylinder on the block.*



Solution Here in this case the normal contact force between the walls of the cylinder and block is providing necessary centripetal force, while the normal contact force by the surface (base) of cylinder on block balances its weight. The free body diagram of the block would be as shown in figure.



For vertical equilibrium,

$$N_2 = mg$$

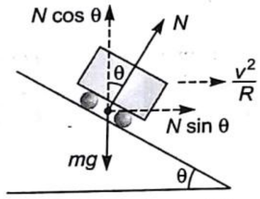
From circular motion dynamics equation,

$$N_1 = \frac{mv^2}{R}$$

$$\Rightarrow \quad N_1 = \frac{2 \times 3^2}{1} = 18 \text{ N}$$

Problem 9. *A circular track of radius 600 m is to be designed for cars at constant speed of 30 ms^{-1}. Determine the banking angle of the track.*
[Take $g = 10 \text{ ms}^{-2}$] Don't consider friction.

Solution Suppose the banking angle is θ, the free body diagram of the vehicle is as shown in figure.



For vertical equilibrium of the vehicle,

$$N\cos\theta = mg$$

From circular motion dynamics equation,

$$N\sin\theta = \frac{mv^2}{R}$$

Dividing above two equations, we get

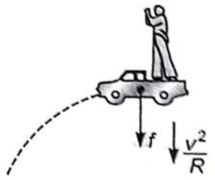
$$\frac{N\cos\theta}{N\sin\theta} = \frac{mg}{mv^2/R} = \frac{Rg}{v^2}$$

$$\tan\theta = \frac{v^2}{Rg} = \frac{(30)^2}{600\times 10} = \frac{3}{20}$$

$$\theta = \tan^{-1}\left(\frac{3}{20}\right)$$

Problem 10. *A motorcyclist weighing* 150 *kg (including the weight of motorcycle) is moving at* 10 ms^{-1} *along a circular turn of radius* 30 *m. What horizontal force on the motorcycle is needed to make the turn possible ?*

Solution Let the horizontal force required to make the turn possible be f, this force will provide the necessary centripetal force. So,



$$f = \frac{mv^2}{R}$$

$$= \frac{150\times(10)^2}{30} = 500\text{ N}$$

Towards Proficiency Problems
Exercise 1

A. Subjective Discussions

1. A car moves at a constant speed, and there are three parts of the motion. It moves along a straight line towards a circular turn, goes around the circular turn and then moves away along a straight line. In which of these three parts, the car is in equilibrium ? Explain your answer with proper reasoning.
2. Is it possible for an object to have an acceleration when the velocity of the object is constant ? When is the speed of the object constant. In each case, give proper reason for your answer.
3. The speedometer of your car shows that you are travelling at a constant speed of 40 ms^{-1}. Is it possible that your car accelerating ?
4. Consider two people on the earth, one on the earth's surface at the equator and the other at the north pole. Which has the largest centripetal acceleration ? Explain. [Assume the earth to be stationary].
5. A small coin is placed on a smooth rotating turntable. Can it perform circular motion *wrt* a stationary observer ?
6. A small coin is placed on a rotating table, it is not moving *wrt* table. From where does the coin get the necessary centripetal force ?
7. In death well (a motorcyclist drives the motorcycle on a vertical wooden well, you may have seen this act in circus), which force provides the necessary centripetal force ?
8. A stone is tied to one end of string and then you start whirling the stone above your head in horizontal plane. Is it possible that the string remains horizontal ?
9. If a particle is moving along a circle with constant angular acceleration, then comment on the linear acceleration of particle.
10. Is it possible for a vehicle to take turns on a smooth horizontal road ?

B. Numerical Answer Types

1. A particle is moving along a circular path whose angular position changes from $\theta_i = \pi/6$ rad to $\theta_f = \pi/2$ rad in 5 s. Determine the angular displacement and the average angular velocity of the particle for this 5 s interval.
2. A particle is moving along a circular path of radius 2 m with constant angular velocity of 3 rad s^{-1}. Determine the angular displacement and the distance travelled by the particle in 3 s.
3. In above question, determine the linear velocity of the particle and acceleration of the particle.
4. A wheel 20 cm in diameter completes 300 revolutions in 3 min. Determine the period of revolution, the angular and linear velocities on the rim of the pulley.
5. A particle initially at rest is moving along a circle of radius 3 m with constant angular acceleration of 2 rad s^{-2}. Determine its linear velocity and the angular velocity at $t = 5$ s. Also determine its radial, tangential and total acceleration at $t = 5$ s.

6. A particle is moving along a circular path of radius 5 m with constant speed 10 ms^{-1}. Determine its linear and angular velocity, time period and acceleration. Also determine the net force acting on the particle. Mass of particle is 2 kg.

7. A wheel starts rotating, making 50 revolutions in first 10 s. Assuming that the rotation is uniformly accelerated, determine the angular acceleration and the final angular velocity.

8. The initial angular velocity of wheel is 10π rad s^{-1}, and after the brakes have been applied its angular velocity decreases to 6π rad s^{-1} in 1 min. Determine the constant angular acceleration of the wheel and the number of revolutions made by it in 1 min.

9. In above question determine the time in which wheel stops and the number of revolutions made by wheel before its coming to stop.

10. A particle starting from rest moves along a circle of radius 20 cm with constant angular acceleration of 2 rad s^{-2}. Determine the time in which the magnitude of tangential acceleration of particle is equal to half of the radial acceleration of particle.

11. A wheel rotates with a constant angular acceleration of 2 rad s^{-2}. The total acceleration of the wheel becomes 13.6 ms^{-2} after 1 s of the beginning of motion. Determine the radius of the wheel.

12. A particle moves along a circle having a radius of 10 cm with constant tangential acceleration of a. Determine the value of a, if velocity of point is 0.792 ms^{-1} at the end of fifth revolutions after the motion has begun.

13. The angular velocity of a wheel is 4800 rev min^{-1}. Determine its angular velocity in rad s^{-1}.

14. A passenger on the outer edge of a merry-go-round 7.5 m from the central axis (centre) experiences that when the merry-go-round is in steady motion, his centripetal acceleration is 3.3 ms^{-2}. Determine the time period of circular motion of merry-go-round.

15. An insect trapped in a circular groove of radius 120 m moves with constant angular velocity along the groove. The insect completes 7 revolutions in 100 s.
(a) What is the angular and linear speeds of the motion ?
(b) Is the acceleration vector a constant vector? Determine its magnitude.

16. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle (on some smooth surface) with constant speed. If the stone makes 14 revolutions in 25 s, determine the magnitude and direction of the acceleration of stone.

17. An aircraft executes a horizontal loop of radius 1 km with a constant speed of 900 kmh^{-1}. Compare its centripetal acceleration with acceleration due to gravity.

18. A cyclist is riding with a speed of 10 ms^{-1}. As he approaches a circular turn on the road of radius 80 m, he applies brakes as a result, its speed decreases at a constant rate of 1 ms^{-2}. Determine the magnitude of the net acceleration of cyclist as he enters on the circular turn.

19. Car A turns on a horizontal circular road of radius 120 m while car B makes a turn on a road of radius 240 m. Both the cars have same the centripetal acceleration. Determine the ratio of velocity of car A to that of car B. Both the cars are moving with constant speed.

20. A skater goes around a turn with a 31 m radius. The skater has a speed of 14 ms^{-1} and experiences a centripetal force of 460 N. What is the mass of the skater ?

21. At what angle should a curve of radius 150 m be banked, so that cars can travel safely at 25 ms^{-1}? Don't consider friction. [Take $g = 10$ ms^{-2}]

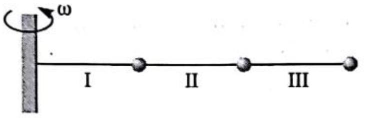
22. A car can go on a horizontal circular path of radius 150 m with maximum safe speed of 25 ms^{-1}. Determine the coefficient of friction between the road and tyres. [Take $g = 10$ ms^{-2}]

23. On an unbanked road (horizontal) a car moves along a turn of radius 200 m, with a constant speed of 15 ms^{-1}. The total mass of the car is 250 kg. Determine the magnitude and direction of net force acting on car.

24. A car has to move on an unbanked turn of radius 45 m. If the coefficient of static friction between the tyre and the road is $\mu_s = 0.5$, then find out the maximum safe speed with which the car can take a turn. [Take $g = 10\ ms^{-2}$]

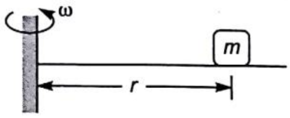
25. A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r. Determine
(a) the speed of particle
(b) the angle made by the string with the vertical
(c) the tension in string
(d) the time period of circular motion

26. Three identical particles are connected with the help of three strings as shown in figure. The free end of string I is fixed to a rigid support. The entire system is placed on a smooth horizontal surface, and is whirled around in a horizontal circle with constant angular speed ω. Determine the ratio of tensions in string I, II and III.



27. A circular road of radius 50 m has the angle of banking equal to 30° . At what speed should a vehicle go on this road so that the friction is not used ?

28. A block of mass m is kept on a rough horizontal plane, and the plane is rotating horizontally with constant angular velocity ω as shown in figure. The coefficient of friction between the block and plane is μ, and the separation between the block and the fixed end is r. Determine



(a) the maximum angular speed ω_0 of plane so that block does not slip.
(b) If $\omega = \frac{\omega_0}{2}$, then find out the force exerted by plane on the block. Analyse the motion.

C. Fill in the Blanks

1. In circular motion the velocity vector of the particle is always along the .... to the path.
2. The acceleration of a particle performing circular motion is ....
3. The acceleration of a particle performing uniform circular motion is .... in magnitude and direction ....
4. In circular motion the speed of particle ....
5. If a particle is performing uniform circular motion with radius R, with angular velocity ω, then magnitude of its acceleration is ....
6. If a particle is performing non-uniform circular motion with constant angular acceleration α, then the linear acceleration of particle as a function of time is .... The particle starts from rest and moves along a circle of radius R.
7. In circular motion the average velocity of a particle over one complete cycle is ....
8. If there is no banking of the road and vehicle is taking a turn, then .... provides the necessary centripetal force.
9. In translatory motion force is necessary to change the state of motion, in same way .... is necessary to change the state of rotational motion.

D. True/False

1. The net acceleration of a particle in uniform circular motion is always along the radius of the circle towards the centre.
2. The net acceleration of particle in circular motion is always along the radius of the circle towards the centre.
3. Kinetic energy of a particle performing circular motion is constant.
4. The net force acting on a particle performing uniform circular motion is constant.
5. The net force acting on a particle performing uniform circular motion is constant in magnitude.
6. It is always necessary that if a particle is performing non-uniform circular motion, then its centripetal acceleration is necessarily equal to its tangential acceleration at a particular instant.
7. Circular motion is always periodic.
8. Uniform circular motion is always periodic.
9. The net force acting on a particle performing uniform circular motion is equal to centripetal force.
10. Uniform circular motion is an example of accelerated motion.

High Skill Questions

Exercise 2

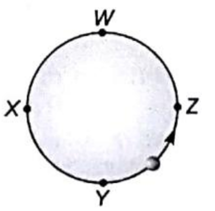
A. Only One Option Correct

1. In uniform circular motion,
   (a) the direction of motion remains the same
   (b) the direction of motion changes continuously
   (c) acceleration is zero
   (d) velocity is constant

2. An object moves in a circle. If the radius of circular path is doubled keeping the speed same, then the centripetal force must be
   (a) twice as great
   (b) half as great
   (c) four times as great
   (d) one-fourth as great

3. An object moves in a circle. If the mass is tripled, the speed halved and the radius remains unchanged, then the centripetal force must change by a factor of
   (a) 3/2 (b) 3/4
   (c) 9/4 (d) 12

4. An object tied to a string, moves in a circle at constant speed on a horizontal surface as shown in figure. The direction of the displacement of this object as it moves from W to X is



   (a) (b) 
   (c)  (d) 

5. A particle moves at a constant speed in a circular path. The instantaneous velocity and instantaneous acceleration vectors are
   (a) both tangent to the circular path
   (b) both perpendicular to the circular path
   (c) perpendicular to each other
   (d) opposite to each other

6. Two objects are travelling around different circular orbits with constant speed. They both have same acceleration but object A is travelling twice as fast as object B. The orbit radius for object A is .... the orbit radius for object B.
   (a) one-fourth (b) one-half
   (c) twice (d) four times

7. A car moves rounds a 20 m radius curve at a constant speed of 10 ms^{-1}. The magnitude of its acceleration is
   (a) Zero (b) 20 ms^{-2}
   (c) 5 ms^{-2} (d) None of these

8. A stone is tied to the end of a string and is whirled with constant speed around a horizontal circle with a radius of 1.5 m. If it makes two complete revolutions each second, its acceleration is
   (a) 0.24 ms^{-2} (b) 2.4 ms^{-2}
   (c) 24 ms^{-2} (d) 240 ms^{-2}

9. An object is travelling along a circular path with increasing speed, then for this situation, mark out the correct statement.
   (a) The radial component of its acceleration is decreasing in magnitude.
   (b) The radial component of its acceleration remains constant in magnitude.
   (c) The tangential component of its acceleration is in the direction of its velocity.
   (d) The tangential component of its acceleration is opposite to the direction of its velocity.

10. An object is moving round a circular orbit with a 12 m radius. At one instant its speed is 6 ms^{-1} and is increasing at 4 ms^{-2}. The magnitude of its acceleration at this instant is

(a) 4 ms^{-2} (b) 3 ms^{-2}
(c) 5 ms^{-2} (d) Zero

11. An object starts from rest and travels around a 6 m radius circular orbit, with its speed increasing at the rate of 8 ms^{-2}. After 0.75 s, the magnitude of its acceleration is

(a) 6 ms^{-2} (b) 8 ms^{-2}
(c) 10 ms^{-2} (d) 12 ms^{-2}

12. Two cars having masses m_1 and m_2 move in a circle of radius r_1 and r_2, respectively. If their time periods are the same, then the ratio of their angular velocity, $\frac{\omega_1}{\omega_2}$ is

(a) $\frac{m_1}{m_2}$ (b) $\frac{r_1}{r_2}$
(c) $\frac{r_2}{r_1}$ (d) 1

13. A particle is kept fixed on a uniformly rotating turntable. As seen from the ground, the particle goes in a circle, its speed is 20 cms^{-1} and acceleration is 20 cms^{-2}. The particle is now shifted to a new position to make the radius half of the original value. The new values of its speed and acceleration will be

(a) 10 cms^{-1}, 20 cms^{-2}
(b) 10 cms^{-1}, 10 cms^{-2}
(c) 20 cms^{-1}, 10 cms^{-2}
(d) 20 cms^{-1}, 20 cms^{-2}

14. A coin placed on a rotating turntable just slips if it is placed at a distance of 4 cm from the centre. If the angular velocity of turntable is doubled, it will just slip at a distance of

(a) 1 cm (b) 2 cm
(c) 4 cm (d) 8 cm

15. A large blade of a helicopter is rotating in a horizontal circle with constant angular velocity. Determine the ratio of acceleration of two particles A and B which are located at a distance of 3 m and 6 m respectively from the centre of the circle.

(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 6

16. A car can safely turn on a horizontal circular road with a maximum speed of $2\sqrt{3}$ ms^{-1}. If the coefficient of friction between the tyres and the road is reduced to one-third of its initial value, then the maximum safe speed for turning is

(a) $2\sqrt{3}$ ms^{-1}
(b) 2 ms^{-1}
(c) 6 ms^{-1}
(d) $\sqrt{3}$ ms^{-1}

17. When a ball is whirling in a circle and the string supporting it suddenly breaks up, then the

(a) ball stops
(b) ball flies away tangentially
(c) ball falls towards the centre
(d) None of the above

B. More Than One Options Correct

1. In uniform circular motion, which of the following remains constant ?

(a) Speed
(b) Magnitude of acceleration
(c) Kinetic energy
(d) Acceleration

2. In uniform circular motion, which of the following changes ?

(a) Velocity
(b) Acceleration
(c) Kinetic energy
(d) Linear momentum

3. In non-uniform circular motion which of the following are not constant ?

(a) Velocity
(b) Acceleration
(c) Kinetic energy
(d) Magnitude of acceleration

4. A particle is moving in a circular path with decreasing speed, then for this situation, mark out the correct statement(s).

(a) The radial component of its acceleration is decreasing in magnitude.
(b) The angular speed of the particle is decreasing.
(c) Tangential component of its acceleration and velocity are in opposite directions.
(d) The particle is performing a non-uniform circular motion.

5. When a particle moves in a circle with constant speed, then mark out the incorrect statement(s).
(a) Its velocity and acceleration are both constant.
(b) Its velocity is constant but acceleration is varying.
(c) Both acceleration and velocity are varying.
(d) Acceleration is constant but velocity is varying.

6. A particle moving in a circular path sweeps out equal areas in equal intervals of time *wrt* the centre of circular path. The particle's
(a) speed is constant
(b) magnitude of acceleration is constant
(c) tangential acceleration is constant
(d) acceleration is always towards the centre

7. Which of the following forces can cause the circular motion in various cases ?
(a) Centripetal force
(b) Tension in string
(c) Spring force
(d) Gravitational force

8. A child is whirling a 0.2 kg ball attached to a string in a horizontal circle of radius 1 m. The time period of circular motion is 1 s. For this situation, mark out the correct statement(s). [Take $\pi^2 = 10$]
(a) The centripetal force acting on the ball is 8 N.
(b) The centripetal force acting on the ball can't be determined from the given information.
(c) If the speed of ball is doubled keeping radius the same, then the centripetal force increases by a factor of four.
(d) If the radius of the ball is changed keeping the speed same, then the time-period of circular motion changes.

9. A body is moving in a circular motion of constant radius, then
(a) the net acceleration of the body may be towards the centre of circle
(b) the net acceleration of the body may not be towards the centre of circle
(c) the velocity of the body is varying
(d) the direction of the acceleration of the body is varying

10. Mark out the incorrect statement(s).
(a) In uniform circular motion, the acceleration is constant.
(b) In non-uniform circular motion, acceleration may be constant.
(c) In non-uniform circular motion, the tangential acceleration is perpendicular to velocity.
(d) In non-uniform circular motion, the radial acceleration is perpendicular to velocity.

11. Mark out the incorrect statement(s) regarding circular motion of a particle.
(a) Particle is in equilibrium
(b) Speed of particle must be constant
(c) Velocity of particle must change
(d) Acceleration of particle may be constant

C. Assertion & Reason

Directions (Q. Nos. 1 to 8) *Some questions (Assertion-Reason type) are given below. Each question contains* ***Statement I (Assertion)*** *and* ***Statement II (Reason)****. Each question has 4 choices (a), (b), (c) and (d) out of which* ***ONLY ONE*** *is correct. So, select the correct choice.*

Choices are

(a) **Statement I** is True, **Statement II** is True; **Statement II** is a correct explanation for **Statement I**

(b) **Statement I** is True, **Statement II** is True; **Statement II** is **NOT** a correct explanation for **Statement I**

(c) **Statement I** is True, **Statement II** is False

(d) **Statement I** is False, **Statement II** is True

1. Statement I The average velocity of a particle performing uniform circular motion over a complete cycle is zero.
Statement II In uniform circular motion, the displacement of the particle in one complete revolution is zero.

2. Statement I The average velocity of a particle performing circular motion over a complete cycle is zero.
Statement II In circular motion, the displacement of particle in one complete revolution is zero.

3. Statement I A car takes a turn of radius 20 m with constant velocity of 10 ms^{-1}.
Statement II In circular motion, velocity can never be constant.

4. Statement I A particle can perform circular motion without having any tangential component of acceleration.
Statement II For circular motion to take place, radial acceleration is a must.

5. Statement I If a car is taking a turn on a banked road, then the normal contact force between car and road is greater than the weight of car. [Neglect friction]
Statement II On a banked road, horizontal component of normal contact force between car and road provides necessary centripetal force, assume friction not there.

6. Statement I For a conical pendulum, if speed of ball is doubled keeping the radius same, then time period of circular motion gets halved.
Statement II For uniform circular motion, time period is given by $\frac{2\pi R}{v}$, where symbols have their usual meanings.

7. Statement I In conical pendulum, the weight of ball is balanced by vertical component of tension in string.
Statement II In conical pendulum, net force acting on the ball in vertical direction is zero.

8. Statement I For a banked road, the correct speed for turning is v_0, if speed of car decreases then car will skid inwards. [Assume no friction]
Statement II For a banked road, the horizontal component of normal contact force between the road and tyre is constant. [Assume no friction]

D. Comprehend the Passage Questions

Passage I

A particle is moving along a circular path of radius 0.7 m with constant tangential acceleration of 5 ms^{-2}. The particle is initially at rest.

Based on above information, answer the following questions :

1. The speed of the particle after 7 s is
(a) 5 ms^{-1} (b) 7 ms^{-1}
(c) 35 ms^{-1} (d) 48 ms^{-1}

2. The radial acceleration of particle at $t = 7$ s is
(a) 1200 ms^{-2} (b) 1750 ms^{-2}
(c) 70 ms^{-2} (d) 250 ms^{-2}

3. The distance travelled by the particle in 7 s is
   (a) 123 m (b) 725 m
   (c) 728 m (d) 426 m
4. The number of revolutions made by the particle in 7 s is
   (a) 27.8 rev (b) 164.8 rev
   (c) 165.52 rev (d) 96.85 rev

Passage II

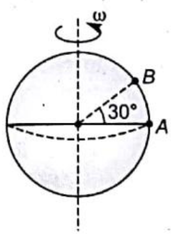
A car starting from rest moves on a circular track of radius 400 m. The car's speed is increasing at a constant rate of 0.5 ms^{-2}. At some point the magnitudes of centripetal and tangential acceleration are same.

Based on above information, answer the following questions :

5. The speed of car is
   (a) 10 ms^{-1} (b) 14.14 ms^{-1}
   (c) 46.74 ms^{-1} (d) None of these
6. The time elapsed is
   (a) 28.28 s (b) 20 s
   (c) 93.48 s (d) None of these
7. The distance travelled is
   (a) 200 m (b) 400 m
   (c) 800 m (d) None of these

Passage III

The earth rotates once per day about an axis passing through the north and south poles, an axis that is perpendicular to the plane containing the equator. Assuming the earth as a sphere of radius 6400 km. The two particles A and B are considered on the surface of earth as shown in figure. The particle A is situated at equator and B is situated at a latitude of 30° north of equator.

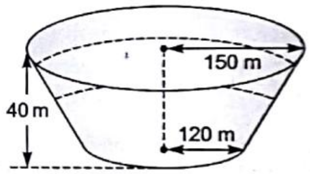


Based on above information, answer the following questions :

8. The speed of particle A is
   (a) 465.4 ms^{-1} (b) 850 ms^{-1}
   (c) 243.6 ms^{-1} (d) 1.675×10^6 ms^{-1}
9. The acceleration of particle A is
   (a) 3.38×10^{-5} ms^{-2}
   (b) 3.38×10^{-2} ms^{-2}
   (c) 6.76×10^{-5} ms^{-2}
   (d) 6.76×10^{-2} ms^{-2}
10. The speed of particle B is
   (a) 403 ms^{-1} (b) 736.12 ms^{-1}
   (c) 210.96 ms^{-1} (d) 1.45×10^6 ms^{-1}
11. The acceleration of particle B is
   (a) 2.927×10^{-5} ms^{-2}
   (b) 2.93×10^{-2} ms^{-2}
   (c) 5.86×10^{-5} ms^{-2}
   (d) 5.86×10^{-2} ms^{-2}

Passage IV

A banked race track is as shown in figure. The radius of the smallest circular path on which racing car can move is 120 m and the largest circular path radius is 150 m. The height of outer wall is 40 m. Neglect friction. The mass of vehicle along with the rider is 2000 kg.
[Take $g = 10$ ms^{-2}]



Based on above information, answer the following questions :

12. The smallest safe speed of car is
   (a) 10 ms^{-1} (b) 30 ms^{-1}
   (c) 33.5 ms^{-1} (d) 40 ms^{-1}
13. The largest safe speed of car is
   (a) 10 ms^{-1} (b) 30 ms^{-1}
   (c) 33.5 ms^{-1} (d) 40 ms^{-1}
14. If the car is moving along a circular path of radius 140 m, then the normal contact force between the track and car is
   (a) 16000 N (b) 25000 N
   (c) 20000 N (d) 22000 N
15. For Q. 14, the safe speed is
   (a) 32.4 ms^{-1} (b) 30 ms^{-1}
   (c) 33.5 ms^{-1} (d) 35 ms^{-1}
16. As radius of circular path increases from 120 m to 150 m, the normal contact force between the car and track
   (a) increases
   (b) decreases
   (c) remains constant
   (d) None of the above

E. Match the Columns

1. Match the entries of Column I with the entries of Column II. Consider all possibilities.

	Column I		Column II
(A)	Circular motion	(P)	Velocity varying
(B)	Uniform circular motion	(Q)	Acceleration varying
(C)	Non-uniform circular motion	(R)	Speed constant
(D)	Uniformly accelerated translatory motion	(S)	Magnitude of acceleration constant

2. In Column I some physical quantities related to translatory motion are given, while in Column II physical quantities associated with rotational motion are mentioned. Match the entries of Column I with the entries of Column II on the basis of analogy.

	Column I		Column II
(A)	Mass	(P)	Angular acceleration
(B)	Velocity	(Q)	Angular velocity
(C)	$v = u + at$	(R)	Moment of inertia
(D)	Linear acceleration	(S)	$\omega = \omega_0 + \alpha t$

3. In Column I some physical quantities are mentioned, while in Column II some information about them. Match the entries of Column I with the entries of Column II for uniform circular motion.

	Column I		Column II
(A)	Velocity	(P)	Magnitude constant
(B)	Acceleration	(Q)	Direction constant
(C)	Momentum	(R)	Magnitude changing
(D)	Kinetic energy	(S)	Direction changing

Answers

Towards Proficiency Problems
Exercise 1

B. Numerical Answer Types

1. $\frac{\pi}{3}$ rad, $\frac{\pi}{15}$ rad s^{-1} **2.** 9 rad, 18 m **3.** 6 ms^{-1}, 18 ms^{-2} **4.** $\frac{3}{5}$ s, $\frac{10\pi}{3}$ rad s^{-1}, $\frac{\pi}{3}$ ms^{-1}

5. 10 rad s^{-1}, 30 ms^{-1}, a_r = 300 ms^{-2}, a_t = 6 ms^{-2}, a = 300.06 ms^{-2} **6.** 2 rad s^{-1}, π s, 40 N

7. $\alpha = 2\pi$ rad s^{-2}, $\omega = 20\pi$ rad s^{-1} **8.** $\frac{\pi}{15}$ rad s^{-2}, 240 rev **9.** 150 and 375 rev^n

10. 1 s **11.** 3.04 m **12.** 0.1 ms^{-2} **13.** 160π rad s^{-1} **14.** 9.47 s

15. (a) 0.44 rad s^{-1}, 52.80 ms^{-1}; (b) No, 23.232 ms^{-2}

16. 9.91 ms^{-2} towards the centre of circle **17.** $\frac{a_r}{g} = 6.25$ **18.** 1.6 ms^{-2}

19. $\frac{1}{\sqrt{2}}$ **20.** 72.755 kg **21.** $\tan^{-1}\left(\frac{5}{12}\right)$ **22.** $\frac{5}{12}$

23. 281.25 N towards the centre of circle **24.** 15 ms^{-1}

25. (a) $r\sqrt{\frac{g}{\sqrt{L^2 - r^2}}}$ (b) $\sin^{-1}\left(\frac{r}{L}\right)$ (c) $\frac{mgL}{\sqrt{L^2 - r^2}}$ (d) $2\pi\sqrt{\frac{\sqrt{L^2 - r^2}}{g}}$

26. 6 : 5 : 3 **27.** 17 ms^{-1} **28.** (a) $\sqrt{\frac{\mu g}{r}}$ (b) $mg\sqrt{1 + \left(\frac{\mu}{4}\right)^2}$

C. Fill in the Blanks

1. Tangent **2.** Changing **3.** Constant, changing **4.** May change **5.** $\omega^2 R$

6. $[\sqrt{(\alpha^2 t^2 R)^2 + (\alpha R)^2}$ **7.** Zero **8.** Friction **9.** Torque

D. True/False

1. T **2.** F **3.** F **4.** F **5.** T **6.** F **7.** F **8.** T
9. T **10.** T

High Skill Questions
Exercise 2

A. Only One Option Correct

1. (b) **2.** (b) **3.** (b) **4.** (c) **5.** (c) **6.** (d) **7.** (c) **8.** (d) **9.** (c) **10.** (c)
11. (c) **12.** (d) **13.** (b) **14.** (a) **15.** (b) **16.** (b) **17.** (b)

B. More Than One Options Correct

1. (a, b, c) **2.** (a, b, d) **3.** (a, b, c, d) **4.** (a, b, c, d) **5.** (a, b, d)
6. (a, b, c, d) **7.** (b, c, d) **8.** (a, c, d) **9.** (a, b, c, d) **10.** (a, b, c, d)
11. (a, b, d)

C. Assertion & Reason

1. (a) 2. (a) 3. (d) 4. (b) 5. (b) 6. (a) 7. (a) 8. (b)

D. Comprehend the Passage Questions

1. (c) 2. (b) 3. (a) 4. (a) 5. (b) 6. (a) 7. (a) 8. (a) 9. (b) 10. (a)
11. (b) 12. (b) 13. (c) 14. (b) 15. (a) 16. (c)

E. Match the Columns

1. (A) → P, Q, R, S; (B) → P, Q, R, S; (C) → P, Q; (D) → P, S
2. (A) → R; (B) → Q; (C) → S; (D) → P
3. (A) → P, S; (B) → P, S; (C) → P, S; (D) → P

Explanations

Towards Proficiency Problems
Exercise 1

Numerical Answer Types

1. $\Delta\theta = \theta_f - \theta_i = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$ rad

$\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\pi/3}{5} = \frac{\pi}{15}$ rad s^{-1}

2. $\theta = \omega t = 3 \times 3$ rad $= 9$ rad

$s = r\theta = 9 \times 2$ m $= 18$ m

3. $v = r\omega = 6$ ms^{-1}

$a_r = \omega^3 r = 18$ ms^{-2}

4. Assume that wheel is rotating with constant angular velocity.

Now, $\theta = 300 \times 2\pi = \omega \times (3 \times 60)$

$\Rightarrow \omega = \frac{10\pi}{3}$ rad s^{-1}

$T = \frac{2\pi}{\omega} = \frac{3}{5}$ s

$v = R\omega = 0.1 \times \frac{10\pi}{3} = \frac{\pi}{3}$ ms^{-1}

5. $\omega = \alpha t = 2 \times 5 = 10$ rad s^{-1}

$v = R\omega = 3 \times 10$ ms^{-1} $= 30$ ms^{-1}

$a_r = \omega^2 R = 300$ ms^{-2}

$a_t = R\alpha = 6$ ms^{-2}

$a = \sqrt{a_r^2 + a_t^2} = 300.06$ ms^{-2}

6. $\omega = \frac{v}{R} = \frac{10}{5} = 2$ rad s^{-1}

$T = \frac{2\pi}{\omega} = \pi$ sec

$F = \frac{mv^2}{R} = \frac{2 \times 100}{5} = 40$ N

7. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$50 \times 2\pi = 0 + \frac{1}{2}\alpha \times 10^2$

$\Rightarrow \alpha = 2\pi$ rad s^{-2}

$\omega = \alpha t = 2\pi \times 10$

$= 20\pi$ rad s^{-1}

8. $\omega = \omega_0 - \alpha t$

$\Rightarrow 6\pi = 10\pi - \alpha \times 60$

$\Rightarrow \alpha = \frac{\pi}{15}$ rad s^{-2}

$n = \frac{\theta}{2\pi} = \frac{[\omega_0 t - 1/2\, \alpha t^2]}{2\pi} = 240$

9. $0 = 10\pi - \alpha t$

$$\Rightarrow \quad t = \frac{10\pi}{\pi/15} = 150\text{ s}$$

$$n_1 = \frac{\omega_0^2/2\alpha}{2\pi} = 375$$

10. $a_t = \frac{a_r}{2}$

$$\Rightarrow \quad r\alpha = \frac{\omega^2 r}{2}$$

As $\quad \omega = \alpha t$

So, $\quad \alpha = \frac{\omega^2}{2} = \frac{\alpha^2 t^2}{2}$

$$\Rightarrow \quad t^2 = \frac{2}{\alpha} \Rightarrow t = 1\text{ s}$$

11. $a = 13.6 = \sqrt{a_r^2 + a_t^2}$

$a_t = r\alpha = 2r$

$a_r = \omega^2 r = (\alpha t)^2 r = 4r$

Solving above equations, we get

$$r = 3.04\text{ m}$$

12. $v = 0.792\text{ ms}^{-1} = r\omega$

where ω is angular velocity at the end of 5^{th} rev^n.

$\omega^2 = 0 + 2\alpha \times 5 \times 2\pi$, and $a = r\alpha$

Solving above equations, we get

$$a = 0.1\text{ ms}^{-2}$$

13. $\omega = 4800$ rpm

$$= \frac{4800 \times 2\pi}{60}\text{ rad s}^{-1} = 160\pi\text{ rad s}^{-1}$$

14. $\omega^2 R = 3.3$

$$\Rightarrow \quad \omega = 0.6633\text{ rad s}^{-1}$$

$$T = \frac{2\pi}{\omega} = 9.47\text{ s}$$

15. $7 \times 2\pi = \omega \times 100$

$\Rightarrow \quad \omega = 0.44\text{ rads}^{-1}$

$v = 120\omega = 52.80\text{ ms}^{-1}$

$a = \omega^2 R = 23.232\text{ ms}^{-2}$

16. $14 \times 2\pi = 25\,\omega$

$\Rightarrow \quad \omega = \frac{88}{25}\text{ rad s}^{-1}$

$$a = \omega^2 r = \left(\frac{88}{25}\right)^2 \times 0.8 = 9.91\text{ ms}^{-2}$$

17. $a_r = \frac{v^2}{R}$

$$= \frac{\left(900 \times \frac{5}{18}\right)^2}{1000} = 62.5\text{ ms}^{-2}$$

$$\frac{a_r}{g} = 6.25$$

18. As cyclist enters the circular turn, he experiences both the centripetal and tangential acceleration, given by

$a_r = \frac{v^2}{R} = \frac{100}{80} = 1.25\text{ ms}^{-2}$

$a_t = 1\text{ ms}^{-2}$

$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1.25)^2 + (1)^2} = 1.6\text{ ms}^{-2}$

19. $a_A = \frac{v_A^2}{120}$

$a_B = \frac{v_B^2}{240}$

$$a_A = a_B \Rightarrow \frac{v_B}{v_A} = \sqrt{2}$$

$$\Rightarrow \quad \frac{v_A}{v_B} = \frac{1}{\sqrt{2}}$$

20. $F_C = \frac{mv^2}{R}$

$$\Rightarrow \quad 460 = \frac{m \times 14^2}{31}$$

$$\Rightarrow \quad m = 72.75\text{ kg}$$

21. The safe speed on a banked road of banking angle θ is given by

$$v = \sqrt{Rg\tan\theta}$$

$$\Rightarrow \quad 25^2 = 150 \times 10\tan\theta$$

$$\Rightarrow \quad \tan\theta = \frac{5}{12}$$

23. The friction force provides the necessary centripetal force and normal contact force balances the gravity force.

$$f = \frac{mv^2}{R} = \frac{250 \times 15^2}{200} = 281.25\text{ N}$$

24. $v_{\max} = \sqrt{\mu_s Rg}$

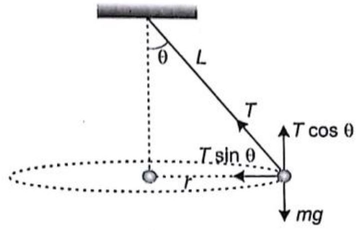
$= \sqrt{0.5 \times 45 \times 10} = 15\text{ ms}^{-1}$

25. FBD of ball is as shown in figure.

$T\cos\theta = mg$ [For vertical equilibrium]

$T\sin\theta = \frac{mv^2}{r}$

[Circular motion dynamics equation]



From diagram

$$\cos\theta = \frac{\sqrt{L^2 - r^2}}{L}$$

$$\sin\theta = \frac{r}{L}$$

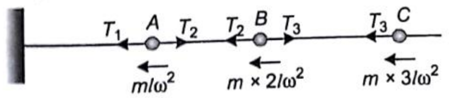
(a) $v = \sqrt{rg\tan\theta} = r\sqrt{\dfrac{g}{\sqrt{L^2 - r^2}}}$

(b) $\theta = \sin^{-1}\left(\dfrac{r}{L}\right)$

(c) $T = \dfrac{mg}{\cos\theta} = \dfrac{mgL}{\sqrt{L^2 - r^2}}$

(d) $T = \dfrac{2\pi r}{v}$

26. Let tension in three strings be T_1, T_2 and T_3, respectively.



For A, $T_1 - T_2 = ml\omega^2$
For B, $T_2 - T_3 = m \times 2l\omega^2$
For C, $T_3 = m \times 3l\omega^2$

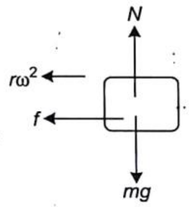
$$T_1 : T_2 : T_3 = 6 : 5 : 3$$

27. $v = \sqrt{Rg\tan\theta} = 17\ \text{ms}^{-1}$

28. The friction force between the block and plane provides necessary centripetal force.

$$N = mg$$

$$f = mr\omega^2$$



(a) For no slipping,

$$f \le f_L$$

$$mr\omega^2 \le \mu\, mg$$

$$\Rightarrow \quad \omega \le \sqrt{\frac{\mu g}{r}}$$

$$\omega_{max} = \sqrt{\frac{\mu g}{r}}$$

(b) $\omega = \dfrac{\omega_{max}}{2} = \dfrac{1}{2}\sqrt{\dfrac{\mu g}{r}}$

$$f = mr \times \frac{1}{4} \times \frac{\mu g}{r} = \frac{\mu\, mg}{4}$$

Required force,

$$R = \sqrt{f^2 + N^2}$$

$$= \left(\sqrt{\frac{\mu^2}{16} + 1}\right) mg$$